Self-organisation in pedestrian dynamics simulation: a stochastic port-Hamiltonian approach

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Abstract In this contribution, we use simulations to explore lane and stripe formation in pedestrian dynamics using a stochastic port-Hamiltonian system. Our approach is minimalist, enabling identification of fundamental modelling components and paving the way for further theoretical investigations.

Keywords Pedestrian dynamics, lane formation, stripe formation, stochastic port-Hamiltonian system

Extended abstract

The modelling of pedestrian dynamics is of great interest in safety, traffic, and civil engineering. Typical applications include simulation-based design and evacuation planning of complex infrastructures (such as train stations or stadiums) or the organisation of large public events (such as festivals or demonstrations). Pedestrian dynamics are mainly governed by local and nonlinear interaction mechanisms. Besides, pedestrian crowds are complex systems that describe self-organising phenomena [7]. For instance, when pedestrians move in opposite directions, they naturally form lanes to accommodate counter flows; while for cross flows, where they move in directions perpendicular to each other, they form stripes. Modelling and analysing the collective behaviour based on pedestrian interaction is not straightforward.

Port-Hamiltonian systems are well established modelling approaches of nonlinear physical systems [9]. The port-Hamiltonian modelling approach, which decomposes the dynamics into skew-symmetric terms, dissipation, input, and output is a meaningful representation of many systems. Interacting particle systems and pedestrian dynamics have also been modelled using port-Hamiltonian frameworks [5, 8].

In this contribution, we explore the dynamics of lane and stripe formation by simulation using stochastic port-Hamiltonian systems. The modelling approach is minimalistic and paves the way for further theoretical investigations. The pedestrian model is a stochastic formulation of a simplified (isotropic) *social force* model [2] given by

$$\begin{cases} \mathrm{d}q_n(t) &= p_n(t)\,\mathrm{d}t, \\ \mathrm{d}p_n(t) &= \gamma\big(u_n - p_n\big)\,\mathrm{d}t + \sum_{m=1}^{N} \nabla\mathcal{U}\big(q_m - q_n\big)\,\mathrm{d}t + \sigma\,\mathrm{d}W_n(t), \end{cases} \tag{1}$$

where $q = (q_n)_{n=1}^N \in \mathbb{R}^{2N}$ and $p = (p_n)_{n=1}^N \in \mathbb{R}^{2N}$ are the positions and velocities of the pedestrians, $\gamma \geq 0$ is the relaxation (dissipation) rate, $u_n \in \mathbb{R}^2$ is the desired velocity, $\mathcal{U}(x) = ab\exp\big(-|x|/b\big)$, $a \geq 0$, $b > 0$, is a short-range repulsive potential on the distance, W_n are standard Wiener process and σ is the noise volatility. The Hamiltonian of the system is

$$\mathcal{H}(z) = \frac{1}{2}\sum_{n=1}^{N} p_n^\top p_n + \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} \mathcal{U}(q_m - q_n), \tag{2}$$

where $z = [q, p]^\top$ describes the system state. The port-Hamiltonian formulation of the system is given by [8, 3]

$$\mathrm{d}z(t) = \begin{bmatrix} 0 & I_N \\ -I_N & -\gamma I_N \end{bmatrix} \nabla\mathcal{H}\big(z(t)\big)\,\mathrm{d}t + \begin{bmatrix} 0 \\ \gamma I_N \end{bmatrix} u\,\mathrm{d}t + \begin{bmatrix} 0 \\ \sigma I_N \end{bmatrix} \mathrm{d}W(t), \tag{3}$$

where $u = (u_1, \ldots, u_N)^\top \in \mathbb{R}^{2N}$. Different parameter settings lead to the formation of various patterns. If $\gamma = 0$, the deterministic system with $\sigma = 0$ is a strictly Hamiltonian colloid where the energy is conserved

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(i.e. the Hamiltonian is constant), while the stochastic system accumulates the energy provided by the noise and diverges. Assume $\gamma > 0$ while $a = 0$, then the pedestrians no longer interact and the system is dissipative. Assume $\gamma, a > 0$ and $u_n = 0$ for all $n \in \{1, \ldots, N\}$, then the system is Hamiltonian-dissipative with stable deterministic crystallisation dynamics. In fact, the self-organization of lanes and stripes observed in pedestrian dynamics requires all port-Hamiltonian components and additionally a high relaxation rate γ and a low noise volatility σ.

In [8], a phase transition from disorder to collective phenomenon occurs in a deterministic framework as the input control parameter γ increases. In preliminary simulation results, we recover a comparable phase transition in the stochastic framework by increasing the noise volatility (see Fig. 1). This phenomenon is known in the literature as the *freezing by heating* effect [1]. Similar behaviour is observed for random initial conditions and for systems initially organised in lanes. This suggests that the system has a unique stationary distribution for a wide range of initial conditions. In contrast, further simulation results show that a *noise-induced ordering* effect [4] arises for stripe formation in 90 degree cross flow for low noise volatility.

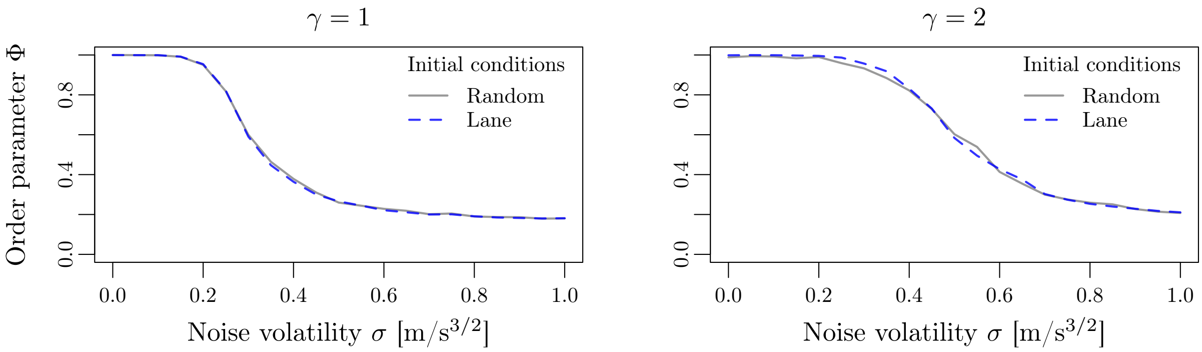

Figure 1: Mean lane formation order parameter Φ for noise volatilities ranging from 0 to 1 in 0.05 steps in a counterflow experiment where $u_n = (-1, 0)$ for half of the pedestrians while $u_m = (1, 0)$ for the remaining pedestrians with $\gamma = 1$ (left panel) and $\gamma = 2$ (right panel). A phase transition occurs as the noise volatility increases. The lower the dissipation rate, the earlier the transition. Simulation results obtained using a leapfrog Maruyama scheme with time step $\delta t = 0.01$. The order parameter is averaged over 20 time units after a simulation time of 200 with an 11×5 periodic system with 32 pedestrians and where $a = 5$ and $b = 0.3$. The results are averaged over 100 simulations for each value of σ. The order parameter for lane formation is given by $\Phi = \frac{1}{N}\sum_{n=1}^{N} \phi_n$ with $\phi_n = \big[(L_n - \underline{L}_n)/(L_n + \underline{L}_n)\big]^2$ where $L_n = \operatorname{card}\big(m,\ |y_n - y_m| < 1/2,\ u_n = u_m\big)$ while $\underline{L}_n = \operatorname{card}\big(m,\ |y_n - y_m| < 1/2,\ u_n \neq u_m\big)$ [6].

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