"Valency" model of pedestrian group behaviour

Francesco Zanlungo*[1] and Zeynep Yücel[2]

[1]University of Palermo, Palermo, Italy
[2]Ca' Foscari University of Venice, Venice, Italy

Abstract We propose a model for social groups of any size in pedestrian crowds. This model allows groups with four of more members to walk in large U formations, while splitting in more stable subgroups (dyads and triads) when needed. The model is not based on a set of rules, but on a single cost function.

Keywords Social groups, mathematical model

Social groups are an important component of pedestrian crowds, and are well known to present characteristic behaviours, such as lower velocities and specific walking formations. For these reasons, they have been the subject of many research works, and are becoming an important component of crowd models and simulators. Most of the research on walking social groups has focused on groups of two (dyads) and three (triads) pedestrians, since these are the most commonly occurring ones, are obviously simpler than larger groups, and they represent the fundamental and dynamically stable components of larger groups [1], which arguably makes them also easier to observe.

A mathematical model of group behaviour, applicable in principle to groups of arbitrary size, is introduced by [2]. The fundamental idea is that pedestrians walking and socially interacting need to keep both their local goal (walking direction) and group partners in their field of view. This necessity may be modelled by introducing a function $U(\vec{\mathbf{r}}) = U(r, \theta)$ that each pedestrian tries to optimise, r being the distance to the partner and θ the angle formed between the relative position of the partner $\vec{\mathbf{r}}$ and the local goal direction (preferred velocity). Although in [2] quantitative computations are performed for a specific form of U and in the framework of a second order differential model, this approach can be used in any model in which pedestrians move to minimise a cost function. Furthermore, any function assuming two minima in $r = r_0, \theta = \pm\theta_0 \approx \pm\pi/2$, r_0 being the distance at which pedestrians prefer to walk, will reproduce the abreast walking of dyads, whose dynamics will be basically determined by the ratio of $\partial_r^2 U$ and $\partial_\theta^2 U$ in the minima. It is also shown that using $\theta_0 > \pi/2$ causes dyads to walk slower than individuals, while preserving abreast walking.

To apply the model to larger groups while preserving realistic behaviour, [2] assumes that pedestrians interact only with their first neighbours. In the case of triads, using the original proposed form leads to reproducing their behaviour (specifically the V formation and velocity) in a quantitative way without the need of a further calibration. When this approach is applied to groups with four or more members, it produces a U formation in agreement with the findings of [3]. Nevertheless, such U formations were not observed in the data set of [1, 2], which on the other hand presented four people groups splitting in two dyads, each of them presenting its typical abreast formation. The discrepancy with the results of [3] may be due to the different environments and crowd compositions of these ecological data sets, but also to the fact that in [1, 2] pedestrians were observed for a longer time, and thus had a higher probability to re-arrange in subgroups. This also suggests that larger groups may be also difficult to observe as that, since they may appear to be multiple dyads and triads.

On the other hand, dyads and triads cannot split as long as the members intend to continue interacting, since if they split, at least one of them would be alone. Basically, each pedestrian in a large group needs to have at least a partner, and must be able to accept another partner. We call this a "valency" model of large groups, since each pedestrian has the possibility to strongly interact with two other people from the large group, which will be positioned in their two U minima. Nevertheless, as soon as one minimum is filled, the pedestrian becomes "neutral", and does not seek further interaction.

Mathematically, the valence model can be defined by modifying the function U (regardless of its specific definition) in the following way. First of all, we define, for each pedestrian i in the group the minimum value

$$U_0^i = \min_j U(\vec{\mathbf{r}}_{ij})$$

*Email of the corresponding author: `francesco.zanlungo@unipa.it`

where $\vec{\mathbf{r}}_{ij}$ is the relative position with respect to group partner j. We then define $\vec{\mathbf{r}}'$ as any vector satisfying the equipotential condition

$$U(\vec{\mathbf{r}}') = U_0^i,\ ||\vec{\mathbf{r}}'|| > r_0,$$

and define the new cost function using a new model parameter Δ_U as

$$U'(\vec{\mathbf{r}}) = \begin{cases} U(\vec{\mathbf{r}}), & \text{if } ||\vec{\mathbf{r}}|| \le r_0 \text{ or } U(\vec{\mathbf{r}}) < U_0^i, \\ U_0^i + \int_{\vec{\mathbf{r}}'}^{\vec{\mathbf{r}}} e^{-\frac{(U(\vec{\mathbf{s}})-U_0^i)^2}{\Delta_U^2}} \vec{\nabla} U(\vec{\mathbf{s}}) \cdot \mathrm{d}\vec{\mathbf{s}}, & \text{otherwise.} \end{cases} \tag{1}$$

Basically this assures that repulsive interactions with partners at a distance closer than r_0, and the interaction with the partner at the minimum value of the cost function will be unchanged, while the interaction (cost function gradient) with the others will be reduced as $\vec{\nabla} U'(\vec{\mathbf{r}}) = e^{-\frac{(U(\vec{\mathbf{r}})-U_0^i)^2}{\Delta_U^2}} \vec{\nabla} U(\vec{\mathbf{r}})$.

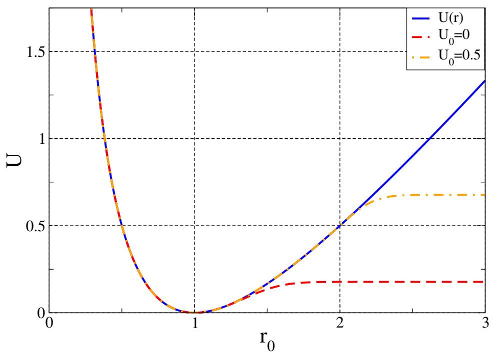

Figure 1: Radial dependence of cost function for first neighbours (blue), and for other partners when the first neighbour is in r_0 (red) or $2r_0$ (orange). Distance measured in multiples of r_0.

This modified cost function, shown in Figure 1 for the only radial dependence of the specific function proposed in [2], assures that while pedestrians will interact strongly with their closest (cost function wise) neighbour, they will be fundamentally neutral with respect to other partners. Nevertheless, partners walking alone will still be strongly seeking for interaction, trying to locate themselves in a free "minimum cost function" position. Once pedestrians have two partners occupying both minima they will strongly interact with both partners, but they will be ready to break one (but not both) link. As a result, large group U formations are possible, but based on initial conditions and/or collision avoidance will in general divide the large group into stable dyads and triads (Figure 2).

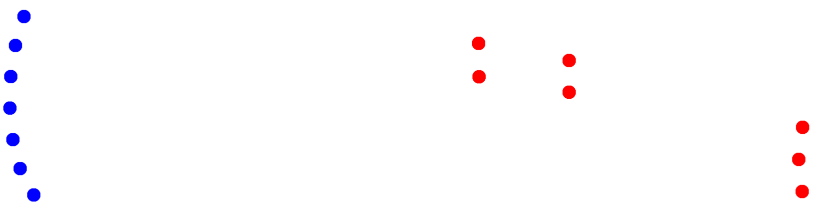

Figure 2: Left, blue: pedestrians walking (from left to right) in a large U formation. Right, red: pedestrians in the same group and using the same model, split in subgroups.

We stress that this is not done by introducing a set of rules, but just a single cost function. Furthermore, although to obtain figures the potential of [2] and a second order model have been used, the proposed approach can be used for any cost function based approach, and for any cost function reproducing the basic behaviour of dyads.

Bibliography

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