# Glimpsing into the Fluid Mechanics of Crowds Featuring Route Choice and Collision Anticipation

Maël LE GARFF[1,2], Jakob CORDES[3], and Alexandre NICOLAS*[1]

[1]Universite Claude Bernard Lyon 1, CNRS, Institut Lumière Matière, 69100 Villeurbanne, France.
[2]École polytechnique, 91128 Palaiseau, France.
[3]Forschungszentrum Jülich, Jülich, Germany.

**Abstract** Collision anticipation plays a key role in crowd dynamics, but was so far absent from the hydrodynamic equations typically used in crowd dynamics. We study how simple microscopic collision avoidance processes translate into the macroscale and unveil new hydrodynamic terms that bear the signature of collision anticipation. The results are validated by comparison to the corresponding agent-based simulations.

**Keywords** crowd hydrodynamics, collision anticipation, crowd theory, decision-making

Over the past few years, captivating crowd phenomena such as the backward propagation of soliton-like density waves through marathon corrals [1] or the oscillating motion of crowd 'patches' in very dense settings at festivals [2] have been successfully reproduced using hydrodynamic equations, in the wake of Toner and Tu's symmetry-based approach for active matter [3]. However, Toner and Tu explicitly stated that their approach holds for particles without 'compass', i.e., with no intrinsic directional preferences. Clearly, this assumption should not be taken for granted for pedestrians in general and we suspect that new relevant terms will arise in the fluid dynamical equations of agents with 'compasses'. Having in mind the goal to probe parts of the foundations of the fluid mechanics of pedestrian crowds, in the wake of [4], we study how *simple* microscopic descriptions accounting for collision anticipation translate into the macroscopic scale and we unveil the new contributing terms, with specific directions in space, that ensue in the hydrodynamic equations and reflect the process of collision anticipation.

## Dimensionless numbers to classify crowd scenarios

First, in the spirit of Fluid Mechanics, we recently introduced two dimensionless numbers [5] that quantify (i) the strength of intrusions into personal spaces and (ii) the avoidance of imminent collisions in any given scenario; the latter is associated with collision anticipation. (Note that the aforementioned hydrodynamic studies took place in settings where settings where distance-based contributions, rendered by the first dimensionless number, are expected to dominate). These two dimensionless number are at the basis of a classification of a vast array of empirical crowd scenarios [6], represented in the left panel of the figure.

## Collision anticipation at the microscopic scale

Then, we showed how the two dimensionless variables can help infer fairly generic *microscopic* equations of motion for pedestrians. In short, the velocity $\boldsymbol{v}_j(t)$ of a pedestrian $j$ can always be expressed as the minimum of a suitably defined, ad-hoc function $\mathcal{C}_j^{(t)}(\boldsymbol{v})$, viz.,

$$\boldsymbol{v}_j(t) = \mathrm{argmin}_{\boldsymbol{v}}\, \mathcal{C}_j^{(t)}(\boldsymbol{v}). \tag{1}$$

Trading accuracy for simplicity, the foregoing reasoning based on dimensionless numbers helps propound a generic form for $\mathcal{C}_j^{(t)}(\boldsymbol{v})$. Following a perturbative approach *à la* Landau, fairly generic equations of motion are then obtained for the individual pedestrian [5]. In these equations, a variant of the anticipated time-to-collision accounts for collision anticipation.

*Email of the corresponding author: `alexandre.nicolas@cnrs.fr`

## Bridging the gap with the macroscopic scale

Finally, we upscale these microscopic agents using Boltzmann's equation of motion, with a derivation that is not rigorous mathematically speaking, but does bridge the gap between the microscopic equation and the macroscopic one without having to posit artificial symmetries. This is achieved, notably, by coarse-graining the different terms into mesoscopic forces, by averaging over the distribution of spacings and velocities of interacting agents, and then into an effective macroscopic force. While the continuity equation (describing the conservation of particle number) does not change, we find that the modified equation describing the mean local velocity $\boldsymbol{v}_\alpha$ of agent type $\alpha$ takes the following schematic form,

$$\frac{\partial \boldsymbol{v}_\alpha}{\partial t} + (\boldsymbol{v}_\alpha \cdot \nabla)\boldsymbol{v}_\alpha = (\dots) + \sum_{\beta \neq \alpha} \rho_\beta \boldsymbol{D}^{(TTC)}_{\alpha,\beta} - \underline{\underline{T^{(TTC)}_{\alpha,\beta}}} \cdot \boldsymbol{\nabla} \rho_\beta, \tag{2}$$

where only the new terms signalling TTC-based collision anticipation are detailed on the right-hand side.

The resulting macroscopic description is tested and validated by comparing its predictions to the output of microscopic agent-based models featuring the same 'ingredients', in the case of intersecting perpendicular flows. This is illustrated in panel B of the figure, where we show the density field of the horizontal pedestrian stream (walking to the right) in the resolution of the hydrodynamic equations and in the agent-based simulations: we notice the same marked deviation of the stream towards the bottom, at the intersection with the perpendicular stream; this marked deviation vanishes if the new hydrodynamic terms exposed above are removed. Accordingly, at least qualitatively, the hydrodynamic contributions that were so far overlooked succeed in capturing the effect of collision anticipation.

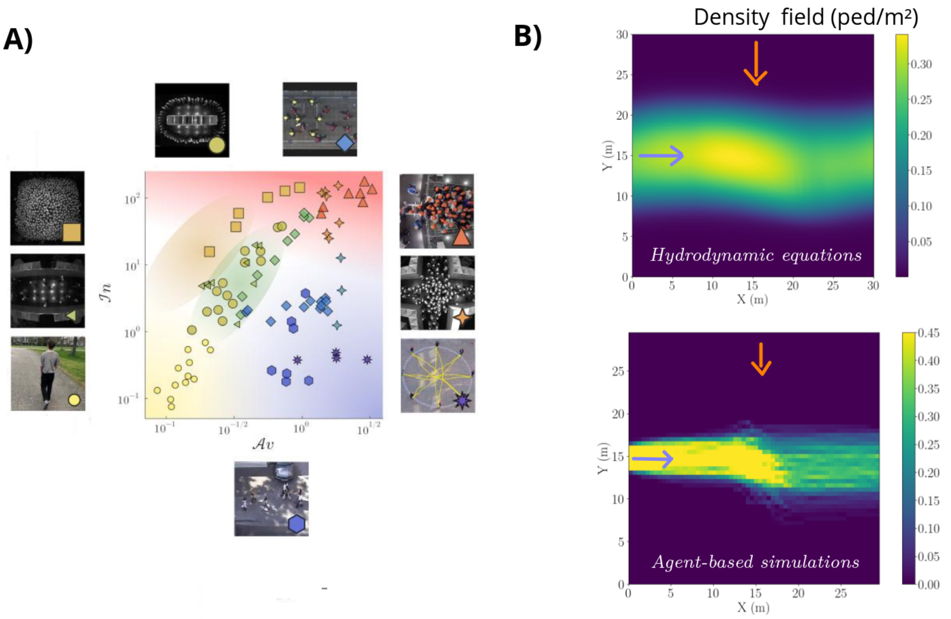


Figure 1: A) Classification of a wide range of empirical crowd scenarios (illustrated by the snapshots on the side) using our two dimensionless numbers, $In$ and $Av$. Adapted from [5]. B) Simulated density field of the horizontal pedestrian stream in a perpendicular intersection scenario, by solving hydrodynamic equations (top) or microscopic agent-based simulations (bottom).

# References


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