A game-theoretic perspective on how pedestrian behaviour changes with crowd density

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Abstract Social and physical interactions between pedestrians significantly influence their behaviour and in scenarios such as evacuation can affect the overall departure efficiency. This study aims to explore such interactions in bottlenecks from a game-theoretic perspective, using a public goods game framework. Simulation and experiments can be developed to validate our model, and the results will reveal different macroscopic patterns of pedestrian movement.

Keywords Bottleneck, Game theory, Public goods game, Nash equilibrium, Movement patterns

Introduction

Game theory is a useful tool to explore behavioural interactions among dynamic and finitely rational agents because this theory can produce self organising behaviours by defining a utility function that can easily be adapted. Some examples include the use of mean field [1, 2] and evolutionary games [3].

We consider the bottleneck evacuation a scenario in which crowds share limited public resources, including exit and available space. Each pedestrian can benefit from these resources that affect the efficiency of evacuation, which we refer to as public goods benefit. At the same time, they can also contribute to these resources because of an aggressive movement strategy (e.g., every move is giving up a certain amount of space for others to occupy). The limited nature of the resources could lead to selfish behaviour and competition, or such constraints could be resolved through cooperation. Here, we thus develop a multi-stage public goods game for pedestrians to explore this tension between individuals maximising their own benefits and regarding others' needs. We can adjust a pedestrian's consideration of their own interests and the overall evacuation efficiency through changing the model parameters and make suggestions based on the corresponding Nash equilibrium results. As the strategies pedestrians choose in our game model relate to movement directions and speeds, we can explore the change of congestion and other crowd behaviours.

Methods

The public goods game model for pedestrians supposes that pedestrian behaviours are multi-stage games consisting of a series of games, denoted by t interactions. Pedestrians are all successfully evacuated through T stages. Importantly, our model should not be interpreted as a physical model, as pedestrians can overlap. Rather, the model should be viewed as describing movement intentions of individuals depending on their surroundings. Our model is outlined in the following.

Agents: the agents of the game are each pedestrian denoted in a set $N = \{1,\ 2,\ 3,\ \cdots,\ n\}$.

Strategies: in interaction t, each pedestrian $i \in N$ decides a movement strategy $\boldsymbol{p}_{i,t}$ (i.e., the target position), which involves a footstep $v_{i,t}^{max}$ (step length) and a movement direction related to θ:

$$v_{i,t}^{max} = v_{max} \cdot \left(1 - \frac{\rho_{i,t}}{\rho_{max}}\right), \tag{1}$$

$$\boldsymbol{p}_{i,t} = (x_{i,t}, y_{i,t}),\ \forall\ i \in N,\ \ t \in T,\ \ \text{and} \parallel \boldsymbol{p}_{i,t} - \boldsymbol{p}_{i,t-1} \parallel \leq v_{i,t}^{max}, \tag{2}$$

where $x_{i,t}, y_{i,t}$ are the spatial position of pedestrians, $v_{i,t}^{max}$ is the maximum footstep of the pedestrian i in t according to the density, v_{max} is the maximum footstep for pedestrians, $\rho_{i,t}$ is the true number of people around the pedestrian i and ρ_{max} is the maximum number of people that can be around pedestrians (the value of this can be set based on physical or psychological considerations). The angle θ is defined as the angle between the direction of pedestrian movement and the direction facing the exit.

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Utility: the possible strategies of pedestrians are associated with a public goods benefit $f(G_t)$, expressed by the following equations:

$$f(G_t) = r \cdot \frac{G_t}{n_t}, \quad G_t = \sum_{i=1}^{n_{t-1}} d^e_{i,t-1} = \sum_{i=1}^{n_{t-1}} \frac{\| \boldsymbol{p}_{i,t-1} - \boldsymbol{p}_{i,t-2} \| \cdot |\cos\theta|}{v_{max}}, \tag{3}$$

where r is a magnification factor (a free parameter), and G_t is the contribution obtained at t from the pedestrian strategy made at $t-1$, and it is related to the effective distance pedestrians have travelled on the shortest path to the exit $d^e_{i,t-1}$.

The utility of pedestrian i in stage t is:

$$u_{i,t}(\boldsymbol{p}_{i,t}, \boldsymbol{P}_{-i,t}) = W_{i,0} + f(G_t) - C_{i,t}(\boldsymbol{p}_{i,t}), \tag{4}$$

where $\boldsymbol{P}_{-i,t} = \{\boldsymbol{p}_{j,t} : \forall j \in N, \text{ and} j \neq i\}$, and $\boldsymbol{P}_{-i,t}$ refers to the strategy for all pedestrians except i. $W_{i,0}$ is the initial benefit value of pedestrians obtained from their initial position (pedestrians closer to the exit will have higher initial benefits). A cost function $C_{i,t}(\boldsymbol{p}_{i,t})$ for pedestrian movement is defined based on the position of pedestrians in space and the density of their surroundings. In general, it contains the cost associated with deviating from the shortest path to the exit and the cost of fatigue of moving to areas of high pedestrian density.

Objectives: each pedestrian i will adopt a Nash equilibrium strategy $\boldsymbol{p}^*_{i,t}$ which is obtained by maximising $u_{i,t}(\boldsymbol{p}^*_{i,t}, \boldsymbol{P}^*_{-i,t})$. At this point, the strategies of the rest pedestrians $\boldsymbol{P}^*_{-i,t}$ are all Nash equilibrium strategies as well.

Results

We now present some initial results. In general, as the acceptable crowd density increases one would expect that the evacuation may also become more efficient in terms of time step for all pedestrians to exit. However, in our model, the effect of the pedestrian willingness to accept the maximum number of people around them (ρ_{max}) is not always linear, as shown in Figure 1. A larger ρ_{max} leads to a higher slope evacuation efficiency, but can also result in a longer congested movement pattern, making evacuation times longer. This may be attributed to the expected increased acceptance of high density crowds by pedestrians, and the increased behaviour of following the flow of the crowd resulting in pedestrians not avoiding the congestion. It reveals the possibility that the evacuation efficiency of a high-density crowd is better than that of a low-density crowd, and it is necessary to explore the mechanisms involved further to understand why this may happen.

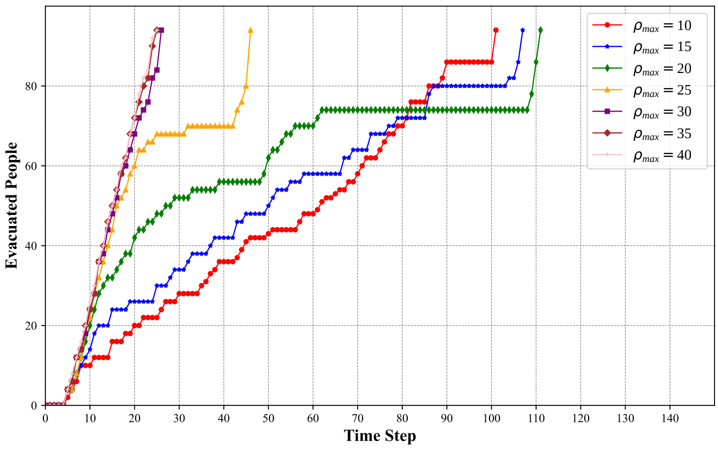

Figure 1: The impact of ρ_{max} on on the number of individuals evacuated in Nash equilibrium. The value of the maximum footstep for pedestrians v_{max}, magnification factor r, the weight κ, the weight α are 0.2, 2, 10, 1, respectively.

Bibliography

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