Variational Modeling for paths through static crowds

Apoorva Singh*[1], Rui M. Castro[2], Maarten Schoukens[3], and Alessandro Corbetta[1]

[1]Department of Applied Physics and Science Education, TU Eindhoven, The Netherlands
[2]Department of Mathematics and Computer Science, TU Eindhoven, The Netherlands
[3]Department of Electrical Engineering, TU Eindhoven, The Netherlands

Abstract We use variational modeling to quantitatively predict the path of an intruder undergoing the N-to-1 interaction while crossing a static crowd. Leveraging large-scale real-life data from Eindhoven Centraal Station, NL, we learn a cost model that stochastically generates trajectories over a discretized domain.

Keywords Pedestrian dynamics, Real-life Data, Variational modeling, Pedestrian paths

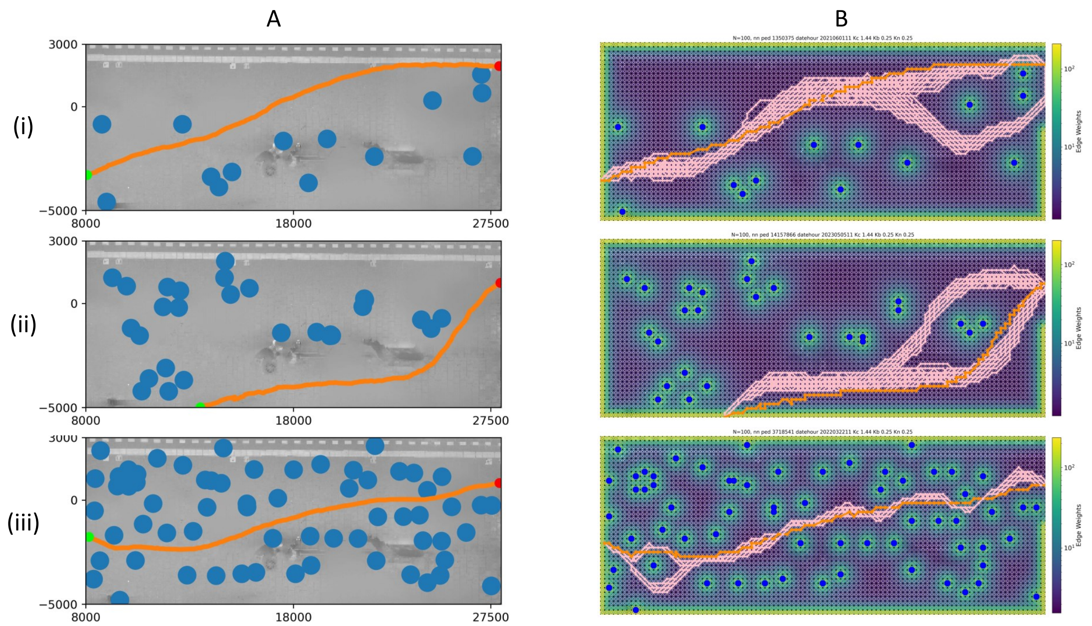

Figure 1: A: Examples of instances of N-to-1 interaction for different sizes of static crowd. Blue dots represent static pedestrians and orange curves represent intruder trajectories. B: Corresponding discrete cost landscapes with original trajectories in orange and stochastically generated trajectories in pink.

The interactions between individuals in a crowd are a major driver of crowd dynamics. In this work, our aim is to develop quantitative, stochastic models for a key type of interaction, which we call the N-to-1 interaction, where an intruder moves through a crowd of stationary pedestrians. Most prior efforts have focused on analyzing laboratory data [1, 2], which is constrained by limited sample size and may not accurately reflect real-world conditions. In contrast, we leverage large-scale, real-life data, which enables us to better understand and quantify the inherent stochasticity in path choice. Empirical evidence suggests that most pedestrians seek to minimize discomfort by avoiding excessive proximity to others while simultaneously reducing effort by choosing shorter, straighter paths [3]. To capture this trade-off, various models for 'rational pedestrians' have been proposed, where pedestrians optimize utility [4]. In these models the pedestrians walk by minimizing a cost function that accounts for factors such as proximity to others, total path length, anticipation of others' movements, etc. [5]. However, without real-life data for validation, such models remain largely qualitative. By utilizing real-world data, we can develop quantitative models that more accurately describe pedestrian behavior.

*Email of the corresponding author: `a.singh2@tue.nl`

The purpose of this study is to understand the factors that inform an intruder's choice of path while crossing, and build a stochastic model that predicts the path. We establish a cost function C that penalizes the following along the intruder's path: total distance walked, interaction with crowd members $I_C(l)$ $(= K_C\, r_{crowd}^{-2}(l))$, and interaction with boundary/obstacles $I_B(l)$ $(= K_B\, r_{obs}^{-3}(l))$. We add a noise $\xi(l) = K_n \mathcal{N}^l(0, 1)$, to account for the stochasticity in choice. $\theta \equiv \{K_C, K_B, K_n\}$ are the model parameters. The intruder chooses a path Υ that minimizes the cost $C(\Upsilon)$ over it:

$$C(\Upsilon) = \int_{\Upsilon} (1 + I_C(l) + I_B(l) + \xi(l))dl \tag{1}$$

One of our research questions is the choice of model for I_C (and I_B), with candidates such as linear superposition (similar to the 'social force' model), nearest-neighbor interaction, and K nearest-neighbor interaction. We start with a crowd interaction term that goes as $1/r^2$ to mimic a repulsion force.

We use pedestrian trajectory data over a region $(20m \times 8m)$ of the platform of the Eindhoven Central train station, The Netherlands (tracking accuracy about 1cm at 10Hz, $\sim$ 3 years of data). From this data we extract instances of N-to-1 interaction at varying densities (Fig.1 A): we build a dataset of 1864 distinct instances. To numerically approximate the path that minimizes the cost in equation (1), we discretize the region of consideration by overlaying a gridded spatial network on it (Fig.2(a)). The weight at any edge is according to the interaction terms in eq. 1, and the cost of a path on the graph is the sum of weights of all the edges it passes through. Using Dijkstra's algorithm we can determine the path with the least cost. Different realizations (with different random noise) generate different paths. Figure 1 shows some instances from the dataset in column A, and the corresponding discrete cost landscapes, created using the nearest-neighbour interaction model with manually chosen parameters, and predicted paths in column B. The predicted paths (in pink) typically diverge into two or more distinct groups of paths, showcasing a meta-stable phenomena, successfully capturing the real-life stochasticity. Fig. 2(b) shows the distribution of costs of the real life trajectories, using manually selected parameters θ, according to different crowd interaction models. It suggests that the nearest-neighbours model (for the chosen parameters) is closest to reality as the cost distribution is minimum. The results highlight the strong quantitative potential of the modeling approach. To get a better fit, the model parameters can be learned from the dataset. The weights and thus cost function can be parameterized in terms of the parameters θ. In this contribution, we will provide a formal model estimation of a parameterized cost function using data driven techniques like system identification and machine learning.

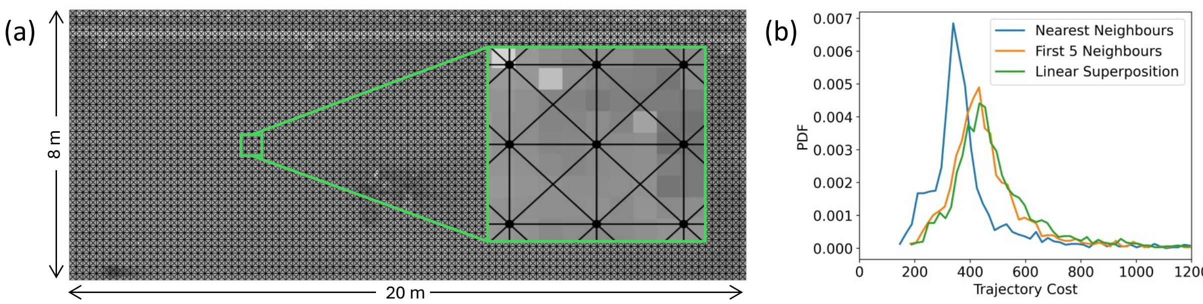

Figure 2: (a) Gridded spatial network overlaid on the region of consideration. (b) Distribution of costs for the real life trajectories from the dataset according to different crowd interaction models.

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