Interaction of 3D pedestrian flow in a congested railway station: Structural estimation based on Mean Field Game theory

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Abstract This paper establishes a new approach for the description and estimation of pedestrian interaction by introducing mean field game (MFG) theory into the network-based route choice model. The mean field dynamics is endogenized in the model and parameters are estimated through repeated structural estimation. The proposed method presents better validation likelihood and shorter computation time than the conventional method.

Keywords pedestrian, 3D route choice model, interaction, mean field game, structural estimation

Introduction

Network-based discrete route choice models are coarser in resolution than simulation-based models in continuous space (Ref. [4]) but can be easily statistically estimated and interpreted with real data based on utility maximization theory. In modeling pedestrian behaviors, description of interaction is important, tackled by several approaches such as social force model (Ref. [3]), cellular automata (Ref. [2]), and discrete choice models (Ref. [1]). Under congested conditions where the number of agents is sufficiently large, considering interactions between each pedestrian is computationally expensive and not realistic. Mean field game (MFG) theory (Ref. [6]) is an effective approach to address this issue. MFG treats agents as a density distribution, reducing calculation costs and enabling simultaneous description of individual strategies and group dynamics (Figure 1). The description is achieved by solving the time evolution of agents' value function and the entire density distribution simultaneously. Ref. [5] introduced MFG into a pedestrian model by Ref. [4], for example, but existing works focus on simulation-based approach and calibrations with real data have not been conducted.

This paper integrates Bellman equation of dynamic route choice and expected pedestrian flow on a discrete network in a MFG framework and constructs structural estimation in which mean field is endogenized by model itself. The contribution of this study is to enable simultaneous estimation of route choice parameters and endogenized mean field using real data by maximum likelihood estimation.

Methodology

To model pedestrians' sequential route choice behaviors, we adopt the discount Recursice Logit (RL) model (Ref. [7]), where each agent is driven by Bellman equation with discount factor β, destination d. In our model, the utility function includes the mean field dynamics M:

$$V^d(k) = E\left[\max_{a\in A(k)}\left\{v(a|k) + \beta V^d(a) + \varepsilon(k)\right\}\right], \qquad v(a|k) = \sum_i \theta_i X_{i,a|k} + \gamma M_{a|k}. \tag{1}$$

The term $\gamma M_{a|k}$ corresponds to the impact of the mean filed on agents moving from node k to a. M is defined as a innor-product of a link vector $\vec{e}$ and average flow vector $\vec{f}$ weighted by an expected density ρ ($M^t_{a|k} = \vec{f}^t_k \cdot \vec{e}_{a|k}$). $\vec{f}$ and ρ are defined at each node and time:

$$\vec{f}^t_k = \sum_{a\in A(k)} \rho(k,t) P^t(a|k;\theta,\gamma,M)\vec{e}_{a|k}, \qquad \rho(k,t) = \sum_{a'\in K(a)} \rho(a',t-1)P^{t-1}(k|a';\theta,\gamma,M). \tag{2}$$

The point here is that M is structurally endogenized in the model instead of given exogenously. Also, according to the concept of MFG, we treat behaviors of other agents as probabilistic distribution P rather than actual choice results. As shown in Figure 1, we estimate parameters by minimizing the log-likelihood function repeatedly until convergence (*structural estimation*). The value function (model parameters) and the mean field are updated iteratively in the process.

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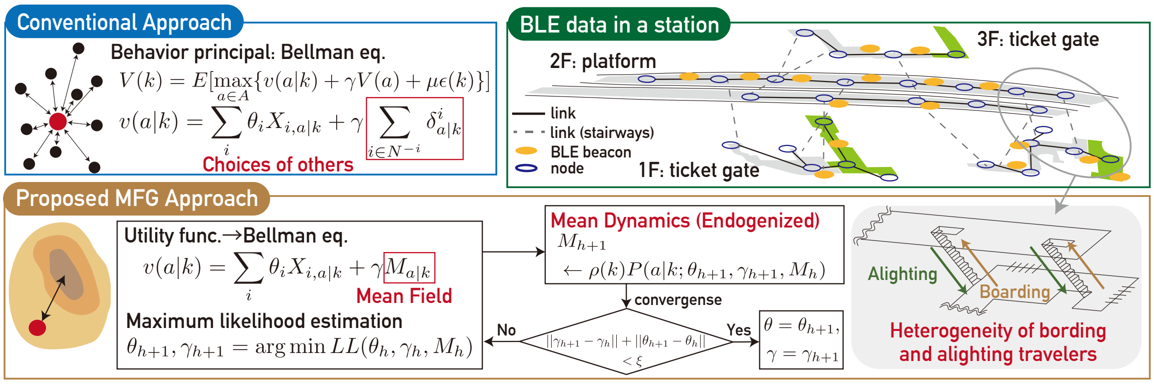

Figure 1: Framework

Table 1: Results

	Sunday morning	Sunday evening	Monday morning	Monday evening
validation LL (not-MFG)	-5.56	-7.69	-6.67	-8.29
validation LL (MFG)	-5.51	-7.75	-7.31	-8.29
time (not-MFG) [s]	7074	1438	1989	1693
time (MFG) [s]	6932	1906	2540	2220
γ_{off} (MFG) (t-val)	0.19 (2.37)	0.16 (2.03)	-0.13 (-0.61)	-0.25 (-1.43)
γ_{on} (MFG) (t-val)	0.16 (0.61)	-0.73 (-2.49)	-0.32 (-1.57)	-0.30 (-1.37)

Results and Discussion

As the case study, we installed BLE beacons to capture BLE signals from travelers' devices in JR Shibuya station, one of the busiest stations in Tokyo, Japan (Figure 1). The data was collected on Sunday and Monday morning (8:30-8:45) and evening (17:30-17:45) on January 29th and 30th, 2023. Agents' footprints are extracted by sorting boarding and alighting trajectories based on MAC addresses, enabling a comparison of interaction effects across groups. Each time slot includes approximately 1,000 agents.

Table 1 shows the comparison of the average validation likelihood (10-fold cross validation) and computation time of the proposed and conventional method as well as the parameter estimates of interaction terms with the MFG-based model. The upper part reveals the MFG-based approach achieves significantly lower calculation cost and equivalent or better validation likelihood (devided the number of path). The lower part presents γ_{off} (interaction parameter for alighting travelers) is significantly positive on Sunday, while that of boarding (γ_{on}) is generally negative or statistically insignificant. Since there are fewer commuters on Sundays, meaning that more passengers are unfamiliar with the station layout than on weekdays, alighting passengers may tend to be guided by others rather than avoiding congestion. It is suggested that the 3D pedestrian flow in a congested station exhibits different effects on boarding and alighting travelers, and these properties dynamically change depending on day and time.

Our result indicates that the MFG-based approach is effective in estimating the impacts of interaction in a space with many agents both in terms of estimation accuracy and computation time. The adaptation of MFG to pedestrian flow control problems is expected as a future work.

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