# Equadiff 2026

## The 16th Czechoslovak Equadiff

book of abstracts

June 28–July 3 ● Prague, Czech Republic

**Under the auspices of**

LEARNED SOCIETY
OF THE CZECH REPUBLIC

**Organizing institutions**

Institute of Mathematics
Czech Academy of Sciences

Faculty of Mathematics and Physics
Charles University

FACULTY OF NUCLEAR SCIENCES AND PHYSICAL ENGINEERING
CTU IN PRAGUE

FACULTY OF CIVIL ENGINEERING
CTU IN PRAGUE

FACULTY OF MECHANICAL ENGINEERING
CTU IN PRAGUE

FACULTY OF APPLIED SCIENCES
UNIVERSITY OF WEST BOHEMIA
IN PILSEN

MUNI
FACULTY OF SCIENCE

**Financial Supporters**

PRAHA
PRAGUE
PRAGA
PRAG

RSJ

*The conference is financially supported by the City of Prague and RSJ, a.s.*
*Conference Agency: AMCA, spol. s r. o., Vyšehradská 320/49, Prague 2*

# Contents

## Modeling Human Brain Function and Pathways of Neurodegeneration

Paola F. Antonietti

Politecnico di Milano, Italy
PAOLA.ANTONIETTI@POLIMI.IT

**Abstract**

Neurodegenerative diseases (NDs) are complex disorders characterized by the progressive structural and functional deterioration of the brain. A common hallmark of many NDs is the accumulation of disease-specific misfolded and aggregated proteins, such as amyloid-beta and tau in Alzheimer's disease, alpha-synuclein in Parkinson's disease, and TDP-43 in Amyotrophic Lateral Sclerosis. In this presentation, we introduce a novel framework that integrates multiphysics and multiscale mathematical modeling, advanced discretization techniques, and computational learning paradigms to elucidate the mechanisms underlying neurodegeneration. The spatio-temporal dynamics of misfolded proteins are modelled using advanced conformational conversion systems discretized with high-order polytopal finite element methods [1, 2, 5]. Additionally, multiphysics mathematical models of cerebral waste-clearance dynamics, a critical factor influencing the onset and progression of NDs, are described, along with an analysis of how pathological processes associated with neurodegeneration may increase epileptiform activity [3, 4]. Finally, a computational learning framework is presented to infer latent disease states and predict personalized trajectories of ND progression from sparse, irregularly sampled, and multimodal clinical data. Extensive patient-specific numerical simulations are provided to validate the proposed methodological approach.

### References

[1] P. F. Antonietti, S. Giani, and P. Houston. hp-version composite Discontinuous Galerkin methods for elliptic problems on complicated domains. *SIAM J. Sci. Comput.*, 35(3):A1417–A1439 (2013)

[2] M. Corti, F. Bonizzoni, P. F. Antonietti, Structure Preserving Polytopal Discontinuous Galerkin Methods for the Numerical Modeling of Neurodegenerative Diseases, *J. Sci. Comput.*, 100:39 (2024)

[3] I. Fumagalli, M. Corti, N. Parolini, P. F. Antonietti, Polytopal discontinuous Galerkin discretization of brain multiphysics flow dynamics, *J. Comput. Phys.*, 513:113115 (2024)

[4] C. B. Leimer Saglio, S. Pagani, P.F. Antonietti, A $p$-adaptive polytopal discontinuous Galerkin method for high-order approximation of brain electrophysiology, *Comput. Methods Appl. Mech. Eng.*, 446:118249 (2025)

[5] V. Pederzoli, M. Corti, D. Riccobelli, P. F. Antonietti, A coupled mathematical and numerical model for protein spreading and tissue atrophy applied to Alzheimer's disease, *Comput. Methods Appl. Mech. Eng.*, 444:118118 (2025)

### A Spectral Approach to Computer-Assisted Proofs in Dynamics

Jan Bouwe van den Berg

Vrije Universiteit Amsterdam, Netherlands
JANBOUWE.VANDEN.BERG@VU.NL

**Abstract**

Finding solutions of problems in nonlinear dynamics often involves simulations. Turning these numerical computations into mathematical theorems requires computer assistance. We will start by explaining the main idea of a computer-assisted proof and give examples of recent results for various ordinary, delay, and partial differential equations. Such computer-assisted proofs focus on specific solutions (or invariant objects more generally) to specific equations. Nevertheless, in this talk we take a more universal viewpoint and describe a relatively simple framework in which many of these problems can be cast. In particular, since these systems and their solutions are usually locally analytic, one can recast the problem into one concerning sequence spaces of rapidly decaying coefficients, say of a Taylor, Fourier, or Chebyshev series. The core of the analysis is then to manipulate such sequences, e.g., evaluating derivatives and nonlinearities, by computer, while keeping track of truncation and rounding errors. We discuss how one can use basic complex and Fourier analysis to accomplish this task for a wide variety of problems in nonlinear dynamics. Such a unified approach can simplify both the analysis and the accompanying implementation.

### Variational Models for Phase Separation in Heterogeneous Media

Irene Fonseca

Carnegie Mellon University, Pittsburgh, USA
FONSECA@ANDREW.CMU.EDU

#### Abstract

Phase Separation in Heterogeneous Media: Modern technologies and biological systems, such as temperature-responsive polymers and lipid rafts, take advantage of engineered inclusions, or natural heterogeneities of the medium, to obtain novel composite materials with specific physical properties. To model such situations using a variational approach based on the gradient theory of phase transitions, the potential and the wells may have to depend on the spatial position, even in a discontinuous way, and different regimes should be considered.

In the critical case case where the scale of the small heterogeneities is of the same order of the scale governing the phase transition and the wells are fixed, the interaction between homogenization and the phase transitions process leads to an anisotropic interfacial energy. The supercritical case for fixed wells is also addressed, now leading to an isotropic interfacial energy. In the subcritical case with moving wells, where the heterogeneities of the material are of a larger scale than that of the diffuse interface between different phases, it is observed that there is no macroscopic phase separation and that thermal fluctuations play a role in the formation of nanodomains.

This analysis is hinged on tools from the Calculus of Variations, including Gamma-convergence, and homogenization theory from multi-scale convergence to unfolding techniques.

## Very Weak Solutions and Asymptotic Behavior of Leray Solutions to the Steady-State Navier-Stokes Equations in Exterior Domains

Giovanni P. Galdi

University of Pittsburgh, USA
galdi@pitt.edu

### Abstract

We show how an appropriate theory of very weak solutions can be employed to advance the resolution of one of the oldest and most extensively studied problems in the theory of steady-state Navier–Stokes equations in unbounded domains: the determination of the sharp asymptotic behavior of Leray solutions.

## AI for Data-driven Simulations in Physics

Siddhartha Mishra

ETH Zürich, Switzerland
SIDDHARTHA.MISHRA@SAM.MATH.ETHZ.CH

### Abstract

Partial Differential Equations (PDEs) are often described as the language of Physics as they describe a wide array of physical phenomena over a vast range of scales. Despite their remarkable success over many decades, numerical methods for approximating PDEs can incur a very high computational cost. This limitation has provided the impetus for the design of fast and accurate Machine Learning/AI based neural PDE surrogates which can learn the PDE solution operator from data. In this talk, we review some latest developments in the field of Neural Operators, which are widely used as an ML paradigm for PDEs and discuss state of the art neural operators based on convolutions or attention. We will discuss graph and transformer based architectures for PDEs on arbitrary domains and conditional Diffusion models for PDEs with chaotic multiscale solutions. Finally, the issue of sample complexity is addressed by the design of general purpose Foundation models for PDEs.

## A Periodic Prey-Predator System

Rafael Ortega

University of Granada, Spain
RORTEGA@UGR.ES

### Abstract

In a paper appeared in 1926 [1], Volterra presented his prey-predator model. He analyzed the system in the plane and he drew the now famous phase portrait having a global center around the equilibrium. Although Volterra dealt with an autonomous system, at the end of his paper he proposed the study of the non-autonomous case: "Side by side with the general theory, we may make various special inquiries. Thus, for example, we may suppose the coefficient of increase of the species to have an annual period, a supposition tending to establish a law of forced fluctuations superposed on the free fluctuations of the biological association considered."

In other words, seasonal effects lead to replace the parameters of the system by time-dependent functions of period one year. In the last fourty years several results on the periodic prey-predator system have been obtained, showing that this system can have an intricate dynamics. In this talk I will discuss some of the known results together with some open questions.

### References

[1] V. Volterra, Fluctuations in the Abundance of a Species considered Mathematically, Nature 118, 558–560 (1926)

### On (In-)Stability Mechanisms in Inverse Problems

Angkana Rüland

Institute of Applied Mathematics, University of Bonn, Germany
RUELAND@UNI-BONN.DE

#### Abstract

Abstract: Inverse problems for PDEs are notoriously ill-posed and often display severe instability properties. In this talk I will identify prototypical instability mechanisms. All of these are based on quantitative compression properties of the forward operator. I will particularly discuss “critical” stability transitions of prototypical PDEs.

### Local Minimizers and Mountain Pass Critical Points

Gabriele Bonanno

Department of Engineering, University of Messina, Italy
BONANNO@UNIME.IT

**Abstract**

In this talk we highlight that the Mountain Pass Theorem should be viewed not only as an existence result but, more fundamentally, as a multiplicity theorem for critical points. A crucial role is played by the presence of a local minimizer: the well–barrier structure generated by a local minimum is precisely what allows the variational framework to produce an additional critical point of saddle type through a minimax argument. We then present an alternative proof of the limiting case of the Mountain Pass Theorem, developed without relying on advanced analytical tools. The proof is based on a new mountain pass lemma, obtained by means of an $\varepsilon$-perturbation technique in the spirit of Brezis–Nirenberg. Applications to differential problems naturally yield the existence of two or three solutions, depending on the growth of the nonlinearity—whether it is superlinear or sublinear. This perspective illustrates how the interplay between a local minimum and a mountain-pass configuration provides a simple yet powerful mechanism for obtaining multiplicity of solutions.

**References**

[1] BONANNO G., CANDITO P., Non-differentiable functionals and applications to elliptic problems with discontinuous nonlinearities, Journal of Differential Equations 244 (2008), 3031-3059.

[2] BONANNO G., A critical point theorem via the Ekeland variational principle, Nonlinear Analysis, 75 (2012), 2992-3007.

[3] BONANNO G., Relations between the mountain pass theorem and local minima, Advances in Nonlinear Analysis, 1 (2012), 205-220.

[4] BONANNO G., LIVREA R., A proof of the Ghoussoub-Preiss theorem by the $\varepsilon-$perturbation of Brezis-Nirenberg, Houston Journal of Mathematics, 47 n.1 (2021), 165-191.

[5] BONANNO G., CANDITO P., Qualitative Analysis of Local Minimizers for Functionals Arising from Nonlinear Differential Problems, work in progress.

## On the Global Well-Posedness of the Stochastic Yang-Mills-Higgs Equations in Two Dimensions

Bjoern Bringmann

Princeton University, Princeton, USA
bringmann@princeton.edu

### Abstract

The Yang-Mills-Higgs (YMH) model plays a central role in several areas of mathematics, including analysis, geometry, and probability theory. In this talk, we implement the stochastic quantization procedure for the YMH model in two dimensions. That is, we construct the two-dimensional YMH measure by proving global well-posedness and uniform-in-time bounds for the two-dimensional stochastic YMH equations. As part of our proof, we first discuss covariant stochastic objects and a covariant para-controlled Ansatz. Then, we discuss a decay mechanism for the stochastic YMH equations near unstable Yang-Mills connections. This is joint work with S. Cao, M. Hairer, and W. Zhao.

### Bifurcations from a Normally Degenerate Cycle in Forced Differential Equations

Adriana Buică

Universitatea Babeş Bolyai, Cluj-Napoca, Romania
florina.buica@ubbcluj.ro

#### Abstract

Bifurcations of periodic solutions from a resonant period manifold in forced systems are intensively studied in the literature, usually assuming that the period manifold is normally nondegenerate. In this talk we consider the situation when the normal degeneracy condition is relaxed, focusing specifically on the case when the period manifold is a cycle. The results are obtained analyzing the Poincaré translation map of the perturbed system.

## Fisher's Infinitesimal Model Features a Nice Convexity Structure

Vincent Calvez

CNRS, Institut Camille Jordan, Lyon, France
VINCENT.CALVEZ@MATH.CNRS.FR

**Abstract**

Fisher's infinitesimal model is a standard model of inheritance in quantitative genetics. Basically, a pair of parents give rise to offspring whose traits are normally distributed around the mean parental trait. We studied the corresponding integro-differential operator which is of collisional type. We identified a remarkable convexity structure which implies asynchronous exponential growth, with a 'spectral gap' (roughly speaking), as measured by some Fisher information, under the simple condition that selection acts through a convex function. We used a natural transformation, converting such a non-linear and non-conservative problem into a linear and conservative problem. The key step is a contraction estimate in the Wasserstein-Kantorovich metric $W_\infty$ for which we shall present a recent, alternative, proof by Khudiakova, Maas, and Pedrotti. [This is a joint work with David Poyato and Filippo Santambrogio.]

## Exploring the Relativistic Kepler Problem: A Journey Through Bounded Orbits and Bifurcation Phenomena

Guglielmo Feltrin

University of Udine, Italy
GUGLIELMO.FELTRIN@UNIUD.IT

### Abstract

The motion of a relativistic particle in a Kepler potential can be described by the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = -\alpha\,\frac{x}{|x|^3}, \quad x \in \mathbb{R}^3 \setminus \{0\},$$

where $m > 0$ represents the mass of the particle, $c$ is the speed of light, and $\alpha > 0$ is a constant. Firstly, we illustrate the Hamiltonian formulation of the problem and we focus our attention on the description of the periodic and quasi-periodic solutions. Secondly, we deal with the perturbed equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = -\alpha\,\frac{x}{|x|^3} + \varepsilon\,\nabla_x U(t,x), \quad x \in \mathbb{R}^3 \setminus \{0\},$$

where $U(t,x)$ is $T$-periodic in the first variable and $\varepsilon \in \mathbb{R}$. We present different kinds of results concerning the existence of periodic orbits obtained in collaboration with Alberto Boscaggin (University of Torino), Walter Dambrosio (University of Torino), and Duccio Papini (University of Modena and Reggio Emilia).

## Landmarks in the History of Linear and Nonlinear Preconditioning

Martin J. Gander

University of Geneva, Switzerland
Martin.Gander@unige.ch

### Abstract

Preconditioning of linear systems remains a highly active area of research, and nonlinear preconditioning has recently emerged as an important new direction. Over the past decade, a unifying principle has become apparent in both the linear and nonlinear settings: effective preconditioners are best developed first as standalone iterative methods, rather than being designed directly as auxiliary components for Krylov or Newton-type methods. Once a robust and well-understood iterative method has been constructed, Krylov methods or Newton's method can be viewed as systematic acceleration mechanisms that enhance performance and improve robustness.

This perspective is strongly reflected in the historical development of iterative methods. I will review key landmarks in the evolution of iterative techniques for both nonlinear and linear problems. The earliest known iterative schemes were developed for nonlinear problems—most notably the Babylonian method for computing square roots—which later evolved into general fixed-point iterations. A major conceptual breakthrough followed with the introduction of Newton's method.

Iterative methods for linear systems trace back to a letter from Gauss to Gerling, which laid the groundwork for stationary schemes such as Jacobi, Gauss–Seidel, and SOR. More powerful physics-based developments include multigrid and domain decomposition methods. In parallel, nonstationary iterations emerged, including extrapolation techniques and, closely related to them, Krylov subspace methods.

In both the linear and nonlinear settings, one observes a natural hierarchy: basic iterative solvers are complemented by systematic acceleration mechanisms—Newton's method in the nonlinear case and Krylov methods in the linear case. I will argue that viewing these accelerators as tools to enhance carefully designed iterative methods provides a historically grounded and conceptually effective framework for advancing research in linear and nonlinear preconditioning.

### References

[1] Martin J. Gander, Philippe Henry and Gerhard Wanner, *Landmarks in the History of Iterative Methods*. SIAM Review, 2025.

[2] Martin J. Gander, *On the Origins of Linear and Non-Linear Preconditioning*. Domain Decomposition Methods in Science and Engineering XXIII, LNCSE, Springer-Verlag, pp. 137–144, 2016.

## New Banach Spaces-Based Mixed Finite Element Methods for Steady-State Flows of Magnetic Fluids

Gabriel N. Gatica

Universidad de Concepción, Chile
ggatica@ci2ma.udec.cl

### Abstract

In this paper we introduce and analyze new mixed finite element methods for solving the steady-state flows of magnetic fluids. To this end, we consider two different ways of deriving the stationary model from the time-dependent problem, namely, either by dropping the time derivatives or by replacing them by a zero order term. In this way, and motivated by our interest in mixed approaches, we are led to systems of equations for the fluid having as unknowns, in the first case, the velocity, its gradient, and a partial stress magnetic tensor, which is given by the sum of the Maxwell and hydrodynamics stress tensors, and the half of the convective term. Similarly, in the second one they turn out to be the strain rate tensor, the velocity, the full stress magnetic tensor, and the vorticity tensor. Next, we introduce a potential unknown so that the magnetic field satisfies a Neumann boundary value problem with the normal component of the applied magnetic field as the corresponding boundary condition. The resulting mixed variational formulations for the fluid fit the structures of a nonlinear saddle-point problem, and a nonlinear perturbation of, in turn, a perturbed twofold saddle-point problem, respectively, whereas the one for the magnetic field is given by a usual saddle-point setting. Hence, fixed-point strategies, along with the generalized Babuška-Brezzi theory in Banach spaces and recent related abstract results, are employed to establish the well-posedness of the associated continuous and discrete schemes. Finally, several numerical results illustrating the performance of our methods are reported. [`This is a joint work with G.A. Benavides, S. Caucao and Y.D. Sobral`].

### References

[1] E. Colmenares, G.N. Gatica and S. Moraga, *A Banach spaces-based analysis of a new fully-mixed finite element method for the Boussinesq problem.* ESAIM Math. Model. Numer. Anal. 54 (2020), no. 5, 1525–1568.

[2] G.N. Gatica, S. Meddahi and R. Ruiz-Baier, *An $L^p$ spaces-based formulation yielding a new fully mixed finite element method for the coupled Darcy and heat equations.* IMA J. Numer. Anal. 42 (2022), no. 4, 3154–3206.

[3] G.N. Gatica, N. Núñez and R. Ruiz-Baier, *New non-augmented mixed finite element methods for the Navier-Stokes-Brinkman equations using Banach spaces.* J. Numer. Math. 31 (2023), no. 4, 343–373.

[4] Y. Wu and X. Xie, *Mixed finite element methods for the ferrofluid model with magnetization paralleled to the magnetic field.* Numer Math Theory Methods Appl 16 (2023), no. 2, 489–510.

### Continuous and Discrete Time Dynamics of Regulatory Networks

Tomas Gedeon

Montana State University, Bozeman, USA
GEDEON@MATH.MONTANA.EDU

#### Abstract

Regulatory networks in cell biology postulate monotonicity structure of interactions between genes and proteins, but not the specific forms of nonlinearities. What types of models are able to provide predictions and insights in this severely underdetermined situation?

Rather than examining individual solutions of particular ODE models, one is drawn to examination of coarse descriptors of dynamics that persist over large sets of parameters and are exhibited by a robust set of nonlinearities. At the same time, since the models of interest have at least 5-15 coupled nonlinear ODEs, these descriptors should be combinatorial, so they can be computed without the need for sampling of parameters or initial conditions.

We discuss one approach to this problem that is based on global approximation of the dynamics of ODE models by models with piece-wise constant functions. We discuss how such approximation yields a finite collection of models for each network, where each model is characterized by a multilevel discrete map which are closely related to monotone Boolean models. Our approach, named DSGRN, provides a direct connection between ODE dynamics and Boolean models.

We examine briefly some consequences of this connection by discussing lattice structure of monotone Boolean functions, algebraic topology that links dynamics predictors computed from discrete and continuous time models, and a potential for combinatorial characterization of bifurcations.

#### References

[1] Cummins, B., Gedeon, T., Harker, S., Mischaikow, K. & Mok, K. Combinatorial Representation of Parameter Space for Switching Systems. *SIAM Journal On Applied Dynamical Systems.* **15**, 2176-2212 (2016)

[2] Gedeon, T., Cummins, B., Harker, S. & Mischaikow, K. Identifying robust hysteresis in networks. *PLoS Comput Bio.* **14**, e1006121 (2018)

[3] Gedeon, T. Multi-parameter exploration of dynamics of regulatory networks. *BioSystems.* **190** pp. 104113 (2020)

[4] Duncan, W., Gedeon, T., Kokubu, H., Oka, H. & Mischaikow, K. Equilibria and their stability in networks with steep sigmoidal nonlinearities. *J. Appl. Dyn. Syst..* **20**, 2108-2141 (2021)

[5] Gedeon, T. Network topology and interaction logic determine states it supports. *Npj Systems Biology And Applications.* **10**, 98 (2024),

### The Hunter–Saxton Equation - Wave Breaking Leads the Way

Katrin Grunert

NTNU Norwegian University of Science and Technology, Trondheim, Norway
katrin.grunert@ntnu.no

**Abstract**

Solutions of the Hunter–Saxton equation might enjoy wave breaking in finite time. This means that even classical solutions in general do not exist globally, but only locally in time since their spatial derivative might become unbounded in finite time, while the solution itself remains bounded. In addition, energy might concentrate on sets of measure zero when wave breaking occurs. As a consequence the prolongation of solutions beyond wave breaking is non-unique and depends heavily on how the concentrated energy is manipulated.

In this talk, we will focus on $\alpha$-dissipative solutions, i.e., solutions where the energy is manipulated at breaking time by taking out an $\alpha$-part of the concentrated energy. Guided by analytical methods and results, we will present a numerical method for $\alpha$-dissipative solutions, which can handle any $\alpha \in [0, 1]$ and in particular the two most prominent type of solutions: conservative, i.e., $\alpha = 0$, and dissipative, i.e., $\alpha = 1$. Convergence is obtained for any admissible initial data, while a convergence rate is derived under some additional constraints on the initial data.

**References**

[1] T. Christiansen and K. Grunert, A numerical view on $\alpha$-dissipative solutions of the Hunter–Saxton equation, ESAIM Math. Model. Numer. Anal. 59, 579–612 (2025).

[2] T. Christiansen and K. Grunert, Rate of convergence for numerical $\alpha$-dissipative solutions of the Hunter–Saxton equation, Numer. Math. 157, 1643–1694 (2025).

### Funny Things Happening in Neural Galerkin Schemes

Olga Mula Hernandez

University of Vienna, Austria
OLGA.MULA.HERNANDEZ@UNIVIE.AC.AT

**Abstract**

Neural Galerkin is a nonlinear counterpart of classical dynamical approximation strategies for time-dependent PDEs. It approximates solutions by neural networks whose weights evolve in time. At first glance, the method is general, elegant, easy to implement, and it indeed has enjoyed notable empirical success. A closer look, however, quickly leads to funny behaviors and troubles: Formulating Neural Galerkin at a proper functional level and connecting it rigorously to practical algorithms raises a number of very subtle challenges. Some stem from sampling issues; others arise from the fact that neural network classes are, mathematically speaking, not particularly well-behaved sets. In this talk, I will illustrate several of these troubles, explain where they come from, and discuss why they are not mere technical nuisances but key conceptual elements to understand when working with neural network approximations.

### Parameterizing Asymptotic Dynamics with Manifolds and Bundles

Jonathan Jaquette

New Jersey Institute of Technology City, Newark, United States
JONATHAN.JAQUETTE@NJIT.EDU

**Abstract**

The parameterization method is a powerful technique for computing high-order approximations of invariant manifolds. In it, one fixes simple internal dynamics for the manifold, which then determines how to solve for the Taylor coefficients, yielding an analytic chart. One can generically show the dynamics are conjugate to the linear flow, but like with the Hartman-Grobman theorem, certain eigenvalue resonances, especially in Hamiltonian systems, can provide an obstruction.

This talk will discuss a parameterization approach towards computing the (un)stable vector bundles attached to invariant manifolds, and are important for a detailed understanding of exponential dichotomies. The first application computes spectral stability of a pulse in the Swift-Hohenberg equation by computing the Maslov index of a homoclinic in a Hamiltonian ODE, and overcomes the problem of eigenvalue resonances. The second application aims to tackle the curse of dimensionality in approximating infinite dimensional stable manifolds arising in PDEs, by constructing a low dimensional slow-stable manifold with attached (un)stable bundles.

### The Strong Unstable Manifold and Periodic Solutions in Differential Delay Equations with Cyclic Monotone Negative Feedback

Anatoli F. Ivanov, Bernhard Lani-Wayda

University of Pennsylvania, USA, and Justus-Liebig-Universität Gießen, Germany
Bernhard.Lani-Wayda@math.uni-giessen.de

#### Abstract

For (N+1)-dimensional systems of differential delay equations with cyclic and monotone negative feedback, we construct a two-dimensional invariant manifold, on which phase curves spiral outward towards a bounding periodic orbit. We assume essentially only instability of the zero equilibrium. Methods of the Poincaré-Bendixson theory due to Mallet-Paret and Sell are combined with techniques used by Walther for the scalar case (N = 0). Statements on the attractor location and on parameter borders concerning stability and oscillation are included. The results apply to models for gene regulatory systems, e.g. the 'repressilator' system.

#### References

[1] Ivanov, A. F., and Lani-Wayda, B.: *The strong unstable manifold and periodic solutions in differential delay equations with cyclic monotone negative feedback.* Commun. Pure Appl. Anal. (CPAA) Vol. **24**, Iss. 9 (2025), 1713-1752. Doi: 10.3934/cpaa.2025057

## Global Solvability of the Q-Tensor Model for Nematic Liquid Crystals

Miho Murata

Shizuoka University, Shizuoka, Japan
MURATA.MIHO@SHIZUOKA.AC.JP

### Abstract

The molecules of nematic liquid crystal flow as in a liquid phase; however, they have an orientation order. The orientation order is described by the symmetric, traceless matrix $\mathbf{Q}$. Beris and Edwards proposed a Q-tensor model, the coupled system of the Navier-Stokes equations and a parabolic-type equation describing the evolution of the order parameter $\mathbf{Q}$, to represent nematic liquid crystal flows. The aim of this talk is to prove the unique existence of the global-in-time solution in the maximal regularity class for the Q-tensor model in the half-space $\mathbb{R}^N_+$, $N \geq 2$. Concerning the existence results for the strong solutions in the maximal regularity class, [1] proved the local well-posedness in $\mathbb{R}^N_+$ for small initial data. The global well-posedness was investigated in $\mathbb{R}^N$ by [4, 2], while that in the bounded domain was studied by [3, 6].

In this talk, we especially discuss the weighted estimates of solutions for the linearized problem in the $L_p$-in-time and $L_q$-in-space setting to obtain the global solvability in $\mathbb{R}^N_+$ for small initial data. The key issue is to estimate the lower order terms by the semigroup theory. This talk is based on [1].

### References

[1] D. Barbera, M. Murata, and Y. Shibata, *The global well-posedness for the Q-tensor model of nematic liquid crystals in the half-space*, submitted. https://doi.org/10.48550/arXiv.2511.03309

[2] D. Barbera and M. Murata, *The $L^p$-$L^q$ maximal regularity for the Beris-Edward model in the half-space*, Ann. Sc. Norm. Super. Pisa Cl. Sci. **56** (2024).

[3] M. Hieber, A. Hussein, and M. Wrona, *Strong well-posedness of the Q-tensor model for liquid crystals: the case of arbitrary ratio of tumbling and aligning effects* $\xi$, Arch. Ration. Mech. Anal. **248 (3)** (2024) Paper No. 40.

[4] M. Murata and Y. Shibata, *Global well posedness for a Q-tensor model of nematic liquid crystals*, J. Math. Fluid Mech. **24 (1)** (2022) Paper No. 34.

[5] M. Schonbek and Y. Shibata, *Global well-posedness and decay for a $\mathbb{Q}$ tensor model of incompressible nematic liquid crystals in* $\mathbb{R}^N$, J. Differential Equations **266 (6)** (2019) 3034–3065.

[6] Y. Xiao, *Global strong solution to the three-dimensional liquid crystal flows of Q-tensor model*, J. Differential Equations **262 (3)** (2017) 1291–1316.

### Multiharmonic Algorithms for Contrast-Enhanced Ultrasound

Vanja Nikolić

Department of Mathematics, Radboud University, Nijmegen, The Netherlands
vanja.nikolic@ru.nl

#### Abstract

Nonlinear acoustic wave models are increasingly important for medical ultrasound and industrial applications where sound propagates at high intensities or frequencies. Harmonic generation, in particular, plays a key role in contrast-enhanced ultrasound, both for imaging and therapeutic applications. In this talk, we will discuss wave–microbubble models, where the acoustic field is governed by a nonlinear Westervelt-type wave equation coupled to a Rayleigh- Plesset-type ODE describing the bubble dynamics. This includes the analytical and computational questions arising from this coupling, such as the existence of time-periodic solutions and handling nonlinearity and differing time scales. Since time-domain approaches are computationally demanding for capturing nonlinear effects, we will introduce an alternative strategy based on a multiharmonic Ansatz applied to the coupled wave–bubble system. A priori error estimates will be presented that characterize the approximation error in terms of the number of retained harmonics and a contribution arising from the fixed-point iteration. Additionally, numerical experiments will illustrate how the number of retained harmonics and the presence of microbubbles influence acoustic wave propagation.

The talk is based on joint works [1, 2].

#### References

[1] V. Nikolić and T. Rauscher, *Mathematical models for nonlinear ultrasound contrast imaging with microbubbles*, SIAM Journal on Applied Mathematics, 85 (2025), pp. 961–982.

[2] V. Nikolić and T. Rauscher, *Multiharmonic algorithms for contrast-enhanced ultrasound*, Journal of Scientific Computing, (to appear).

## Dynamics of Non-Equilibrium Systems: Homoenergetic Flows for the Boltzmann Equation

Alessia Nota

Gran Sasso Science Institute, L'Aquila, Italy
alessia.nota@gssi.it

### Abstract

Kinetic equations have been used for more than a century to describe the collective behavior of many-particle systems, providing a bridge between the microscopic laws of classical dynamics and macroscopic fluid mechanics. One of the most famous examples of a kinetic equation is the Boltzmann equation, which describes the evolution of a rarefied gas. In this talk, I will consider a particular class of solutions of the Boltzmann equation, known as homoenergetic solutions, which were introduced by Galkin and Truesdell in the 1960s. These are a particular type of non-equilibrium solutions of the Boltzmann equation, useful for modeling the dynamics of Boltzmann gases subjected to mechanical deformations (such as shear, expansion, or compression), thereby yielding insight into the behavior of open systems. While the well-posedness theory for these solutions shares similarities with that of homogeneous solutions, their long-time behavior differs significantly due to their far-from-equilibrium nature. In particular, their long-time asymptotics often cannot be described by equilibrium Maxwellian distributions; the landscape of possible long-time asymptotics is extremely rich and diverse. I will present different possible long-time asymptotics for homoenergetic solutions to the Boltzmann equation in the case of shear deformations. Additionally, I will discuss some conjectures and open problems in this direction.

**On the Dispersive Effect of Internal Gravity Waves in Two-Phase Incompressible Viscous Flows**

Hirokazu Saito

The University of Electro-Communications, Tokyo, Japan
HSAITO@UEC.AC.JP

**Abstract**

In this talk, we consider the dispersive effect of internal gravity waves in two-phase incompressible viscous flows in the $N$-dimensional Euclidean space $\mathbb{R}^N$ for $N \geq 2$. The two fluids are separated by a horizontal interface, and surface tension is taken into account at the interface. We call the fluids above and below the interface the upper fluid and lower fluid, respectively. If the upper fluid is heavier than the lower fluid, the well-known Rayleigh–Taylor instability occurs. In this study, on the other hand, we focus on the stable case where the upper fluid is lighter than the lower fluid. Under this stable setting, the boundary symbol associated with the linearized problem admits two zeros $\lambda_\pm$ satisfying $\lambda_\pm = \pm ic_1|\xi'|^{1/2} - c_2(1 \pm i)|\xi'|^{5/4} + o(|\xi'|^{5/4})$ as $|\xi'| \to 0$, where $i = \sqrt{-1}$ and $\xi' \in \mathbb{R}^{N-1}$ is the Fourier transform variable corresponding to the tangential direction $x' = (x_1, \ldots, x_{N-1})$. Here, $c_1$ and $c_2$ are positive constants depending on the densities, the viscosity coefficients, and the acceleration of gravity. The leading terms $\pm ic_1|\xi'|^{1/2}$ represent dispersive waves known as internal gravity waves. By utilizing both the dispersive effect of these waves and the fractional dissipation $-c_2|\xi'|^{5/4}$, we establish $L_p$-$L_q$ decay estimates for the two-phase Stokes semigroup. If time permits, we shall discuss applications of these estimates to the nonlinear problem. This is based on joint work with Xin Zhang (Tongji University).

## Potential and Flux Reconstructions for Optimal a Priori and a Posteriori Error Estimates

Martin Vohralík

Inria & Ecole nationale des ponts et chaussées, Paris, France
MARTIN.VOHRALIK@INRIA.FR

### Abstract

Given a scalar-valued discontinuous piecewise polynomial, a **potential reconstruction** is a piecewise polynomial that is **trace continuous**, i.e., $H^1$-conforming. It is best obtained via a conforming finite element solution of **local** homogeneous **Dirichlet problems** on vertex patches, using the finite element method. Similarly, given a vector-valued discontinuous piecewise polynomial not having the target divergence, a **flux reconstruction** is a piecewise polynomial that is **normal-trace continuous**, i.e., $\boldsymbol{H}(\mathrm{div})$-conforming, and has the target divergence. It is best obtained via **local** homogeneous **Neumann problems**, using the mixed finite element method.

The concepts of potential and flux reconstructions are known to lead to guaranteed, locally efficient, and polynomial-degree-robust **a posteriori error estimates**. Such use is based on the Prager–Synge equality and piecewise polynomial extension operators that we recall here.

We show that potential and flux reconstructions can also be used to obtain novel results in a priori error analysis. They actually allow to devise **stable local commuting projectors** that lead to $p$-robust **equivalence** of **global-best approximation** over the whole computational domain by a continuous (and constrained) finite element space with **local-best approximations** over individual mesh elements without any continuity requirement along mesh faces (and without any divergence constraint). Therefrom, optimal $hp$ **approximation/a priori error estimates** under minimal elementwise Sobolev regularity follow. The main additional tools include here $p$-stable local decompositions.

### References

[1] A. Ern, M. Vohralík, Polynomial-degree-robust a posteriori estimates in a unified setting for conforming, nonconforming, discontinuous Galerkin, and mixed discretizations, *SIAM J. Numer. Anal.* **53** (2015), 1058–1081.

[2] A. Ern, M. Vohralík, Stable broken $H^1$ and $\boldsymbol{H}(\mathrm{div})$ polynomial extensions for polynomial-degree-robust potential and flux reconstructions in three space dimensions, *Math. Comp.* **89** (2020), 551–594.

[3] L. Demkowicz, M. Vohralík, $p$-robust equivalence of global continuous constrained and local discontinuous unconstrained approximation, a $p$-stable local commuting projector, and optimal elementwise $hp$ approximation estimates in $\boldsymbol{H}(\mathrm{div})$, *Math. Comp.* (2026), accepted for publication.

[4] M. Vohralík, $p$-robust equivalence of global continuous and local discontinuous approximation, a $p$-stable local projector, and optimal elementwise $hp$ approximation estimates in $H^1$, HAL Preprint 04436063, (2025).

## Cut Finite Element Methods

Sara Zahedi

KTH Royal Institute of Technology, Stockholm, Sweden
sara.zahedi@math.kth.se

### Abstract

Cut Finite Element Methods (CutFEM) provide an attractive alternative to standard finite element discretizations by allowing interfaces and boundaries to intersect the computational mesh arbitrarily, while retaining the accuracy and stability properties of conforming finite element methods [1]. In this talk, I introduce CutFEM and discuss recent developments in this class of unfitted finite element methods. I begin with the Stokes interface problem and present stabilization techniques that preserve the incompressibility constraint, as well as approaches to achieve pressure robustness [2, 3]. Starting from stationary problems, I then describe our strategy for time-dependent PDEs in evolving domains [4, 5]. I also present recent developments of CutFEM within a discontinuous Galerkin formulation for hyperbolic PDEs [6, 7].

### References

[1] E. Burman, P. Hansbo, M. G. Larson, and S. Zahedi. Cut finite element methods. *Acta Numerica*, 34:1–121, 2025.

[2] T. Frachon, P. Hansbo, E. Nilsson, and S. Zahedi. A divergence preserving cut finite element method for *Darcy* flow. *SIAM Journal on Scientific Computing*, 46(3):A1793–A1820, 2024.

[3] T. Frachon, E. Nilsson, and S. Zahedi. Divergence-free cut finite element methods for *Stokes* flow. *BIT Numer. Math.*, 64(4):39, 2024.

[4] P. Hansbo, M. G. Larson, and S. Zahedi. A cut finite element method for coupled bulk-surface problems on time-dependent domains. *Comput. Methods Appl. Mech. Engrg.*, 307:96–116, 2016.

[5] S. Myrbäck and S. Zahedi. A high-order conservative cut finite element method for problems in time-dependent domains. *Comput. Methods Appl. Mech. Engrg.*, 431:117245, 2024.

[6] P. Fu, T. Frachon, G. Kreiss, and S. Zahedi. High order discontinuous cut finite element methods for linear hyperbolic conservation laws with an interface. *J. Sci. Comput.*, 90(3):Paper No. 84, 39, 2022.

[7] P. Fu, G. Kreiss, and S. Zahedi. A bound preserving cut discontinuous *Galerkin* method for one-dimensional hyperbolic conservation laws. *ESAIM: Math. Model. Numer. Anal.*, 58(5):1651–1680, 2024.

## Slow-Fast Solutions of Abel Equations

María Jesús Álvarez, Peter De Maesschalck

Universitat de les Illes Balears, Palma de Mallorca, Spain
Unversiteit Hasselt, Hasselt, Belgium
chus.alvarez@uib.cat, peter.demaesschalck@uhasselt.be

### Abstract

In this talk, we present an overview of the new results we have obtained concerning trigonometric slow–fast Abel equations.

Specifically, we study the equation

$$\varepsilon \frac{dx}{d\theta} = A_\varepsilon(\theta)x^3 + B_\varepsilon(\theta)x^2,$$

or equivalently the system

$$\begin{cases} \dot{\theta} = \varepsilon, \\ \dot{x} = A_\varepsilon(\theta)x^3 + B_\varepsilon(\theta)x^2. \end{cases}$$

Our main goal is to characterize the limit cycles near the slow–fast regime, proving along the way the existence of canard periodic solutions. To achieve this, we analyze the behavior of solutions both near the critical curves and near infinity. This analysis requires the use of the blow-up technique.

As a corollary of this study, we recover the result in [1], where the authors proved that the maximum number of limit cycles of the linear trigonometric Abel equation is three.

### References

[1] X. Yu, J. Huang, C. Liu. Maximum number of limit cycles for Abel equation having coefficients with linear trigonometric functions. Journal of Differential Equations, 410 (2024), 301–318.

### A Systematic Study of Two-Layer Canard Cycles

De Maesschalck Peter

Hasselt University, Belgium
PETER.DEMAESSCHALCK@UHASSELT.BE

### Abstract

The talk discusses a published paper [1] that is joint work with F. Dumortier and R. Roussarie. We deal with 2-layer canard cycles, extending well-known results of 1-layer canards such as the one appearing in the classical Van der Pol system. We provide precise results on the number and the bifurcations of the limit cycles that can be created from the canard cycles, confirming that both the type of the layers and the nature of the connections between the layers make a difference. We also prove an upperbound on the cyclicity of a large class of 2-layer canard cycles.

### References

[1] P. De Maesschalck, F. Dumortier, and R. Roussarie. A systematic study of two-dimensional 2-layer canard cycles. *Nonlinearity*, 38(5), 2025.

## Computer Assisted Study of a Perturbed Planar Vector Field in the Quadratic Case

Zbigniew Galias*, Warwick Tucker†, Alvaro Fernandez-Mora†

*AGH University of Kraków, Kraków, Poland
†Monash University, Melbourne, Australia
GALIAS@AGH.EDU.PL

### Abstract

Determining how many limit cycles a planar polynomial system of differential equations can have is a remarkably hard problem. In this work, we study the weak infinitesimal Hilbert 16th problem which asks for the number of limit cycles can a polynomial vector field of degree $n$ have if it is close to a Hamiltonian vector field.

We consider cubic Hamiltonians in the normal form

$$H(x,y) = \frac{1}{2}(x^2+y^2) - \frac{1}{3}x^3 + axy^2 + \frac{1}{3}by^3.$$

A perturbed system is defined as

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\partial H(x,y)}{\partial y} + \varepsilon f(x,y), \quad \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\partial H(x,y)}{\partial x} + \varepsilon g(x,y),$$

where $f$ and $g$ are polynomials of degree 2. Let $\gamma_h \subset \{(x,y)\colon H(x,y)=h\}$ with $h \in (0,1/6)$ be an oval surrounding the origin.

In this work, we present a method to carry out a computer assisted proof that for fixed $(a,b)$ the number of limit cycles bifurcating from ovals $\gamma_h$ for $h \in (0,1/6)$ is less than three. The method is based on the relation between the number of such limit cycles and the number of isolated zeros of the Abelian integrals $I(h) = \oint_{\gamma_h} f(x,y)\mathrm{d}y - g(x,y)\mathrm{d}x$ for $h \in (0,1/6)$.

## Isolated Periodic Orbits in Behavioristic Diffusion Models

Robert Kooij

Faculty of Electrical Engineering, Mathematics and Computer Science,

Delft University of Technology, the Netherlands,
&
Unit ICT, Strategy & Policy, TNO,
(Netherlands Organisation for Applied Scientific Research), the Netherlands
R.E.KOOIJ@TUDELFT.NL

### Abstract

In 1992, Feichtinger [1] discussed an advertising diffusion model showing isolated periodic behavior as a result of the occurrence of the Andronov-Hopf bifurcation. We extend his result to a model accounting for different kinds of interaction diffusion and prove existence and uniqueness of the limit cycle with the Zhang Zhifen theorem. Then we unfold a particular instance of the model equations leading to a system that describes the influence of government policy regarding drug addicts. Again, only a single stable regime is possible. The proof requires the more complicated theorem of Coppel - Zeng Xianwu in this case. All models remain within the class of those having two state variables, giving rise to families of planar vector fields.

### References

[1] G. Feichtinger, Hopf bifurcation in an advertising diffusion model. *Journal of Economic Behavior & Organization*, 17(3):401–411, 1992.

### A Geometric Approach to Exponentially Small Splitting Phenomena

Kristian Uldall Kristiansen

Technical University of Denmark, 2800 Kgs. Lyngby, Denmark
krkri@dtu.dk

#### Abstract

In this talk, I will present a geometric and dynamical-systems-based method for exponentially small splitting phenomena within the context of unfoldings of real-analytic zero-Hopf bifurcations. I will consider the generic co-dimension two case, where our approach improves existing results, but also degenerate cases with higher co-dimension. As a novel aspect, we relate the exponential splitting to the lack of analyticity of invariant manifolds of generalized saddle-nodes. The blowup method plays an important technical role as a systematic way to relate dynamics on different orders of magnitude. Moreover, as our approach takes place in the (complexified) phase space, we do not rely on explicit time-parametrizations of the unperturbed heteroclinic connections.

### Unfoldings of Limit Cycles in Families of Centers

Joan Torregrosa

Universitat Autònoma de Barcelona - Cerdanyola del Vallès (Barcelona) - Spain
joan.torregrosa@uab.cat

### Abstract

Classical bifurcation mechanisms, such as degenerate Hopf bifurcation, Melnikov theory, and the averaging method, provide lower bounds for the number of limit cycles that a planar polynomial vector field of fixed degree can exhibit. Typically, the analysis focuses on a single center. In this talk, we show how exploiting families of centers can increase these lower bounds. In particular, we investigate how the cyclicity depends on the specific point within a connected component of centers. We present several examples, in both smooth and nonsmooth settings, illustrating this phenomenon under different perturbation techniques, near center points and heteroclinic connections.

## Center Conditions and Cyclicity for Generic Planar Polynomial Vector Fields

Warwick Tucker

School of Mathematics, Monash University, Melbourne, Australia
WARWICK.TUCKER@MONASH.EDU

**Abstract**

We study the center-focus problem for planar polynomial vector fields, which can be viewed as a local version of Hilbert's 16th problem. Based on a Lyapunov function approach, we establish novel results regarding the center-focus conditions. More precisely, under generic conditions, and for any degree of a polynomial vector field, we find an upper bound on the size of the Bautin ideal generated by the Lyapunov constants. This also provides an upper bound on the cyclicity of the systems we consider. This is joint work with Yovani Villanueva, Universidade Federal de Goiás, Goiás, Brasil.

## $C^1$-Perturbations of a Continuum of Critical Points

Rafael Ortega and Antonio J. Urena

University of Granada, Granada, Spain
ajurena@ugr.es

### Abstract

Given a real-valued function having a nondegenerate compact manifold of critical points, some of these points survive under small $C^2$-perturbations. This is a well-known result in critical point theory. In 1986, Weinstein obtained the analogous conclusions when the perturbation is only $C^1$ and the ambient space is a finite-dimensional manifold. In this work we present a complete proof for $C^1$ perturbations in infinite-dimensional Hilbert spaces. This is a joint work with R. Ortega.

### References

[1] Ortega, R., Urena, A.J., *$C^1$-perturbations of a continuum of critical points.* Calc. Var. **65**, 31 (2026). https://doi.org/10.1007/s00526-025-03200-5

## Spectral Element Methods for Boundary-Value Problems of Functional Differential Equations

Alessia Andò

Department of Mathematics, Computer Science and Physics – University of Udine, Udine, Italy
alessia.ando@uniud.it

### Abstract

We prove convergence of the spectral element method for piecewise polynomial collocation applied to periodic boundary value problems BVP for functional differential equations. In particular, we prove that the numerical collocation solution approximates the true solution with accuracy of order $e^{-\eta m}$ for some $\eta > 0$ and increasing degree $m$ of the polynomials for a case that is common in applications: differential equations where the right-hand side depends on a finite number of delayed arguments with parametric delays and real analytic coefficients. For state-dependent delays the spectral element method also converges under mild regularity assumptions, but the geometric convergence of the collocation solution depends on the properties of the true solution, which may in general not be real analytic even for analytic coefficients.

We introduce the concept of extended local Lipschitz continuity of the right-hand side, which allows us to extend our convergence results to the case of state-dependent delays. In particular, we can arrive at the same geometric convergence statement by assuming, in this case, analyticity of the solution rather than of the right-hand side.

## A Nonautonomous Chemostat System with Delays

Teresa Faria

Department of Mathematical Sciences and CEMS.UL, University of Lisbon, Lisbon, Portugal
teresa.faria@fc.ul.pt

### Abstract

We consider a nonautonomous delayed chemostat system modelling $n$ species in competition. Conditions for the extinction versus persistence of all the species are established. When the coefficients and delays are periodic in time, criteria for the existence of (at least) $n$ nontrivial and nonnegative periodic solutions are given. Further results guarantee that the system admits at least one strictly positive periodic solution. For the model with a simple microorganism, a refinement of the previous analysis provides the uniform persistence, as well as the global attractivity of a positive periodic solution.

These results generalise and enhance recent achievements in the literature, see [1, 2].

### References

[1] T. Faria, Periodic solutions for a delayed competitive chemostat model with periodic nutrient input and rate, *Nonlinearity* 38 (2025), paper 055023, pp. 1-22.

[2] T. Faria, J.G. Mesquita, A competitive chemostat model with time-dependent delays, *Nonlinear Anal. RWA* 88 (2026), paper 104476, pp. 1-16.

## Discrete Lyapunov Functional for Cyclic Systems of Differential Equations with Time-Variable or State-Dependent Delay

Ábel Garab

Bolyai Institute, University of Szeged, Hungary
garab@math.u-szeged.hu

**Abstract**

We consider nonautonomous cyclic systems of delay differential equations (DDEs) with variable delay. Under suitable feedback assumptions, we define an (integer valued) Lyapunov functional related to the number of sign changes of the coordinate functions of solutions. We prove that this functional possesses properties analogous to those established by Mallet-Paret and Sell for the constant delay case and by Krisztin and Arino for the scalar case. This may serve as an effective tool in the study of the global dynamics of DDEs with variable delays. For example, the results can be applied to cyclic systems of DDEs with state-dependent delays to obtain a Morse-decomposition of the global attractor.

This is a joint work with István Balázs (U. Szeged).

### References

[1] I. Balázs, Á. Garab, Discrete Lyapunov functional for cyclic systems of differential equations with time-variable or state-dependent delay, *J. Differential Equations* **452**(2026), 113768.

[2] I. Balázs, Á. Garab, T. Rauscher, Morse decomposition for semi-dynamical systems with an application to systems of state-dependent delay differential equations, *J. Dynam. Differential Equations*, published online, 2025.

### Connecting Orbits for Some Mackey–Glass Type Equations

Tibor Krisztin

University of Szeged, Szeged, Hungary
KRISZTIN@MATH.U-SZEGED.HU

#### Abstract

Consider the delay differential equation

$$y'(t) = -ay(t) + bf(y(t-1)) \tag{1}$$

where $b > a > 0$, and $f(\xi) = \frac{\xi^k}{1+\xi^n}$ with $k > 0$ and $n > 0$. The case $k = 1$ is the famous Mackey–Glass equation, the case $k > 1$ appears in population models with the Allee effect, and $k \in (0,1)$ arises in some economic growth models.

We consider an unstable stationary point of (1), and its unstable set. The unstable set contains connecting orbits from the stationary point to a subset $\mathcal{A}$ of the phase space. The aim is to describe the structure of $\mathcal{A}$, which can be complicated for some parameter values, see [1].

In case $k \in (0,1)$ and the stationary point is 0, the invariant manifold technique is not applicable, since the nonlinearity $f$ is not differentiable at 0. Nevertheless, we construct a positive heteroclinic solution from 0, in this singular case as well.

#### References

[1] G. Benedek, T. Krisztin, Connecting orbits for delay differential equations with unimodal feedback, J. Diff. Equations, Volume 465, 2026, 114252, ISSN 0022-0396, https://doi.org/10.1016/j.jde.2026.114252.

## On the Geographic Spread of Chikungunya between Brazil and Florida: A Multi-patch Model with Time-delay

Jaqueline Mesquita

Universidade Estadual de Campinas, Brazil
jgmesquita@ime.unicamp.br

### Abstract

Chikungunya (CHIK) is a viral disease transmitted to humans through the bites of Aedes mosquitoes infected with the chikungunya virus (CHIKV). CHIKV has been imported annually to Florida in the last decade due to Miami's crucial location as a hub for international travel, particularly from Central and South America including Brazil, where CHIK is endemic. This paper presents a comprehensive mathematical model for the geographic spread of CHIKV, incorporating pivotal factors such as human movement, temperature, rainfall, vertical transmission, and incubation period. Central to the model is the integration of a multi-patch framework, considering human movement between endemic Brazilian states and Florida. We establish crucial correlations between the mosquito reproduction number Rm and the disease reproduction number R0, thereby advancing our understanding of CHIKV transmission dynamics in complex multi-patch environments. Through numerical simulations, validated with real population, temperature and rainfall data, it is possible to understand the disease dynamics under many different scenarios and make future projections, offering crucial insights for devising effective control strategies. This is a joint work with A. Gondim, X. Huo and S. Ruan.

## Blow-up and Convergence of Solutions to a Differential Equation with Delayed Negative Feedback

Yukihiko Nakata

College of Science and Engineering, Aoyama Gakuin University
ynakata@math.aoyama.ac.jp

### Abstract

There have been studies on the existence of finite-time blow-up solutions for delay differential equations, see [1, 2, 3, 4, 5]. For those equations, identifying blow-up solutions, estimating their blow-up times and characterizing the blow-up behaviour are generally challenging problems. In this talk, we discuss the dynamics of the following delay differential equation:

$$x'(t) = x(t)^2 - x(t-\tau), \tag{2}$$

where $\tau > 0$. Eq (2) is a simple model incorporating "self-amplification" and a delayed negative feedback. We discuss the blow-up phenomena of (2), comparing it to (2) with $\tau = 0$, and provide sufficient conditions on the initial function for finite-time blow-up solution. We also investigate the convergence of the solution to the zero solution. This study is based on the collaboration work with T. Ishiwata (Shibaura Institute of Technology).

### References

[1] J.A.D. Appleby, D.D. Patterson, Blow-up and super exponential growth in superlinear Volterra equations. Disc. Cont. Dyn. Syst. A 38(8), pp. 3993–4017 (2018)

[2] A. Eremin, E. Ishiwata, T. Ishiwata, Y. Nakata, Delay-induced blow-up in a planar oscillation model. Jpn. J. Indust. Appl. Math. 38, pp. 1037–1061 (2021)

[3] K. Ezzinbi, M. Jazar, Blow-up results for some nonlinear delay differential equations. Positivity 10(2), pp. 329–341 (2006)

[4] I. Győri, Y. Nakata, G. Röst, Unbounded and blow-up solutions for a delay logistic equation with positive feedback. Comm. Pure Appl. Anal. 17(6), pp. 2845–2854 (2018)

[5] T. Ishiwata, Y. Nakata, A note on blow-up solutions for a scalar differential equation with a discrete delay, Jpn. J. Indust. Appl. Math. 39, pp. 959–971 (2022) (Correction: 39, p. 1109 (2022))

### On Simplicity of Solution Manifolds

Hans-Otto Walther

Mathematisches Institut,Universität Gießen, Gießen, Germany
HANS-OTTO.WALTHER@MATH.UNI-GIESSEN.DE

**Abstract**

Differential equations with state-dependent delays define a semiflow of continuously differentiable solution operators in general only on the associated *solution manifold* $X \subset C^1([-h, 0], \mathbb{R}^n)$. For systems with discrete state-dependent delays we construct a diffeomorphism on a neighbourhood of $X$ which takes $X$ to an open subset of the subspace given by $\phi'(0) = 0$. This is in line with earlier work on the nature of solution manifolds. The present approach, however, is new and dismisses all hypotheses beyond smoothness which have been instrumental so far. Compared to a recent case study it is more direct in the sense that theory of *algebraic delay systems* is avoided.

#### References

[1] A direct approach to simplicity of solution manifolds. Preprint, arxiv.org/abs/2601.00642, 21 pp, 2026.

**Impact of Spatial Heterogeneity in Developmental Delay on Population Dynamics**

Jianhong Wu

York University, Canada
WUJH@YORKU.CA

**Abstract**

Developmental delay plays a crucial role in shaping population dynamics, particularly for species whose maturation processes depend strongly on environmental conditions. In heterogeneous environments, the duration of developmental stages may vary across space, leading to complex patterns in population distribution. In this talk, we present recent studies based on an integro-differential equation model incorporating spatially varying developmental delay, which introduces a nonlocal feedback mechanism into the population dynamics of individuals undergoing maturation. In this framework, the delay represents the time required for individuals to develop from one life stage to another, while its spatial variability reflects environmental heterogeneity across the habitat. Using tick populations as a case study, we demonstrate the high sensitivity of population persistence to the speed of environmental change. In particular, slow environmental changes may lead to population extinction, whereas rapid environmental changes can facilitate population survival by redistributing individuals across space. Furthermore, we show that for a prescribed spatial distribution pattern, it is always possible to construct a corresponding spatially varying developmental delay such that the resulting stable positive equilibrium exhibits the desired spatial mode.

## Nonlinear Effects in Linear Delay Equations and Linear Approach to Nonlinear Models with Delay Mortality

Elena Braverman

Department of Mathematics and Statistics, University of Calgary, Calgary, Alberta, Canada
maelena@ucalgary.ca

### Abstract

In the first part of the talk, we consider a scalar linear mixed differential equation with several terms, both delayed and advanced arguments and a bounded right-hand side. Assuming that the deviations of the argument are bounded, we present sufficient conditions when there exists a unique bounded solution on the positive half-line. Explicit tests are obtained when a bounded solution of a homogeneous equation decays exponentially. Existence of exponentially decaying solutions for this class of differential equations has not been studied before. We show that the standard approach when convergence of all solutions is stated does not work for mixed equations; in addition to an exponentially decaying, there may be a growing solution. All the coefficients and the mixed arguments are assumed to be Lebesgue measurable functions, not necessarily continuous. Though the equation is linear, some properties, as well as the methods applied, are more typical for nonlinear models, for example, fixed-point theorems used in the proofs.

In the second part, we explore existence of positive solutions, persistence, and boundedness of solutions for the Nicholson blowflies model with delayed mortality term $-\delta N(h(t))$. Two global stability tests for the positive equilibrium are obtained based on a linearized global stability method, reducing stability of a non-linear model to a specially constructed linear equation. The first test extends the absolute stability result to the case of delayed mortality, and the second one is delay-dependent. This is a joint work with Leonid Berezansky ( Ben-Gurion University of the Negev, Israel) and Sandra Pinelas (Academia Militar, Amadora, Portugal).

### References

[1] L. Berezansky, E. Braverman, Nicholson's blowflies differential equations with a small delay in the mortality term, *Nonlinear Analysis: Real World Applications* **81** (2025), Paper # 104193, 10 pp.

[2] L. Berezansky, E. Braverman, and S. Pinelas, Exponentially decaying solutions for models with delayed and advanced arguments: nonlinear effects in linear differential equations, *Proc. AMS* **151** (2023), 4261–4277.

## Spectral Characterization of the Constant Sign Derivatives of Green's Functions Related to Two-Point BVPs

Alberto Cabada[1, 2]

[1] CITMAga, 15782, Santiago de Compostela, Galicia, Spain.
[2] Department of Statistics, Mathematical Analysis and Optimization, Faculty of Mathematics, University of Santiago de Compostela, Spain.

ALBERTO.CABADA@USC.ES

### Abstract

This talk is devoted to the study of the following $n$-order linear differential equation

$$T_n[M]\,u(t) := u^{(n)}(t) + M\,u(t) = 0\,,\ \ t \in [a,b]\,, \tag{3}$$

where $M \in \mathbb{R}$ is a real parameter, coupled to the two-point boundary conditions

$$u^{(\sigma_1)}(a) = \quad \cdots \quad = u^{(\sigma_k)}(a) \quad = 0, \tag{4}$$

$$u^{(\varepsilon_1)}(b) = \quad \cdots \quad = u^{(\varepsilon_{n-k})}(b) = 0, \tag{5}$$

with $k \in \{1,\dots,n-1\}$ and the sets of indices $\{\sigma_1,\dots,\sigma_k\}$, $\{\varepsilon_1,\dots,\varepsilon_{n-k}\} \subset \{0,\dots,n-1\}$, satisfying that

$$0 \le \sigma_1 < \sigma_2 < \cdots < \sigma_k \le n-1\,, \quad 0 \le \varepsilon_1 < \varepsilon_2 < \cdots < \varepsilon_{n-k} \le n-1\,.$$

More concisely, it is characterized the set of parameters $M$ for which certain partial derivatives of the Green's function $g_M$, related to problem (3)–(5), are of constant sign, without relying on its explicit expression. It is proved that this set is an interval whose endpoints are characterized by the first eigenvalues of the operator $T_n[M]$ coupled to suitable boundary conditions. As a consequence, some sufficient conditions are provided ensuring that these derivatives cannot be nonpositive (or nonnegative). The obtained results are compiled in [1] and extend, for the particular case of operator $T_n[M]$, the ones proved in [2] for the constant sign of the Green's functions of the general $n$-th order linear differential operator, with non constant coefficients, and coupled to the boundary conditions (4)–(5).

### References

[1] Cabada, A., López-Somoza, L., Yousfi, M., Spectral Characterization of the Constant Sign Derivatives of Green's Function Related to Two Point Boundary Value Conditions. Qual. Theory Dyn. Syst. 24, 38 (2025), 1–57.

[2] Cabada, A., Saavedra, L., Characterization of constant sign Green's function for a two-point boundary-value problem by means of spectral theory. Electron. J. Differential Equations, 2017 (2017), 146, 1–95.

## Radial Solutions to Equation with Inhomogeneous Operator

Zuzana Došlá

Department of Mathematics and Statistics, Masaryk University, Brno, Czech Republic
DOSLA@MATH.MUNI.CZ

### Abstract

A nonlinear and non-autonomous second-order differential equation of the form

$$(a(t)\Phi(x'))' + b(t)F(x) = 0, \quad t \geq t_0,$$

is considered. Here, the function $\Phi(u) = \Phi_p(u)L(u)$ generalizes the classical $p$-Laplacian $\Phi_p(u) = |u|^{p-2}u$ for $p > 1$ and $L$ is an even, continuous and positive function that is slowly varying at zero in the sense of Karamata, and defined in a neighborhood of $u = 0$. Several applied examples of such $\Phi$ are presented and, in particular, a specific differential equation involving an inhomogeneous operator used to model material deformation under stress is given.

Moreover, the existence of global positive bounded solutions on the half-line with the Neumann type boundary conditions is studied by means of an abstract fixed point theorem and certain properties of an associated half-linear equation. The results do not require the explicit form of the inverse operator of $\Phi$, and are completed by an asymptotic analysis of these solutions near infinity.

This is a joint research with Mauro Marini and Serena Matucci, University of Florence, Italy.

### References

[1] Z. Došlá, M. Marini, S. Matucci, Global positive bounded solutions for equations with regularly varying operator, J. Math. Anal. Appl. (2026), https://doi.org/10.1016/j.jmaa.2026.130443

[2] Z. Došlá, M. Marini, S. Matucci, Zero-convergent solutions for equations with generalized relativistic operator: a fixed point approach, J. Nonlinear Convex. Anal. **8** (2025) 2583–2599.

[3] Z. Došlá, S. Matucci, Ground state solutions to nonlinear equations with p-Laplacian, Nonlinear Anal. **184** (2019) 1–16.

[4] M. Bocea, M. Mihailescu, On a family of inhomogeneous torsional creep problems, Proc. Amer. Math. Soc. **145** (2017), 4397–4409.

### Existence Results for Systems of First-Order Stieltjes Differential Equations

Marlène Frigon

University of Montreal, Montreal, Canada
MARLENE.FRIGON@UMONTREAL.CA

#### Abstract

In the last years, there has been quite an interest in the theory of Stieltjes differential equations which have the important advantage of providing a unified framework to differential equations, discrete equations, dynamic equations on time scales and differential equations with impulses at fixed times. They are particularly useful for modeling evolution processes in which sudden changes and stationary periods occur.

In this talk, we present existence results for systems of first-order Stieltjes differential equations with a periodic boundary condition or an initial value condition. No growth condition is imposed on the the right-hand side $f$. Our results rely on the method of $g$-solution-regions. Examples and an application are also presented.

## Maximum Principle for the Hill Equation: Eigenvalue Approach

Gabriela Holubová

University of West Bohemia in Pilsen, Pilsen, Czech Republic
gabriela@kma.zcu.cz

### Abstract

We consider the linear second-order periodic problem

$$-u'' + p(t)u = h(t), \quad u(0) = u(\omega),\ u'(0) = u'(\omega)$$

with a sign-changing coefficient $p$ and provide new possible answers to one of its related fundamental questions:

*For which $p$ does a nonnegative $h$ result in a nonnegative $u$?*

In other words, for which $p$ does the maximum principle hold. We can find several necessary and/or sufficient conditions on the coefficient $p$ in the extensive relevant literature. Optimality and especially applicability and verifiability of these conditions are crucial for further studies of related nonlinear problems.

Inspired by our previous results concerning the fourth-order problems, we state alternative series of (optimal and/or verifiable) conditions on $p$ based on the principal weighted eigenvalue of the corresponding linear operator. We also present particular examples and comparison with some so far known conditions.

### References

[1] G. Holubová: Optimal conditions for the maximum principle for second-order periodic problems, Electron. J. Diff. Eqns., Special Issue 02 (2023), pp. 151–160.

[2] G. Holubová, J. Janoušek: Extending the threshold values for inverse-positivity of a linear fourth order operator, Positivity 25 (2021), no. 5, 1721–1735.

[3] A. Lomtatidze: Theorems on differential inequalities and periodic boundary value problem for second-order ordinary differential equations, Mem. Differ. Equ. Math. Phys. 67 (2016), 1–129.

**Are Nonlinear First Order Caputo Fractional Differential Equations Solvable?**

Kunquan Lan

Toronto Metropolitan University 350 Victoria Street Toronto, Ontario Canada, M5B 2K3
KLAN@TORONTOMU.CA

**Abstract**

Consider the first order Caputo fractional differential equation (FDE)

$$(D^{1-\alpha}_{C,a^+}u)(x) := (I^{\alpha}_{a+}u')(x) = f(x, u(x)) \quad \text{for almost every } x \in [a, b],$$

where $\alpha \in (0, 1)$, $I^{\alpha}_{a+}$ is the Riemann-Liouville fractional integral, $u'$ is the traditional first-order derivative and $f : [a, b] \times [0, \infty) \to \mathbb{R}$ is a function. The Caputo FDE can be a single equation or a system of equations.

It was claimed in some previous papers that if $f$ satisfies the locally Lipschitz condition in the second variable, then the Caputo FDE has a unique solution. However, the result would be incorrect, see the open question below.

The above result has been widely used in the literature to obtain the existence and uniqueness of solutions of a variety of models such as disease models and predator-prey models published, for example in **Scientific Reports, PLoS One, Epidemics, Communications in Nonlinear Science and Numerical Simulation**. However, these previous results which applied the above claimed result to obtain the existence and uniqueness of solutions of the models would not be correct unless one can prove that the locally Lipschitz condition implies the necessary condition for the Caputo FDE to have solutions:

$$Fu \in I^{\alpha}_{a+}(L^1[a, b]) \quad \text{for all } u \in S,$$

where $S$ is a ball in $C_+[a, b]$ and $(Fu)(x) = f(x, u(x))$ for almost every $x \in [a, b]$.

**The open question** is under what conditions on the nonlinearity $f$, does the above necessary condition hold?

It is noted that if the nonlinearity $f$ satisfies the locally Lipschitz condition in the second variable or is infinitely differentiable, it is unknown whether $f$ satisfies the necessary condition.

## On the Existence of Positive Solutions to Boundary Value Problems with Riemann-Liouville Fractional Derivatives

Satoshi Tanaka

Tohoku University, Sendai, Japan
satoshi.tanaka.d4@tohoku.ac.jp

### Abstract

We investigate the existence, non-existence, uniqueness, and multiplicity of positive solutions to the following problem

$$\begin{cases} D^{\alpha}_{0+}u + h(t)f(u) = 0, \quad 0 < t < 1, \\ u(0) = u(1) = 0, \end{cases}$$

where $D^{\alpha}_{0+}$ is the Riemann-Liouville fractional derivative of order $\alpha \in (1, 2]$. Firstly, by characterizing the first eigenvalue $\lambda_1(\alpha)$ of the associated eigenvalue problem, we establish the existence of positive solutions for both sublinear and superlinear cases relative to $\lambda_1(\alpha)$, thereby extending previously known results. Secondly, we address the uniqueness of these solutions. In the sublinear case, we impose certain monotonicity conditions on $f$, while for the superlinear case, we assume a specific condition on $h$ to ensure uniqueness at $\alpha = 2$. For values of $\alpha$ near 2, uniqueness is established by leveraging the non-degeneracy of the unique solution. Finally, we apply this methodology to Hénon-type problems to demonstrate the existence of at least three positive solutions. This is a joint work with Inbo Sim (University of Ulsan).

### References

[1] I. Sim and S. Tanaka, Positive solutions for fractional-order boundary value problems with or without dependence of integer-order ones, *Fract. Calc. Appl. Anal.* **29** (2026), no. 1, 66–100.

## Multiple Solutions of Dirichlet Problems to Implicit Differential Equations and Inclusions in Billiard Spaces

Jan Tomeček

Palacký University in Olomouc, Czechia
JAN.TOMECEK@UPOL.CZ

### Abstract

This is a joint work with Jan Andres (Palacký University in Olomouc, Czechia). We investigate a Dirichlet problem for implicit differential inclusion of the second order with state-dependent impulses in the form

$$\begin{aligned}
&x'' \in f(t, x, x', x'') \quad \text{if } t \in [0,T],\ x \in \operatorname{int} K,\\
&x(s+) = \dot{x}(s) + I(x(s), \dot{x}(s)), \quad \text{if } x(s) \in \partial K,\\
&x(0) = A, \quad x(T) = B, \qquad A, B \in \operatorname{int} K,
\end{aligned}$$

where $K$ is an interval in $\mathbb{R}^n$, $f : [0,T] \times K \times \mathbb{R}^{2n} \multimap \mathbb{R}^n$ is the Marchaud mapping, the impulse function $I$ corresponds with the absolutely elastic impact of the ball at the boundary of the "billiard table" $K$, $T > 0$. We give existence and multiplicity result for solutions with prescribed number of impacts.

## Nonlocal Differential Problems in Abstract Spaces

Irene Benedetti

Department of Mathematics and Computer Science, University of Perugia, Perugia, Italy
IRENE.BENEDETTI@UNIPG.IT

### Abstract

This talk presents results on the existence and localization of solutions for nonlocal differential problems in abstract spaces. The differential equations under consideration involve a term governed by an $m$-dissipative maximal monotone operator, which may also be nonlinear. The proposed approach is based on fixed point theorems combined with so-called transversality conditions, offering a unifying framework for the study of diffusion models in various settings. This method covers both periodic and more general nonlocal initial conditions, such as multipoint or integral-type conditions, and can handle nonlinearities with superlinear growth, including cubic-type terms or nonlinearities depending on the integral of the solution, thus capturing behaviors characteristic of nonlocal diffusion phenomena. The talk is mainly based on the papers [1], [2], [3].

### References

[1] Benedetti I., Ciani S. (2022). Evolution equations with nonlocal initial conditions and superlinear growth. *Journal of Differential Equations*, 318, 270–297.

[2] Benedetti I., Loi N. V., Taddei V. (2017). Nonlocal diffusion second order partial differential equations. *Discrete and Continuous Dynamical Systems A*, 37(6), 2977–2998.

[3] Benedetti I., Malaguti L., Monteiro M.D.P. (2024). Differential equations with maximal monotone operators, *Journal of Mathematical Analysis and Applications*, 539 (1), 128484.

## On Impulsive Problems Driven by Second Order Differential Inclusions

Giulia Duricchi

University of Modena and Reggio Emilia
Department of Sciences and Methods for Engineering,
Reggio Emilia, Italy
G.DURICCHI@UNIMORE.IT

### Abstract

The talk will be about new results, obtained in [1], [2] in collaboration with Tiziana Cardinali (Univ. of Perugia) and Valentina Taddei (Univ. of Modena and Reggio Emilia) on the existence of impulsive mild solutions for a problem driven by the following non-autonomous semilinear second order differential inclusion

$$x''(t) \in A(t)x(t) + F(t, x(t), x'(t)), \tag{6}$$

where $\{A(t)\}_t$ is a suitable family of operators generating a fundamental system (see [3]).
To establish this goal, we first investigate the existence of mild solutions for a Cauchy problem governed by the same differential inclusion (6) with initial value belonging to a suitable space $E_\gamma$, $\gamma \geq 0$. We introduce this space, by extending a classical set presented by Travis and Webb in order to study autonomous problems. In this setting we obtain continuously differentiable mild solutions even in absence of compactness or strong regularity assumptions on the multivalued nonlinearity.
Then, the impulsive problem is treated by constructing a subspace $E \subset E_\gamma$, and by requiring an evolution property on the fundamental system generated by the family $\{A(t)\}_t$. This assumption plays a key role in ensuring the regularity of the solutions and in using the gluing method to achieve the existence of impulsive mild solutions.
As an application of our results we study an impulsive problem, subjected to a control action given by a multimap $U$, monitored by the following hyperbolic integro-differential equation

$$\begin{aligned}\frac{\partial^2 w}{\partial t^2} =& \frac{\partial^2 w}{\partial \xi^2} + b(t,\xi)\frac{\partial w}{\partial \xi} + \lambda(t,\xi)w \\ &+ p\left(t, \xi, \int_0^1 h(\xi,s)w(t,s)ds, \int_0^1 k(\xi,s)\frac{\partial w}{\partial t}(t,s)ds\right) + u(t,\xi),\end{aligned}$$

with $u(t,\xi) \in U(t,\xi)$.

### References

[1] Cardinali T., Duricchi G., Taddei V. *On second order nonautonomous semilinear differential inclusions with the nonlinear part depending on speed*, Submitted.

[2] Cardinali T., Duricchi G., Taddei V. *Modelling impulsive wave propagation over a finite/infinite time interval*, Submitted.

[3] Kozak M., *A fundamental solution of a second-order differential equation in Banach space*, Univ. Iagel. Acta Math., **32**, 275-289, 1995.

## Controllability of Semilinear Second-Order Differential Equations in Abstract Spaces

Martina Pavlačková

Dept. of Appl. Mathematics, Moravian Business College Olomouc,
tř. Kosmonautů 1288/1,77900 Olomouc, Czech Republic
pavlackovam@centrum.cz

### Abstract

This talk is based on joint work with Valentina Taddei (University of Modena and Reggio Emilia, Italy).

We study the controllability of semilinear second-order problems in abstract spaces in the case when the right-hand side depends on the first derivative. At first, the definition of controllability for second-order problems in Banach spaces that considers both the solution and its derivative at the final point using a unique control is introduced. Afterwards, sufficient conditions for such controllability are obtained. The results are achieved by combining the Schauder fixed point theorem with the approximation solvability method and weak topology. This combination enables getting the results under easily verifiable and non-restrictive conditions imposed on the cosine family generated by the linear operator and without any requirements for compactness of the right-hand side. The talk concludes by applying the obtained results to a system governed by the Klein-Gordon equation.

### References

[1] M. Pavlačková, V. Taddei, On a new concept of controllability of second-order semilinear differential equations in Banach spaces. *Mathematical Control and Related Fields* 15 (3) (2025), 1150-1173. https://doi.org/10.3934/mcrf.2025002

## Evolution Problems with Nonlocal Equations and Constraints: a Survey

Radu Precup

Babeş–Bolyai University and Tiberiu Popoviciu Institute of Romanian Academy,
Cluj-Napoca, Romania
r.precup@ictp.acad.ro

### Abstract

Nonlocal evolution problems arise naturally in the study of dynamical systems where the present state depends not only on instantaneous data but also on past or globally distributed information. Such nonlocal features may appear both in the evolution equation, through delay or history–dependent terms, and in the associated constraints, including multipoint, integral, or functional conditions. This survey presents an overview of abstract evolution problems involving various types of nonlocal equations and nonlocal constraints, with particular emphasis on the notion of the support of a nonlocal condition and its influence on solvability. Existence results are obtained by means of general fixed point techniques combined with the use of suitable equivalent Bielecki-type norms. For systems of equations, vectorial methods based on matrix estimates are employed, allowing componentwise information on the behavior of solutions. The presentation refers to some of the results obtained jointly with several collaborators, and suggests some perspectives for further developments.

### References

[1] A. Boucherif, R. Precup: *Semilinear evolution equations with nonlocal initial conditions.* Dynamic Systems Appl. 16, 2007

[2] T. Cardinali, P. Rubbioni: *A unified existence theory for evolution equations and systems under nonlocal conditions*, J. Math.Anal.Appl. 432, 2015

[3] R. Bunoiu, R. Precup: *Vectorial approach to coupled nonlinear Schrödinger systems under nonlocal Cauchy conditions.* Appl. Anal. 95, 2016

[4] O. Bolojan, R. Precup: *Hybrid delay evolution systems with nonlinear constraints.* Dynamic Systems Appl. 27, 2018

[5] I. Benedetti, T. Cardinali, R. Precup: *Fixed point–critical point hybrid theorems and application to systems with partial variational structure.* J. Fixed Point Theory Appl. 23, 2021

[6] D. Brumar, R. Precup: *Kirchhoff type parabolic equations with nonlocal in space and time diffusion coefficients.* Rend. Circ. Mat. Palermo II 75, 2026

[7] R. Precup, A. Stan: *On equilibrium in control problems with applications to evolution systems.* Submitted

### Quasi-Periodic Breathers for Klein-Gordon Chains and Discrete Nonlinear Schrödinger Equations with Quasi-Periodic Potential

Chuanfang Ge

Nanjing University of Information Science and Technology, Nanjing, PRC
003376@nuist.edu.cn

#### Abstract

We study whiskered quasi-periodic breathers for nonlinear Klein-Gordon chains with tangent potential or Maryland potential. Remarkably, nonlinear Klein–Gordon chains not only lose gauge invariance of the perturbation, but also they have infinite normally-hyperbolic directions. Using orthogonal transformation technique of linear operator and Kolmogorov-Arnold-Moser(KAM) iteration mechanism together with decay property of perturbation, we obtain the whiskered quasi-periodic breathers with positive measurable subset of spatial parameter. We also investigate multi-dimensional, discrete nonlinear Schrödinger equations, which carry quasi-periodic, analytic potentials(non-constant) with general form. By using KAM approach, the nonlinear Schrödinger equations with small perturbations admit the existence of small-amplitude quasi-periodic solutions.

## Quasiperiodic Motion in a Planar Sun–Jupiter–Saturn System

Àlex Haro

Universitat de Barcelona (UB) and Centre de Recerca Matemàtica (CRM), Barcelona, Spain
ALEX.HARO@UB.EDU

### Abstract

In this talk, we present evidence for the stability of a simplified model of the Solar System: a planar (Newtonian) Sun–Jupiter–Saturn system with realistic parameters, namely the planetary masses, semi-major axes, eccentricities, and precession rates close to the observed ones. The evidence is based on numerical computations indicating that a KAM theorem applies to the Hamiltonian equations of the model, leading to quasiperiodic motion on an invariant torus.

This is joint work with Jordi-Lluís Figueras (Uppsala University).

### References

[1] J.-L. Figueras and À. Haro, A modified parameterization method for invariant Lagrangian tori for partially integrable Hamiltonian systems, Phys. D **462** (2024), Paper No. 134127, 21 pp.

[2] J.-L. Figueras and À. Haro, Sun-Jupiter-Saturn system may exist: a verified computation of quasiperiodic solutions for the planar three-body problem, J. Nonlinear Sci. **35** (2025), no. 1, Paper No. 13, 20 pp.

# Ejection-Collision Solutions and KAM Tori in Restricted $N$-Body Problems

Jesús F. Palacián

Departamento de Estadística, Informática y Matemáticas, Universidad Pública de Navarra, Pamplona, Spain
palacian@unavarra.es

## Abstract

We study ejection-collision solutions in several cases of the $N$-body problem, starting with the restricted three-body problem in the planar case. In these solutions the particle of negligible mass passes several times near one of the primaries before colliding with it. In the so-called lunar regime, the Hamiltonian is a perturbation of the Kepler problem. In a first step, applying the Han-Li-Yi theorem, we prove the persistence of KAM tori filled in with quasi-periodic motions of the small particle. This is achieved by using some techniques of Hamiltonian dynamics including regularisation of collision orbits, normalisations and symplectic reductions. Second, from the rectilinear quasi-periodic motions we identify those corresponding to the ejection-collision solutions. The analysis of these solutions in the planar case of the restricted three-body problem has been performed since the 1980s and more recently by M. Ollé and collaborators from both an analytical and a numerical point of view. Our approach is generalised to the planar restricted $N$-body problem and also to the spatial case of the restricted three-body problem. In the restricted $N$-body problem normally four families are found, but under some extra symmetries and degeneracies related to the configurations of the primaries, more families can be obtained. In the spatial restricted three-body problem, in addition to the four families found in the planar case, two new families of ejection-collision orbits are identified, that correspond to vertical motions of the negligible particle. The analysis of the spatial restricted three-body problem has been performed mainly numerically, hence the interest in applying the techniques mentioned for the search for analytical results.

This is a joint work with Aitor Ayape, Xabier Larequi and Patricia Yanguas.

### References

[1] A note on the existence of invariant punctured tori in the planar circular restricted three-body problem. A. Chenciner, J. Llibre. Ergodic Theory & Dynamical Systems (1988) 8*, 63–72.

[2] Generalized analytical results on $n$-ejection-collision orbits in the RTBP. Analysis of bifurcations. T. M-Seara, M. Ollé, Ó. Rodríguez, J. Soler, *Journal of Nonlinear Science* (2023) 33:17.

[3] Ejection-collision solutions in the planar restricted $N$-body problems. A. Ayape, J.F. Palacián, P. Yanguas, *in preparation* (2026).

## Quasi-Periodic Solutions to the Nonlinear Klein-Gordon Equations in Two Dimensions

Wei-Min Wang

CNRS, Paris
WEI-MIN.WANG@MATH.CNRS.FR

### Abstract

We construct time quasi-periodic solutions to the nonlinear Klein-Gordon equation with polynomial nonlinear terms on the two dimensional torus:

$$\frac{\partial^2 u}{\partial t^2} - \Delta u + u + u^{p+1} = 0.$$

To our knowledge, this is the first such result for the Klein-Gordon equation above dimension one. The analysis explores in an essential way the submodule structure on a dual Fourier lattice. We shall discuss the main ideas of the approach.

### Nekhoroshev Estimates for Multi-Scale Hamiltonian Systems

Lu Xu

Jilin University, Changchun, China
xulu@jlu.edu.cn

#### Abstract

Motivated by applications arising in celestial mechanics, we give some Nekhoroshev stability estimates for multi-scale nearly-integrable Hamiltonian systems. Particular attention is paid to two special cases: (I) The integrable part at each scale is $(\alpha, K)$ completely non-resonant; and (II) the resonance only occurs in the highest scale of the integrable part. By considering non-resonance and obtaining normal forms via a finite number of steps of an averaging process, not only do we obtain the Nekhoroshev estimates for both cases, but also we reveal some impacts of multi-scales on the nature of stability. In particular, the estimates of the stability time are determined by the smallness of the perturbation in the normal form, while the stability radius is also influenced by the resonant gap in the second case. Moreover, in the non-resonant case, the estimates of each component are relatively independent, while in the resonant case, the stability time is determined by both non-resonant and resonant terms. Some examples of multi-scale Hamiltonian systems arising as perturbations of the Kepler problem are given to illustrate the application of our results.

## Co-Orbital Solutions in the Planar Three-Body Problem

Patricia Yanguas

Departamento de Estadística, Informática y Matemáticas, Universidad Pública de Navarra, Pamplona, Spain
yanguas@unavarra.es

### Abstract

The purpose of this work is to provide a qualitative explanation of the co-orbital motion of two small moons orbiting a common planet.

The two moons revolve around the planet along nearly circular trajectories with almost equal radii. The system is modelled as a planar three-body problem, whose Hamiltonian function is developed as a perturbation of two decoupled Kepler Hamiltonians. Techniques from averaging theory are combined with normal forms, symplectic scalings, and Hamiltonian reduction, together with the application of a suitable KAM theorem for highly degenerate cases.

The goal is to establish the persistence of 4-dimensional KAM tori and quasi-periodic solutions associated with the co-orbital motion of the moons. A region of the reduced phase space (which is the Cartesian product of a two-sphere and a two-sheeted hyperboloid of revolution) must be selected, and within this portion of phase space the Hamiltonian is expressed in suitable action-angle coordinates at successive orders of perturbation, up to fourth order, in order to apply KAM theory. One of the action variables is related to the angular momentum and can take any value, thereby extending previous work by the same authors.

Finally, applications to the motion of Janus and Epimetheus around Saturn are discussed, as well as the generalisation to the spatial three-body problem.

This work is a collaboration with Josép María Cors and Jesús F. Palacián.

### References

[1] On co-orbital quasi-periodic motion in the three-body problem. J.M. Cors, J.F. Palacián, P. Yanguas, *SIAM Journal on Applied Dynamical Systems* (2019) 18, 334–353.

### Quasi-Periodic Breathers in Newton's Cradle

Yingfei Yi

University of Alberta & Jilin University
YINGFEI@UALBERTA.CA

#### Abstract

Quasi-periodic breathers are important coherent structures in infinite-dimensional Hamiltonian systems that are spatially localized and temporally quasi-periodic. In the real analytic case, they are well understood, but the finitely smooth case, including granular chains and Newton's cradles with Hertzian contact potentials, had been long-open. This talk will present a recent result on the existence of quasi-periodic breathers in a broad class of finitely smooth Hamiltonian lattices and networks. To overcome obstacles in applying KAM method due to the finite smoothness of potentials and to obtain a sharp estimate of the localization rate of the quasi-periodic breathers, the proof of the result uses the Jackson-Moser-Zehnder analytic approximation technique but with refined estimates on error bounds, depending on the smoothness and dimension, which provide crucial controls on the convergence of KAM iterations.

### Spontaneous Particle Aggregation with Memory

Jan Haskovec

King Abdullah University of Science and Technology, 23955 Thuwal, KSA
JAN.HASKOVEC@KAUST.EDU.SA

**Abstract**

We present a nonlinear and nonlocal model of spontaneous biological aggregation. On the microscopic scale it is represented as an agent-based stochastic model where each individual modulates its random movement based on the perceived local density of its neighbours. Memory is introduced via a chain of internal variables, allowing agents to retain past environmental information. With appropriate parameter settings the model exhibits emergent formation of particle clusters. We present results of systematic stochastic simulations, showing that short-term memory promotes cluster coarsening, while long-term memory disrupts aggregation, increasing the number of outliers and instances with no clustering. Statistical analysis shows that memory inhibits the particles' responsivity to environmental cues, specifically the perceived density of their neighbours, explaining the reduced clustering tendency at higher values of the memory length.To gain deeper insights into the formation and shape of particle clusters, we derive the Fokker-Planck equation in the macroscopic limit, characterize its steady states and provide results of numerical simulations.

#### References

[1] R. Erban, J. Haskovec: *Impact of memory on clustering in spontaneous particle aggregation.* SIAM J. Life Sciences, accepted (2026). Preprint arxiv.org/abs/2510.15335.

## Dynamics of a Piecewise Smooth Ghil-Zaliapin-Thompson ENSO Model

Tony Humphries

McGill University, Montreal, Canada
tony.humphries@mcgill.ca

### Abstract

The Ghil–Zaliapin–Thompson (GZT) model, a scalar delay differential equation with periodic forcing and time-delayed feedback, captures key features of the El Niño–Southern Oscillation (ENSO) phenomenon. Numerical studies have revealed parameter regimes of very complex dynamics, but its analytical treatment remains challenging. To bridge this gap, we propose [1,2] a piecewise smooth version of the GZT model

$$h'(t) = -\text{sign}(h(t-\tau)) + c\cos(2\pi t),$$

with piecewise constant delayed feedback and continuous periodic forcing.

For this piecewise smooth GZT model we explicitly construct $n{:}s$-periodic orbits of odd integer period $n$ with $s$ upward crossings of zero, and derive existence and stability conditions for them, and identify the bifurcations that delimit their regions of existence in the $(\tau, c)$-parameter plane.

For $c \gg 0$ we show that there is a unique stable 1:1-periodic orbit (seasonal forcing dominates), which loses stability along a bifurcation curve that we identify explicitly. For most values of $\tau$ this occurs in a torus bifurcation. For small $c$ the dynamics is altogether more interesting with overlapping resonances tongues composed of $n{:}s$ and $n{:}(s+2)$-periodic orbits with $n$ and $s$ odd and $s \leq (n-1)/2$. All of the period $n$ orbits that we study satisfy $h(t) = -h(t+n/2)$, and this symmetry is central to our constructions and analysis.

This is joint work with Sam Bolduc-St-Aubin of the University of Auckland, New Zealand.

### References

[1] S. Bolduc-St-Aubin and A.R. Humphries Seasonal Forcing Dominated Dynamics of a piecewise smooth Ghil-Zaliapin-Thompson ENSO model. arXiv:2510.27084 [math.DS] doi: 10.48550/arXiv.2510.27084 (2025)

[2] S. Bolduc-St-Aubin and A.R. Humphries Resonant Dynamics of a piecewise smooth Ghil-Zaliapin-Thompson ENSO model. In preparation (2026)

## Complex Bifurcations Generated by Waning and Boosting of Immunity in Infectious Disease Dynamics

Gergely Röst

HCEMM / University of Szeged, Hungary
rost@math.u-szeged.hu

### Abstract

We investigate a class of epidemiological models that incorporates waning of immunity at the individual level and boosting of the immune system upon re-exposure to the pathogen. When immunity is fully restored upon boosting, the system can be expressed as an SIRS-type model with discrete and distributed delays. We conduct a numerical bifurcation analysis varying the boosting force and the maximum period of immunity (in the absence of boosting), while keeping other parameters fixed at values representative of a highly infectious disease like pertussis. The stability switches of the endemic equilibrium, identified numerically, are validated using an established analytical approach, confirming that the equilibrium is unstable in a bounded parameter region, and stable outside this region. Using recently developed continuation methods for models with discrete and distributed delays, we explore periodic solutions and their bifurcations. Our analysis significantly extends previous findings and reveals a rich dynamical landscape, including catastrophic bifurcations of limit cycles, torus bifurcations, and bistability regions where two stable periodic solutions coexist, differing in amplitude and period. These complex bifurcations have critical public health implications: perturbations—such as non-pharmaceutical interventions—can shift the system between attractors, leading to long-term consequences on infectious disease dynamics from short-term measures.

Joint work with Francesca Scarabel, Mónika Polner, Daniel Wylde, Maria Vittoria Barbarossa.

### References

[1] Scarabel, F., Polner, M., Wylde, D., Barbarossa M.V., Röst G., Bistability and complex bifurcation diagrams generated by waning and boosting of immunity. J. Math. Biol. 91, 30 (2025). https://doi.org/10.1007/s00285-025-02264-3

[2] Barbarossa MV, Polner M, Röst G (2017) Stability switches induced by immune system boosting in an SIRS model with discrete and distributed delays. SIAM J Appl Math 77(3):905–923

[3] Barbarossa MV, Polner M, Röst G (2018) Temporal evolution of immunity distributions in a population with waning and boosting. Complexity 2018(1):9264743

[4] Barbarossa MV, Röst G (2015) Immuno-epidemiology of a population structured by immune status: a mathematical study of waning immunity and immune system boosting. J Math Biol 71:1737–1770

## Homogeneous Boltzmann-Type Equations on Graphs

Andrea Tosin

Politecnico di Torino, Torino, Italy
ANDREA.TOSIN@POLITO.IT

### Abstract

In this talk, we present recent results on homogeneous (i.e., space-independent) Boltzmann-type collisional kinetic equations for interacting multi-agent systems on graphs. The graph structure encodes preferential interaction patterns among agents, departing from the classical "all-to-all" interaction framework.

We focus, in particular, on mathematically consistent approaches for embedding graph structures into homogeneous Boltzmann-type equations, considering both finite and infinite graphs, as well as their limiting graphons. Relevant motivating applications include kinetic models for the spread of infectious diseases and for opinion formation dynamics.

### References

[1] M. Fraia, N. Loy, A. Tosin. Linear inelastic kinetic equations modelling the spread of fake news and its interplay with personal awareness, *Kinet. Relat. Models*, 19:1-34, 2026

[2] G. Taricco, A. Tosin. Homogeneous Boltzmann-type equations and graphons. In preparation

[3] N. Loy, A. Tosin. Essentials of the kinetic theory of multi-agent systems, 2025. Preprint (doi:10.13140/RG.2.2.22407.30882)

[4] M. Nurisso, M. Raviola, A. Tosin. Network-based kinetic models: Emergence of a statistical description of the graph topology, *European J. Appl. Math.*, 2024. doi:10.1017/S0956792524000020

[5] N. Loy, A. Tosin. A viral load-based model for epidemic spread on spatial networks, *Math. Biosci. Eng.*, 18(5):5635-5663, 2021

## Period Two Implies Chaos for a Class of ODEs Revisited

Jan Andres

Palacký University, Olomouc, Czech Republic
JAN.ANDRES@UPOL.CZ

### Abstract

It is well known that neither the celebrated Sharkovsky cycle coexistence theorem, nor Li-Yorke's "Period three implies chaos" can be applied in their standard forms to differential equations. On the other hand, Obersnel and Omari (cf. [3]) formulated, in reaction to our earlier results, the Sharkovsky-type theorem for the scalar ODEs, but without Sharkovsky's ordering and no exceptional cases. Nevertheless, despite a rather provocative title of [3], there is no related information about chaos at all.

The main aim of our talk is therefore to deduce criteria of chaos in terms of the associated multivalued Poincaré translation operators along the trajectories of given scalar ordinary differential equations and inclusions. Besides other things, we will show that such criteria essentially depend on the definitions of various sorts of chaos. For instance, topological entropy can be at the same time infinite in the sense of one definition and zero in the sense of another.

As far as we know, some particular results along these lines were obtained just by Pireddu (cf. [4]) in terms of the Bebutov flows, in [1] via single-valued chaotic selections and more recently in [2] for hidden orbits of discontinuous maps.

### References

[1] Andres, Jan. Period two implies chaos for a class of multivalued maps: a naive approach. *Comput. Math. Appl.* **64** (2012), no. 7, 2160–2165.

[2] Jeffrey, Mike R.; Avrutin, Viktor. Periods 1 + 2 imply chaos in steep or nonsmooth maps. *Nonlinearity* **35** (2022), no. 12, 6014–6041.

[3] Obersnel, Franco; Omari, Pierpaolo. Period two implies chaos for a class of ODEs. *Proc. Amer. Math. Soc.* **135** (2007), no. 7, 2055–2058.

[4] Pireddu, Marina. Period two implies chaos for a class of ODEs: a dynamical system approach. *Rend. Istit. Mat. Univ. Trieste* **41** (2009), 43–54 (2010).

## Stability of Linear Systems and Two-Term Equations with Prabhakar Derivatives: Spectral Criteria and Computations

Eva Kaslik

West University of Timisoara, Romania
eva.kaslik@e-uvt.ro

**Abstract**

We investigate stability and instability properties for (i) linear fractional systems and (ii) two-term linear fractional differential equations incorporating Prabhakar-type derivatives. Using Laplace-transform techniques, we derive the corresponding characteristic equations, connecting asymptotic stability properties to the location of characteristic roots in suitable complex sectors. The analysis distinguishes stability properties that depend on the fractional orders from those that are robust with respect to order variations, highlighting how the Prabhakar parameters deform classical stability wedges known from Caputo-type counterparts.

Numerical investigations are also presented to complement the theoretical results: we compute stability maps and root/eigenvalue loci across fractional orders and Prabhakar parameters and validate predicted regimes through time-domain simulations. Our aim is to provide unified analytical and computational tools for the stability analysis of Prabhakar-type fractional-order systems.

### References

[1] R. Garrappa, E. Kaslik, Stability of fractional-order systems with Prab- hakar derivatives, Nonlinear Dynamics 102 (1) (2020) 567–578.

[2] O. Brandibur, A. Jhinga, E. Kaslik, Exploring stability and instability criteria in two-term fractional differential equations involving Prabhakar derivatives, submitted.

## Fractional Initial and Boundary Value Problems: from Qualitative Analysis to Applications

Kateryna Marynets

Delft Institute of Applied Mathematics, Delft University of Technology, The Netherlands
K.Marynets@tudelft.nl

### Abstract

In recent years fractional initial and boundary value problems have attracted much attention in the field of mathematical modeling due to the ability of fractional operators to capture non-local and memory effects. Such effects are relevant in many physical processes, thus making fractional differential equations (FDEs) a suitable modeling tool. Despite of the mentioned advantages of fractional operators, the inherent nonlocality of fractional calculus introduces several difficulties. Closed-form solutions to FDEs – particularly those encountered in engineering and applied sciences – are rarely available, especially when nonlinearities are involved. Traditional analytical methods tend to have limited applicability or require substantial adaptation to handle fractional formulations. In addition, the computational cost of numerical techniques for FDEs is typically higher than for classical integer-order models.

In my talk I will explain how a suitable parametrization technique and construction of functional sequences can not only ensure existence and uniqueness of solutions to the studied FDEs, but also enables approximation of these solutions with high precision. I will demonstrate efficiency of the presented approach on concrete examples of fractional initial and boundary value problems arising from applied sciences. The presented results will be based on the recent papers with Dona Pantova [1] and Niels Goedegebure [2].

### References

[1] K. Marynets, D. Pantova, Fractional boundary value problems with parameter-dependent and asymptotic conditions, *Fractal and Fractional* (2025), 9 (7), 462; doi: 10.3390/fractalfract9070462.

[2] N. Goedegebure, K. Marynets, A numerical Bernstein splines approach for nonlinear initial value problems with Hilfer fractional derivative, *preprint* (2025), doi: 10.48550/arXiv.2503.22335.

## Asymptotic Stability of Linear Differential Equations with Discrete and Distributed Delays

Hideaki Matsunaga

Osaka Metropolitan University, Osaka, Japan
hideaki.matsunaga@omu.ac.jp

**Abstract**

This study summarizes recent results on the asymptotic stability of linear differential equations with discrete and distributed delays. It is important to clarify the stability dependence on parameters of delay autonomous equations. Stability results are classified into two types: results on stability regions in the parameter space of the coefficients under fixed delays, and results on delay-dependent stability conditions under fixed coefficients. The aim of this study is to derive the latter stability conditions.

### References

[1] C. Huang and S. Vandewalle, An analysis of delay-dependent stability for ordinary and partial differential equations with fixed and distributed delays, *SIAM J. SCI. Comput.* **25** (2004), no. 5, 1608–1632.

[2] M. Qu and H. Matsunaga, Exact stability criteria for linear differential equations with discrete and distributed delays, *J. Math. Anal. Appl.* **539** (2024), no. 2, Paper No. 128663, 15 pp.

## From Fractional Differential to Difference Equations: A Stability Perspective

Luděk Nechvátal

Brno University of Technology, Brno, Czechia
NECHVATAL@FME.VUTBR.CZ

### Abstract

The fractional calculus has become increasingly popular in modelling dynamical systems with memory and hereditary properties. However, despite achieving many important results, there are still challenges in the fundamental qualitative analysis of important classes of differential or difference equations. Those are mainly due to the non-locality of the fractional operator (which depends on the initial point). Consequently, autonomous equations in fact do not behave like autonomous (the group property of the flow $\phi_{s+t} = \phi_s \circ \phi_t$ is not valid any more).

Here, we present several results on the stability and asymptotic properties of some classes of fractional differential equations, with a focus on their discretisation.

### References

[1] Čermák, J., Nechvátal, L., On a problem of linearized stability for fractional difference equations, Nonlinear Dyn. 104 (2021), 1253–1267.

[2] Čermák, J., Kisela, T., Nechvátal, L., The Lambert function method in qualitative analysis of fractional delay differential equations, Fract. Calc. Appl. Anal. 26 (2023), 1545–1565.

### Asymptotic Behavior and Stability of Nonautonomous Linear Differential Equations with Kirchhoff Coefficients

Mihály Pituk

University of Pannonia, Veszprém, Hungary
PITUK.MIHALY@MIK.UNI-PANNON.HU

#### Abstract

In this talk, we consider nonautonomous linear ordinary differential equations with Kirchhoff coefficients. We give sufficient conditions for the global asymptotic stability of the solutions. Under appropriate additional assumptions, we give an asymptotic description of the solutions in terms of the Perron vectors of the coefficient matrices. This is a joint work with Professors Josef Diblík (Brno University of Technology, Czech Republic) and Gábor Szederkényi (Pázmány Péter Catholic University and HUN–REN Institute for Computer Science and Control, Hungary).

## Tempered Ψ-Fractional Calculus: Integral Inequalities and Their Applications

Michal Pospíšil

Faculty of Mathematics, Physics and Informatics, Comenius University in Bratislava, Bratislava, Slovakia
Mathematical Institute, Slovak Academy of Sciences, Bratislava, Slovakia
MICHAL.POSPISIL@FMPH.UNIBA.SK

### Abstract

The notion of the tempered Ψ-Hilfer fractional integral was recently introduced to establish a connection between the tempered Riemann–Liouville fractional integral and Ψ-Riemann–Liouville fractional integral. By modifying the classic integral inequalities using a desingularization method for integrals with weakly singular kernels, new nonlinear integral inequalities with the tempered Ψ-Hilfer fractional integrals are derived. Results for differential equations involving the tempered Ψ-Caputo fractional derivative, such as Ψ-exponential stability, nonexistence of blow-up solutions, and the existence of a unique solution, are derived with the aid of these inequalities.

### References

[1] H. M. Fahad, A. Fernandez, M. ur Rehman, M. Siddiqi, Tempered and Hadamard-type fractional calculus with respect to functions, *Mediterr. J. Math.* **18**(2021), No. 4, Art. No. 143, 28 pages.

[2] M. Medveď, E. Brestovanská, Differential equations with tempered Ψ-Caputo fractional derivative, *Math. Model. Anal.* **26**(2021), No. 4, 631–650.

[3] M. Medveď, M. Pospíšil, E. Brestovanská, Nonlinear integral inequalities involving tempered Ψ-Hilfer fractional integral and fractional equations with tempered Ψ-Caputo fractional derivative, *Fractal Fract.* **7**(2023), No. 8, Art. No. 611, 1–17.

[4] M. Medveď, M. Pospíšil, E. Brestovanská, A new nonlinear integral inequality with tempered Ψ-Hilfer fractional integral and its application to a class of tempered Ψ-Caputo fractional equations, *Axioms* **13**(2024), No. 5, Art. No. 301, 16 pages.

[5] M. Medveď, M. Pospíšil, E. Brestovanská, Nonlinear integral inequalities involving Ψ-Hilfer fractional integrals and iterated fractional integrals, with applications to Ψ-Caputo fractional differential equations, *Electron. J. Qual. Theory Differ. Equ.*, (2025), No. 30, 1–24.

[6] M. Pospíšil, L. Pospíšilová Škripková, Existence results for differential equations with tempered Ψ-Caputo fractional derivatives, *Axioms* **13**(2024), No. 10, Art. No. 680, 16 pages.

## Fractional Integration and Differentiation of Asymptotic Relations and Applications in Fractional Differential Equations

Pavel Řehák

Brno University of Technology, Brno, Czech Republic
rehak.pavel@fme.vutbr.cz

**Abstract**

In the first part of the talk, we will discuss how asymptotic relations are preserved under integration and differentiation in the sense of fractional operators. We will examine fractional analogues/extensions of Karamata's integration theorem, the monotone density theorem, the generalized L'Hospital rule, and related results. The second part will be devoted to applications of these techniques in the asymptotic theory of fractional differential equations. Special attention will be devoted to phenomena that have no integer analogue.

### References

[1] P. Řehák, Fractional integration and differentiation of asymptotic relations and applications, Math. Methods Appl. Sci. 48 (2025), 6381–6395.

[2] P. Řehák, Regular variation and fractional operators, Fract. Calc. Appl. Anal. (2026).

[3] S. Matucci, P. Řehák, Long-time behavior of solutions to a nonlinear fractional differential system, submitted.

## Differentiability of the Solution Operator of a Sweeping Process

Martin Brokate

TU München, WIAS Berlin
BROKATE@MA.TUM.DE

### Abstract

We show that the solution operator for a sweeping process (equivalently, the stop operator of hysteresis, or a certain energetic system) possesses a directional derivative if the convex constraint has a smooth boundary or is a polyhedron of a certain type. The derivative satisfied a variational system where the Kurzweil integral crucially enters. We indicate applications to optimal control.

### References

[1] M. Brokate, C. Christof: Directional differentiability for solution operators of sweeping processes with smooth convex admissible sets, manuscript in preparation.

[2] M. Brokate, C. Christof: Directional differentiability for solution operators of sweeping processes with convex polyhedral admissible sets, submitted.

[3] M. Brokate, C. Christof: Strong stationarity conditions for optimal control problems governed by a rate-independent evolution variational inequality, SIAM J. Control Optim. 61 (2023), 2222–2250.

## Koopman Semigroups on Lebesgue Spaces

Pedro J. Miana

Universidad de Zaragoza, Spain
pjmiana@unizar.es

**Abstract**

Composition operators in dynamic systems are often referred to as Koopman operators, in honor of the French-American mathematician Bernard Osgood Koopman (1900–1981). In this talk, we present a brief introduction to Koopman semigroups. We then introduce three examples of weighted Koopman semigroups defined on fractional Lebesgue-Sobolev spaces on the real ray.

We are also interested in connecting Koopman semigroups in Lebesgue spaces of functions $L^p(\mathbb{R}^+)$ and semigroups in Lebesgue spaces of sequences $\ell^p$ for $1 \leq p < \infty$. To achieve this, we use a certain Poisson transformation $\mathcal{P} : L^p(\mathbb{R}^+) \to \ell^p$ and its adjoint $\mathcal{P}^*$, which allows us to transfer the semigroup properties from one space to another. Two Koopman semigroups in $\ell^p$ are presented and related to the canonical Koopman semigroup in $L^p(\mathbb{R}^+)$.

In the last part of the talk, we introduce operators that extend Cesáro operators (called Chen-type integral operators) subordinate to these Koopman semigroups in $L^p(\mathbb{R}^+)$ and $\ell^p$.

The first results of this article are joint work with Verónica Poblete from the University of Chile and are published in Monatshefte für Mathematik, 206, (2025). The second part of the talk contains results from a preprint available on the Arxiv platform.

### References

[1] P. J. Miana and V. Poblete, *Three weight Koopman semigroups on Lebesgue space,* Monatsh. fur Math. (2025) https://doi.org/10.1007/s00605-024-02041-2

[2] P. J. Miana, *Two Koopman semigroups on discrete Lebesgue spaces.* ArXiv:2511.13868 (2025).

## The Kurzweil Integral in Hysteresis Modeling

Giselle Antunes Monteiro

Institute of Mathematics of the Czech Academy of Sciences, Prague, Czechia
GAM@MATH.CAS.CZ

### Abstract

Many systems in science and engineering show hysteresis – a memory effect in which the response of a system depends not only on its current input but also on its history. The fact that certain memory effects is independent of the speed naturally leads to an extension of hysteresis operators to processes with jump discontinuities. Notably, the distinctive features of the theory of integration due to J. Kurzweil make it particularly well suited for studying such discontinuous processes. In this talk we discuss how the use of the Kurzweil integral has proven valuable in modeling hysteresis.

*Acknowledgments:* This work was supported by GACR project GA24-10586S.

## The Kernel of the Stieltjes Derivative in the Analysis of Cauchy Problems

F. Adrián F. Tojo

CITMAGA & Universidade de Santiago de Compostela,
Santiago de Compostela, Spain
FERNANDOADRIAN.FERNANDEZ@USC.ES

### Abstract

In this talk, we investigate the existence and uniqueness of solutions to first-order Stieltjes differential problems, with particular emphasis on the structural role of the Stieltjes derivative and its kernel. We provide a complete characterization of this kernel by establishing necessary and sufficient conditions for a function to possess a vanishing Stieltjes derivative.

A key feature distinguishing the Stieltjes setting from the classical framework is the nontriviality of the kernel of the associated differential operator. As a consequence, solutions to first-order Stieltjes differential equations are, in general, not unique. To properly address this phenomenon, we introduce a suitable functional space that allows us to recover a well-posed analytical framework for these problems. This approach highlights the structural differences with respect to classical differential equations and provides the appropriate tools to handle the intrinsic lack of uniqueness.

### References

[1] Fernández, F. J., Márquez Albés, I., Tojo, F. A. F., & Villanueva Mariz, C. (2025). *On the kernel of the Stieltjes derivative and the space of bounded Stieltjes-differentiable functions*. Electronic Journal of Qualitative Theory of Differential Equations, 2025, 1-41.

## Exponential Stability of Second Order Delay Differential Equations through Floquet Theory with Application to Angular Stabilization of Drone Flight

Alexander Domoshnitsky

Department of Mathematics, Ariel University, Ariel-40700, Israel
ADOM@ARIEL.AC.IL

**Abstract**

In this talk, we obtain results on exponential stability of second order delay differential equations, which are based on a version of the Floquet theory for delay differential equations of the second order we proposed. Our version allows researchers to preserve the order of equation and to obtain analogues of the classical results of the Floquet theory known for ordinary differential equations. On the basis of our version of the Floquet theory, new original unexpected results on the exponential stability are proposed.

Our results do not assume smallness of the product of delay and gain. We demonstrate that choosing period of coefficients and delays of the gain in corresponding intervals allows to achieve the exponential stabilization in the cases considered as impossible when the standard technique was applied. Then we use obtained stability tests in a model of angular stabilization of drone's flight.

Results of the talk were obtained in joint paper with O.Kupervasser, S.Malev and T.Shavit.

*Key Words and Phrases*: Delay Equations, Floquet theory, Exponential stability without assumption on smallness the product of gain and delay

## Generalized Oscillation Theory for Canonical Differential Systems on Hybrid Time Domains

Roman Šimon Hilscher

Masaryk University, Brno, Czechia
HILSCHER@MATH.MUNI.CZ

### Abstract

We will present the development of the notion of a generalized focal point and its multiplicity for matrix-valued solutions of canonical differential systems on continuous, discrete, and arbitrary hybrid time domains (time scales). Such a notion leads to proper Sturm separation theorems, both in the local and global sense. The main results were obtained in a joint work with Peter Šepitka (Masaryk University) and Vera Zeidan (Michigan State University).

### References

[1] W. Kratz, Discrete oscillation, J. Difference Equ. Appl. 9 (2003), no. 1, 135–147.

[2] W. Kratz, Definiteness of quadratic functionals, Analysis (Munich) 23 (2003), no. 2, 163–183.

[3] P. Šepitka, R. Šimon Hilscher, Generalized focal points and local Sturmian theory for linear Hamiltonian systems, Discrete Contin. Dyn. Syst. 43 (2023), no. 12, 4139–4173.

[4] P. Šepitka, R. Šimon Hilscher, Contributions to generalized oscillation theory of linear Hamiltonian systems, Results Math. 79 (2024), no. 8, Article No. 263, 47 pp.

[5] P. Šepitka, R. Šimon Hilscher, V. M. Zeidan, Oscillation theory on hybrid time domains: Local oscillation properties, J. Differential Equations 457 (2026), 114080, 42 pp.

[6] P. Šepitka, R. Šimon Hilscher, V. M. Zeidan, Oscillation theory on hybrid time domains: Global oscillation properties, submitted (2025).

[7] M. Wahrheit, Eigenvalue problems and oscillation of linear Hamiltonian systems, Int. J. Difference Equ. 2 (2007), no. 2, 221–244.

## On an Optimal Strict Sturm Majorant Condition for Second Order Linear Differential Equations

Peter Šepitka

Masaryk University Brno, Czech Republic
SEPITKAP@MATH.MUNI.CZ

### Abstract

This is a joint work with Roman Šimon Hilscher. Given a real interval $\mathcal{I}$ and continuous functions $\hat{r}, \hat{p}, r, p : \mathcal{I} \to \mathbb{R}$ we consider two second order linear differential equations

$$\big(\hat{r}(t)\hat{x}'\big)' + \hat{p}(t)\hat{x} = 0, \quad \big(r(t)x'\big)' + p(t)x = 0, \quad t \in \mathcal{I}. \tag{7}$$

together with the majorant condition

$$\hat{p}(t) \leq p(t), \quad \hat{r}(t) \geq r(t) > 0, \quad t \in \mathcal{I}. \tag{8}$$

In this talk we present a new type of condition (8) for equations in (7) on a general noncompact (possibly unbounded) interval $\mathcal{I}$ derived in [6]. This can be viewed as an-extension of the classical result by Coppel in [3] for the compact interval $\mathcal{I}$ as well as an-extension of the result of Aharonov and Elias in $[1,2]$ for the noncompact interval $\mathcal{I}$ under assumption $r(t) = \hat{r}(t)$ on $\mathcal{I}$. We will apply a singular Sturmian theory for possibly uncontrollable linear Hamiltonian systems, which was recently developed in $[4,5]$

### References

[1] D. Aharonov, U. Elias, *Singular Sturm comparison theorems*, J. Math. Anal. Appl. **371** (2010), no. 2, 759–763.

[2] D. Aharonov, U. Elias, *Sturm comparison theorem in singular situations*, Funct. Differ. Equ. **18** (2011), no. 3–4, 171–175.

[3] W. A. Coppel, *Comparison theorems for canonical systems of differential equations*, J. Math. Anal. Appl. **12** (1965), no. 2, 306–315.

[4] P. Šepitka, R. Šimon Hilscher, *Singular Sturmian comparison theorems for linear Hamiltonian systems*, J. Differential Equations **269** (2020), no. 4, 2920–2955.

[5] P. Šepitka, R. Šimon Hilscher, *Sturmian comparison theorems for completely controllable linear Hamiltonian systems in singular case*, J. Math. Anal. Appl. **487** (2020), no. 2, Art. 124030, 14 pp.

[6] P. Šepitka, R. Šimon Hilscher, *Note on singular Sturmian comparison theorem and strict majorant condition*, J. Math. Anal. Appl. **538** (2024), no. 2, 1–16.

### Renormalized Oscillation Theory for Hamiltonian Pencils

Alim Sukhtayev

Miami University, Oxofrd, USA
SUKHTAA@MIAMIOH.EDU

#### Abstract

Eigenvalue problems arising in hydrodynamics, quantum mechanics, and magnetohydrodynamics (MHD) often admit formulations as linear Hamiltonian systems with suitable boundary conditions. In many physically relevant settings, these systems involve singular boundary conditions and may depend nonlinearly on the spectral parameter.

In this talk, we present a general framework for analyzing a broad class of such systems. We obtain renormalized oscillation results through the Maslov index associated with suitably chosen paths of Lagrangian subspaces. We conclude by applying our framework to analyze the spectrum in each of our motivating examples.

## Blow-up Problems to Some Delay Differential Equations

Tetsuya Ishiwata

Shibaura Institute of Technology, Saitama, Japan
TISIWATA@SHIBAURA-IT.AC.JP

### Abstract

Time delays frequently arise in biological and social phenomena and are often modeled mathematically by delay differential equations. It is also well known that many such phenomena exhibit nonlinear behavior. In this talk, we consider how the interaction between time delay and nonlinearity influences the behavior of solutions, focusing in particular on the phenomenon of finite-time blow-up. After briefly reviewing several results on blow-up problems in equations with constant delay, we present our main results on finite-time blow-up in systems with distributed time delays and with state-dependent delays. This talk is based on joint works with Y. Nakata and J. Nishiguchi.

### References

[1] A. Eremin, E. Ishiwata, T. Ishiwata, Y. Nakata, Delay-induced blow-up in a planar oscillation model, Jpn. J. Ind. Appl. Math., 38(2021), pp. 1037–1061.

[2] K. Ezzinbi, M. Jazar, Blow-up results for some nonlinear delay differential equations, Positivity, 10(2) (2006) pp. 329–341.

[3] I. Győri, Y. Nakata, G. Röst, Unbounded and blow-up solutions for a delay logistic equation with positive feedback, Comm. Pure Appl. Anal., 17(6) (2018) pp. 2845–2854.

[4] T. Ishiwata and Y. Nakata, A note on blow-up solutions for a scalar differential equation with a discrete delay, Jpn. J. Ind. Appl. Math., 39(2022), pp. 959–972.

[5] K. Yagasaki, Existence of finite time blow-up solutions in a normal form of the subcritical Hopf bifurcation with time-delayed feedback for small initial functions, Disc. Cont. Dyn. Syst., Ser. B, 27(2022), pp. 2621–2634.

## Existence and Uniqueness of $L^1$-Solutions to Time-Fractional Nonlinear Diffusion Equations

Tatsuki Kawakami

Ryukoku University, Otsu, Japan
KAWAKAMI@MATH.RYUKOKU.AC.JP

### Abstract

Consider the Cauchy problem for time-fractional porous medium type nonlinear diffusion equations

$$\begin{cases} \partial_t \left(g_\alpha * [u - u_0]\right) = \Delta |u|^{m-1}u, & x \in \mathbb{R}^N, \quad t > 0, \\ u(x,0) = u_0(x), & x \in \mathbb{R}^N, \end{cases} \tag{P}$$

where $N \geq 1$, $\alpha \in (0,1)$, $m > 0$, and $u_0$ is an integrable function in $\mathbb{R}^N$. Here

$$g_\alpha(t) := \frac{t^{-\alpha}}{\Gamma(1-\alpha)},$$

which is so-called the Riemann-Liouville kernel, and $\partial_t^\alpha (u - u_0)$ is so-called the $\alpha$-th order Caputo derivative of $u$ if it is sufficiently smooth. In this case, the equation (P) is so-called the fast diffusion equation if $0 < m < 1$, the heat equation if $m = 1$, and the porous medium equation if $m > 1$. In this talk, under the condition $(K)$ and $(L)$, we establish the global existence and uniqueness of $L^1$-solutions to problem (P). Furthermore, we give the mass conservation law for $L^1$-solutions to time-fractional fast diffusion equations, namely $0 < m < 1$, and prove that the finite-time extinction does not occur for any nonnegative $L^1$-solutions.

This talk is based on a joint work with Mikiya Kametaka (Ryukoku University).

## Fractional Model of Danckwerts Tubular Catalytic Reactor

Jaromir Kukal, Michal Benes

FNSPE, Czech Technical Univesity in Prague, Czech Republic
JAROMIR.KUKAL@FJFI.CVUT.CZ

### Abstract

Modeling of anomalous diffusion in closed domain $\mathcal{D} \subset \mathbb{R}^d$ is a complex problem with many chemical, physical, and mathematical aspects. When the isotropic anomalous diffusion is unlimited, the fundamental solution of corresponding fractional PDE has to be radially symmetric. Previous condition motivates us to use only Riesz derivative as a model of fractional Laplacian and Feller derivative as a model of fractional gradient also in one-dimensional case of the anomalous tubular catalyst reactor. Danckwerts in 1950's supposed catalytic bed which is fixed inside the tube of chemical reactor. The fluid (gas or liquid) pass thorough the catalyst bed of fixed length with constant flow rate and therefore the advection occurs in front of, inside, and behind the catalyst bed. But there are two significant suppositions: both chemical reaction and the traditional diffusion occurs only inside the bed. The classical model is represented by parabolic PDE with well defined boundary conditions.

We suppose one-dimensional anomalous diffusion of single substance inside the catalyst bed with the first order chemical reaction and study various formulations of Fractional Advective Diffusion Equation (FADE). First, using Feller fractional gradient instead the traditional one and mass balance, we obtain fractional PDE with Riesz fractional Laplacian and fractional boundary conditions which is not useful for analytic or numeric solution. The second case is pure stochastic model of particle movement with spatially constrained anomalous diffusion part and unlimited particle translation via advection. The third model combines the exact solution of motionless anomalous diffusion with Neumann boundaries with spatial shift $h \to 0_+$. Resulting integro-differential equation enables to study the properties of fractional diffusive, advection, and reaction terms. The fourth model is based on the Finite Volume Method (FVM) and is suitable for direct numerical simulation including nonlinear chemical kinetics. The catalyst bed is decomposed into $N$ identic segments to obtain resulting system of ODE using only the mass balance and Feller derivative as model of fractional gradient. First numerical results will be presented at the end of presentation.

## On the Singularities in Area-Preserving Curvature Flows of Convex Symmetric Immersed Closed Plane Curves

Takeo Ushijima

Tokyo University of Science, Noda, Japan
USHIJIMA_TAKEO@RS.TUS.AC.JP

### Abstract

In this talk, we consider the motion of the convex immersed plane curves, which is described by the following differential equation with a nonlocal term $\Lambda_p(t)$;

$$v_t(\theta,t) = v(\theta,t)^p\big(v_{\theta\theta}(\theta,t) + v(\theta,t) - \Lambda_p(t)\big), \quad \theta \in (-m\pi, m\pi),$$

$$\Lambda_p(t) := \left(\int_{-m\pi}^{m\pi} \frac{1}{v(\theta,t)^{p-1}}\, d\theta\right)^{-1} \int_{-m\pi}^{m\pi} \frac{1}{v(\theta,t)^{p-2}}\, d\theta.$$

Here, $p$ is a parameter, and the relation between $v$ and the curvature $k$ of the curve is given by $v(\theta,t) = (p-1)^{-\frac{1}{p}} k(\theta,t)^{\frac{1}{p-1}}$. Due to the effect of the nonlocal term, the enclosed area of the curve is conserved. This motion of the curve is called area-preserving curvature flow. This problem was first considered in [2] and the development of the finite time singularity was conjectured ($p=1$). For Abresch-Langer type curves with highly symmetric properties, it has been proved that the maximum of curvatures blows up at a finite time under some assumptions [4, 3] ($p \geq 1$). In this case, we consider the blow-up rate.

This talk is based on joint work with Koichi Anada and Tetsuya Ishiwata [1].

### References

[1] Koichi Anada and Tetsuya Ishiwata and Takeo Ushijima, Type II singularities in area-preserving curvature flows of convex symmetric immersed closed plane curves, Journal of Differential Equations 437 (2025) 113348.

[2] M.E. Gage, On an area-preserving evolution equation for plane curves, Nonlinear Problems in Geometry, Contemp. Math., Amer. Math. Soc. 51 (1985) 51–62.

[3] N. Sesum, D.H. Tsai, X.L. Wang, Evolution of locally convex closed curves in the area-preserving and length-preserving curvature flows, Comm. Anal. Geom. 28 (2020) 1863–1894.

[4] X.L. Wang, L.H. Kong, Area-preserving evolution of nonsimple symmetric plane curves, J. Evol. Equ. 14 (2014) 387–401

## Computer-Assisted Proofs for PDEs: a Matrix-Free Approach

Maxime Breden

Ecole polytechnique, Palaiseau, France
MAXIME.BREDEN@POLYTECHNIQUE.EDU

### Abstract

Computer-assisted proofs have become more and more popular in the study of differential equations and other dynamical systems [1]. Many of them revolve around one central idea: first obtain numerically an approximate solution of your problem, and then prove, via a fixed point argument, the existence of a true solution nearby. To that end, it is often crucial to build an accurate approximation of the inverse of the linearized operator, which is obtained by combining quantitative compactness estimates and the numerical computation of a finite dimensional approximate inverse. Once quantitative compactness estimates are available, one should *in principle* be able to perform a computer-assisted proof. However, *in practice*, one must still be able to deal with the finite dimensional approximate inverse. For 2D or 3D PDEs, this is often where the bottleneck of such proofs lies.

In this talk, I will describe in more details the general strategy outlined above, and then present a new matrix-free approach to deal with the finite dimensional part, which uses only standard ideas from scientific computing and numerical linear algebra, but scales better than the usual approach used in most computer-assisted proofs. This is joint work with Hugo Chu (Ecole polytechnique).

### References

[1] J. B. van den Berg and J.-P. Lessard. Rigorous numerics in dynamics. *Notices Amer. Math. Soc.*, 62(9), 2015.

### Periodic Solutions in State-Dependent Delay Differential Equations

Matthieu Cadiot

Ecole Polytechnique, Palaiseau, France
MATTHIEU.CADIOT@POLYTECHNIQUE.EDU

#### Abstract

In this talk, I will present a computer-assisted methodology for constructively proving the existence of periodic solutions to state-dependent delay differential equations. The proof relies on a Newton–Kantorovich framework applied to a carefully formulated zero-finding problem. By combining a detailed analytical study with precise Fourier series estimates, we derive sufficient conditions ensuring the existence of a true solution in a neighborhood of a numerically computed approximation. These conditions can be effectively verified in practice using rigorous numerics. Applications to non-trivial examples will be presented to illustrate the scope and effectiveness of the method. This is joint work with J.B. van den Berg, M. Breden, K. Church and J.P. Lessard.

## An Introduction to Computer-Assisted Proofs in Differential Equations

Jean-Philippe Lessard

McGill University, Canada
jp.lessard@mcgill.ca

**Abstract**

Computer-assisted proofs (CAPs) have undergone remarkable development in recent years, fueled by advances in functional-analytic frameworks for infinite-dimensional dynamical systems, reliable interval-arithmetic numerical methods, and high-performance computing. These techniques now make it possible to rigorously validate equilibria, periodic orbits, invariant manifolds, connecting orbits, and even chaotic dynamics in ordinary, delay, and partial differential equations, often in regimes far beyond the reach of classical analytical methods. In this introductory talk, I will present the general philosophy and methodology behind CAPs, focusing on a Newton–Kantorovich framework for the validation of solutions. The emphasis will be on the core ideas: how numerical approximations can be combined with functional analysis and rigorous error bounds to produce mathematically certified results. The goal is to provide a clear conceptual roadmap of the approach, illustrated with representative examples.

### Validated Numerics for Convolutions in the Borel Plane

J.D. Mireles James

Florida Atlantic University, Boca Raton, Florida, United States
JMIRELESJAMES@FAU.EDU

#### Abstract

I will discuss computer assisted arguments for constructive proof of solutions to certain fixed point problems in the Borel plane. The Laplace transform of the constructed solution can solve a degenerate ordinary differential equation, or represent a parabolic center manifold for an equilibrium solution. A standard feature of such problems is the appearance of complex convolution in the nonlinear terms. This operation is nonlocal, and requires special care. Another feature of these arguments is that solutions must be continued along a ray to infinity, where the Laplace transform will be applied. This continuation argument combinines pen-and-paper arguments with earlier computer assisted steps. This talk will illustrate the procedure for several examples, and is joint work with Juan Miranda.

## A MLMC-VE Method for Uncertainty Quantification of Elliptic PDEs

Paola F. Antonietti, Francesca Bonizzoni, Ilaria Perugia and Marco Verani

MOX, Department of Mathematics, Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milano, Italy
FRANCESCA.BONIZZONI@POLIMI.IT

### Abstract

In this talk, we consider the following stochastic elliptic PDE: find $u\colon \Omega \times D \to \mathbb{R}$ such that, almost surely in $\Omega$,

$$\begin{cases} -\nabla \cdot (a \nabla u) = f & \text{in } D, \\ u = 0 & \text{on } \partial D, \end{cases} \tag{9}$$

where $D \subset \mathbb{R}^2$ is an open bounded domain, $f \in L^2(D)$ and the diffusion coefficient is a random field $a\colon \Omega \times D \to \mathbb{R}$ satisfying $0 < \operatorname{ess\,inf}_{x \in D} a(\omega, x) \leq a(\omega, x) \leq \operatorname{ess\,sup}_{x \in D} a(\omega, x) < \infty$ a.e. in $D$, and with realizations in $L^\infty(D)$. We are interested in deriving reliable approximations for the expected value of the solution $\mathbb{E}[u]$, as well as of some quantity of interest (QoI) of the solution $\mathbb{E}[\mathcal{Q}(u)]$, where $\mathcal{Q}$ is a linear functional of the form $\mathcal{Q}(u) = \int_D u\, q\, dx$ for $q \in L^2(D)$.

The deterministic PDE obtained for a fixed realization of the diffusion coefficient $a(\omega, x)$ is discretized by means of the conforming Virtual Element (VE) method [2]. Novel error estimates for the VE approximation of QoI of the solution are proved.

To discretize the integral in probability, we employ the Multi Level Monte Carlo (MLMC) method [3] that considers multiple levels of spatial resolution: the estimator is first computed on a coarse resolution level, and successive correction terms are then added to improve accuracy. We analyze the $h$-version of the MLMC-VE method, which relies on a sequence of nested polygonal meshes of the domain $D$. In practice, such sequence is constructed using a mesh-agglomeration strategy [1]. Computable estimators for both $\mathbb{E}[u]$ and $\mathbb{E}[\mathcal{Q}(u)]$ are designed, and error estimates are derived.

Numerical tests that verify the theoretical error estimates and validate the proposed methodology will be presented.

### References

[1] Antonietti P. F., Caldana M., Mazzieri I., Re Fraschini A., *MAGNET: an open-source library for mesh agglomeration by graph neural networks*, Engineering with Computers, Vol. 41, pp 4825–4850, 2025.

[2] Beirão da Veiga L., Chernov A., Mascotto L., Russo, A., *Basic principles of hp virtual elements on quasiuniform meshes*, Math. Models Methods Appl. Sci., Vol. 26 (8), pp 1567–1598, 2016.

[3] Giles M. B., *Multilevel Monte Carlo methods*, Acta Numerica, Vol. 24, pp 259–328, 2015.

## A Priori Error Analysis for a Nonconforming Virtual Element Method for the Monge-Ampère Equation

Scott Congreve

Charles University, Faculty of Mathematics and Physics, Prague, Czech Republic
congreve@karlin.mff.cuni.cz

### Abstract

The Monge-Ampère equation is a well known example of a fully nonlinear second-order partial differential equation. By use of the vanishing moment method [1] it is possible to approximate the solution as a fourth-order quasilinear partial differential equation. In this talk, we consider a $C^1$-nonconforming virtual element method (VEM) approximation of this quasilinear PDE that is $C^0$-conforming. We show the existence of the solution of this approximation, and derive optimal a priori error estimates in the $H^2$-, $H^1$-, and $L^2$-norms.

### References

[1] X. Feng and M. Neilan. Mixed finite element methods for the fully nonlinear Monge–Ampère equation based on the vanishing moment method. *SIAM Journal on Numerical Analysis*, 47(2):1226–1250, 2009.

## A High-Order Discontinuous Galerkin Method for the Numerical Modeling of Neuronal Electro-Metabolic Dynamics

Francesco Daniele[a], Caterina B. Leimer Saglio[a], Niteen Kumar[a], Stefano Pagani[a] and Paola F. Antonietti[a]

[a]Politecnico di Milano - MOX - Dipartimento di Matematica, 20133, Milano, Italy.
FRANCESCO.DANIELE@POLIMI.IT

### Abstract

Numerical approximation of traveling wave-like phenomena represents a challenging problem in the simulation of multiscale processes. These kinds of models characterize various biological scenarios, such as electrical propagation in excitable tissues, and they require numerical methods that combine high spatial and temporal accuracy with geometric flexibility, due to the presence of heterogeneous tissues and sharp wavefronts. High-order discontinuous Galerkin methods on polytopal and polyhedral meshes (polyDG) [1, 4] provide a particularly suitable framework for the discretization of these problems. In particular, the use of high-order polynomials enables an accurate resolution of steep and fast-propagating wavefronts, while the element-wise structure of DG methods naturally supports p-adaptive strategies [3], allowing the local polynomial degree to be adjusted dynamically where the solution exhibits steep gradients, in order to improve computational efficiency. Furthermore, this formulation offers significant geometric flexibility for the treatment of complex computational domains. A relevant application of these models arises in the electrophysiological modeling of brain tissue. At the tissue scale, neuronal activity can be described by the monodomain equation, coupled with appropriate ionic and metabolic models that account for underlying metabolic dynamics and ion-channel mechanisms [2]. Within this framework, we investigate how oxygen availability influences cellular energy production and ionic pump activity under pathological hypoxic conditions, providing a mathematical description of the interaction between electrical activity and metabolic processes.

### References

[1] P. F. Antonietti, S. Bonetti, M. Botti, M. Corti, I. Fumagalli, I. Mazzieri, *lymph: Discontinuous Polytopal Methods for Multi-Physics Differential Problems*, ACM Transactions on Mathematical Software, 51 (2025).

[2] F. Daniele, C. B. Leimer Saglio, S. Pagani, P. F. Antonietti, *Mathematical and numerical modeling of coupled oxygen dynamics and neuronal electrophysiology*, arXiv preprint arXiv:2509.22863, 2025.

[3] C. B. Leimer Saglio, S. Pagani, P. F. Antonietti, *A p-adaptive polytopal discontinuous Galerkin method for high-order approximation of brain electrophysiology*, Computer Methods in Applied Mechanics and Engineering, 446 (2025), 118249.

[4] C. B. Leimer Saglio, S. Pagani, P. F. Antonietti, *A massively parallel non-overlapping Schwarz preconditioner for PolyDG methods in brain electrophysiology*, arXiv preprint arXiv:2512.19536, 2025.

## A Reylonds-Semi-Robust $H(\text{div})$-Conforming Method for Unsteady Incompressible Power-Law Flows

L. Beirão da Veiga*, D.A. Di Pietro† and K.B. Haile*

* Dipartimento di Matematica e Applicazioni Università di Milano Bicocca, Milano, Italy
LOURENCO.BEIRAO@UNIMIB.IT, K.HAILE@CAMPUS.UNIMIB.IT

† IMAG, Univ. Montpellier, CNRS, Montpellier, France
DANIELE.DI-PIETRO@UMONTPELLIER.FR

### Abstract

In this work, we address the numerical approximation of unsteady systems of p-Navier–Stokes type equations that model incompressible fluid flows with non-Newtonian rheology. The interplay between the nonlinearity of the non-coercive convective term, typical of Navier-Stokes, and the nonlinearity introduced by the non–Newtonian constitutive law presents interesting challenges. In the present talk, based on [3], we introduce a new H(div)-conforming, ”convectionsemi-robust” and pressure-robust method for solving unsteady systems of p-Navier–Stokes type equations. A numerical scheme is considered ”convection-semi-robust” if its velocity error estimates are independent of the inverse of the kinematic viscosity, while pressure robustness ensures that the velocity error estimates are unaffected by the pressure. These two conditions are considered critically important in modern literature (see e.g., [1]). To derive sharp local estimates, the complex interaction between the nonlinear convective term and the nonlinear diffusion term must be carefully analyzed. This interaction cannot be addressed through a straightforward combination of existing techniques, as the estimation of each term depends on the local flow regime. The peculiarity of our error estimates is that they track the dependence of the local contributions to the error on local Reynolds numbers, in the spirit of [2] developed for the scalar problem, which combines p-laplacian diffusion with convection and reaction terms. In the present talk, after presenting the model and the numerical method, we will outline the theoretical results. Finally, a set of numerical tests supporting the theory will be shown.

### References

[1] V. John, A. Linke, C. Merdon, M. Neilan, and L. G. Rebholz, On the Divergence Constraint in Mixed Finite Element Methods for Incompressible Flows, SIAM Review (2017) 59.3: 492–544.

[2] L. Beirão da Veiga, D. A. Di Pietro, and K. B. Haile, A Péclet-robust discontinuous Galerkin method for nonlinear diffusion with advection, Math. Models Methods Appl. Sci. (2024) 34.9: 1781–1807.

[3] L. Beirão da Veiga, D. A. Di Pietro, and K. B. Haile, A pressure and convection robust Finite Element Method for non-newtonian Navier-Stokes system, Submitted, Preprint available at https://arxiv.org/abs/2505.08708.

## On Positivity Preservation of HDG Methods for the Diffusion Equation on Hypergraphs

Petr Knobloch

Charles University, Faculty of Mathematics and Physics, Prague, Czech Republic
KNOBLOCH@KARLIN.MFF.CUNI.CZ

**Abstract**

Structure preserving discretizations of the diffusion equation need to be locally mass conservative and, in the case of nonnegative data, they should also preserve the positivity of the solution. This is particularly important in applications where the solution represents a physical quantity, such as temperature or concentration, which is nonnegative by definition. The most direct approach to obtain local mass conservation for the diffusion equation comprises dual, mixed methods, which discretize the flux explicitly. A way to bypass the saddle-point structure of the corresponding linear system lies in the concept of hybridization, which has been generalized to cover a large class of so-called hybrid discontinuous Galerkin (HDG) methods in [1]. These methods are locally mass conservative not only for classical domains but even if they discretize the diffusion equation posed on graphs or networks of surfaces, which naturally appear as singular limits of thin structures [2]. The positivity preservation of such HDG methods has not been fully established, which is the focus of this contribution.

It will be shown that only a few low-order HDG methods on hypergraphs are positivity preserving. For nonobtuse simplicial meshes, this will be proved for the lowest-order Raviart–Thomas discretization with the stabilization parameter $\tau = 0$ and for piecewise constant approximations of all variables if $\tau = \mathcal{O}(1)$. For general meshes (also nonsimplicial ones), positivity preservation for piecewise constant approximations can be restored if $\tau = \mathcal{O}(1/h)$ is selected with $h$ denoting the mesh width. This choice of $\tau$ leads to positivity preservation also for a few other discretizations. However, for most HDG methods, the positivity can be violated for any choice of $\tau$ as can be shown by appropriate counterexamples. The theoretical findings will be ilustrated by numerical experiments.

This is a joint work with Philip L. Lederer (University of Hamburg) and Andreas Rupp (Saarland University).

### References

[1] B. Cockburn, J. Gopalakrishnan, R. Lazarov: Unified hybridization of discontinuous Galerkin, mixed, and continuous Galerkin methods for second order elliptic problems. *SIAM J. Numer. Anal.*, 47(2):1319–1365, 2009.

[2] A. Rupp, M. Gahn, G. Kanschat: Partial differential equations on hypergraphs and networks of surfaces: Derivation and hybrid discretizations. *ESAIM: Math. Model. Numer. Anal.*, 56(2):505–528, 2022.

## A Higher Order Polytopal Method for Contact Mechanics with Tresca Friction

Raman Kumar

IMAG, Univ. Montpellier, CNRS, Montpellier, France
RAMAN.KUMAR@UMONTPELLIER.FR

**Abstract**

In this work, we design and analyze a Discrete de Rham (DDR) scheme for a contact mechanics problem involving fractures along which a model of Tresca friction is considered. Our approach is based on a mixed formulation involving a displacement field and a Lagrange multiplier, enforcing the contact conditions, representing tractions at fractures. The approximation space for the displacement is made of vectors values attached to each vertex, edge, face, and element, while the Lagrange multiplier space is approximated by piecewise constant vectors on each fracture face. The displacement degrees of freedom allow reconstruct piecewise quadratic approximations of this field. We prove a discrete Korn inequality that account for the fractures, as well as an inf-sup condition (in a non-standard $H^{-1/2}$-norm) between the discrete Lagrange multiplier space and the discrete displacement space. We provide an in-depth error analysis of the scheme and show that, contrary to usual low-order nodal-based schemes, our method is robust in the quasi-incompressible limit for the primal variable (displacement). An extensive set of numerical experiments confirms the theoretical analysis and demonstrate the practical accuracy and robustness of the scheme.

### References

[1] A higher order polytopal method for contact mechanics with Tresca friction, Droniou, Jerome and Kumar, Raman and Masson, Roland and Singla, Ritesh, arXiv preprint arXiv:2601.07586 (2026).

## Maxwell Compactness for Polytopal Methods

Simon Lemaire

INRIA, Univ. Lille, France
SIMON.LEMAIRE@INRIA.FR

### Abstract

Real-world, coupled and multiphysics problems are often nonlinear. As such, their analysis (existence of solutions to the PDE model, convergence of numerical approximations) often relies on compactness arguments.

At the continuous level, for PDE models based on the de Rham complex, these compactness results are well established, and due to Rellich (for $H^1$) and to Weck/Weber (for $\boldsymbol{H}(\mathbf{curl})$ and $\boldsymbol{H}(\mathrm{div})$). The last two results are often referred to, in the literature, as Maxwell compactness, as they naturally appear in the study of models in electromagnetism.

At the discrete level, even for conforming finite element approximations on standard meshes, Maxwell compactness results do not directly follow from their infinite-dimensional counterpart. This is even more true for polytopal methods, which hinge on fully algebraic spaces and discrete operator and potential reconstructions.

In this talk, we will discuss Maxwell compactness results in the case of two different polytopal methods, namely the HHO (Hybrid High-Order) and the DDR (Discrete De Rham) approaches. The former is non-conforming and non-compatible, whereas the latter is conforming (in a sense that will be made precise) and compatible.

The material of this talk is based on joint works (i) with S. Pitassi (Univ. Montpellier) on the one hand [1], and (ii) with T. Chaumont-Frelet (INRIA Lille) and J. Droniou (CNRS, Univ. Montpellier) on the other hand [2].

### References

[1] S. Lemaire and S. Pitassi. *Discrete Weber inequalities and related Maxwell compactness for hybrid spaces over polyhedral partitions of domains with general topology*, Found. Comput. Math., 25(3) (2025), 725–763.

[2] T. Chaumont-Frelet, J. Droniou, and S. Lemaire. *Commuting quasi-interpolators and Maxwell compactness for a polytopal de Rham complex*. Preprint `arXiv:2510.18835`, Oct. 2025.

### A High-Order Localized Orthogonal Decomposition for Stokes Equations

Alexei Lozinski

Université Marie et Louis Pasteur, Besançon, France.
ALEXEI.LOZINSKI@UMLP.FR

#### Abstract

We shall present a high-order multiscale method introduced in [1] for heterogeneous Stokes problems. It is based on the Localized Orthogonal Decomposition methodology and achieves optimal convergence orders under minimal structural assumptions on the coefficients. The method can be set on meshes with elements of arbitrary shape, and precomputes problem specific basis functions on patches of cells, associating them to both faces and cells of the mesh. A key feature of our approach is the careful design of so-called quantities of interest, defining functionals of the solution whose values the multiscale approximation aims to reproduce. Their selection is particularly delicate in the context of Stokes problems due to potential conflicts arising from the divergence-free constraint. The exponential decay of the problem-adapted basis functions can be proven, justifying their localized computation in practical implementations. A rigorous *a priori* error analysis predicts high-order convergence for both velocity and pressure, if the basis supports grow logarithmically with the desired accuracy.

This is a joint work with Moritz Hauck (Karlsruhe).

#### References

[1] Hauck, M., & Lozinski, A. (2025). A High-Order Localized Orthogonal Decomposition Method for Heterogeneous Stokes Problems. arXiv:2511.22684.

## Level-set Approximation of Noisy Functions

M. Croci, A.-L. Haji-Ali, I. C. J. Powell

BCAM - Basque Center for Applied Mathematics, Ikerbasque, and UPV/EHU, Spain
mcroci@bcamath.org

### Abstract

In this talk, we consider the problem of numerically approximating the level set of a noisy multivariate function which is only accessible via pointwise sampling. A key application is the estimation of failure probability regions: given a random PDE that depends on a set of deterministic parameters and a failure criterion, determine the parameter region where the probability of failure exceeds a given safety threshold.

We present a level-set estimation algorithm for Lipschitz-continuous functions which is adaptive in terms of both parameter-space approximation and sampling, and automatically increases accuracy close to the level set. This algorithm is compatible with general Monte Carlo estimators and achieves improved cost complexity rates with respect to non-adaptive approximations. We provide numerical experiments based on random PDEs and computer vision in support of our theoretical findings.

### References

[1] M. Croci, A.-L. Haji-Ali, and I. C. J. Powell. An adaptive sampling algorithm for level-set approximation. *Preprint*, 2025. `https://arxiv.org/abs/2509.14896`

**Long-Time Behavior of Some Stochastic Evolution Equations on the Sphere and Their Numerical Discretizations**

Björn Müller

Chalmers & University of Gothenburg, Gothenburg, Sweden
bjornmul@chalmers.se

### Abstract

We study the stochastic wave and stochastic free Schrödinger equations on the unit sphere $\mathbb{S}^2$, driven by a mean-zero square-integrable additive $Q$-Lévy noise. We show linear growth of the expected energy for both equations, as well as of the expected mass for the stochastic Schrödinger equation. We further investigate the long-time behavior of these quantities in the exponential Euler, Euler–Maruyama and backward Euler–Maruyama time-stepping schemes. While the exponential Euler method recovers the correct linear growth, the Euler–Maruyama and backward Euler–Maruyama exhibit too fast and too slow growth, respectively. Our theoretical predictions are confirmed through numerical experiments. This is joint work with David Cohen and Andrea Papini.

### Multigrid Monte Carlo Revisited: Fast Sampling of Gaussian Random Fields

Nils Friess[1], Yoshihito Kazashi[2], Eike Müller[3], Robert Scheichl[1]

[1]Heidelberg University, Germany; [2]University of Manchester, United Kindom;
[3]University of Bath, United Kingdom
E.MUELLER@BATH.AC.UK

**Abstract**

The fast simulation of Gaussian random fields plays a pivotal role in Uncertainty Quantification, data science and spatial statistics. In theory, it is a well understood problem, but in practice the efficiency and performance of traditional sampling procedures degenerates quickly when the random field is discretised on a fine grid. Most existing algorithms, such as Cholesky factorisation samplers, do not scale well on large-scale parallel computers. On the other hand, stationary, iterative approaches such as the Gibbs sampler become extremely inefficient at high grid resolution since they perform inherently local updates. In the late 1980s Goodman & Sokal [1] showed that the Gibbs sampler can be accelerated using multigrid ideas. The key observation is the intricate connection of random samplers, such as the Gibbs method, with iterative solvers for linear systems also pointed out in [2]: both methods have very similar convergence properties. Based on this observation, Goodman & Sokal proposed the so-called multigrid Monte Carlo (MGMC) method for hierarchical sampling and applied it to problems in quantum physics. However, MGMC does not overcome critical slowing down in simulations of field theories near phase transitions.

In [3] we extend the MGMC algorithm to the important setting of a Gaussian random field conditioned on noisy data via a Bayesian approach; in this case, the precision matrix is a finite-rank perturbation of some differential operator. By using a bespoke variant of the Gibbs sampler with a low-rank update on each level of the multigrid hierarchy we are able to generate Markov chains with a fixed, grid-independent integrated autocorrelation time. Our theoretical analysis is confirmed by numerical experiments for sampling rough Gaussian random fields in two and three dimensions. At high resolution MGMC is able to produce independent samples at a faster rate than a Cholesky sampler. We also extend MGMC to a purely algebraic setting to handle problems formulated on graphs and unstructured grids; in this case our implementation leverages existing building blocks from the well-established PETSc iterative solver library.

**References**

[1] Jonathan Goodman and Alan D Sokal. Multigrid Monte Carlo Method. Conceptual Foundations. *Physical Review D*, 40(6):2035, 1989.

[2] Colin Fox and Albert Parker. Accelerated Gibbs sampling of normal distributions using matrix splittings and polynomials. *Bernoulli*, 23(4B):3711 – 3743, 2017.

[3] Yoshihito Kazashi, Eike H Müller, and Robert Scheichl. Multigrid Monte Carlo Revisited: Theory and Bayesian Inference. *to appear in Foundations of Computational Mathematics; arXiv:2407.12149*, 2024.

## Dirichlet-Neumann Averaging: Efficient SPDE-Based Gaussian Process Simulation

Robert Scheichl

Universität Heidelberg, Heidelberg, Germany
R.SCHECHL@UNI-HEIDELBERG.DE

### Abstract

Gaussian processes (GPs) and Gaussian random fields (GRFs) are essential for modelling spatially varying stochastic phenomena. Yet, the efficient generation of corresponding realisations on high-resolution grids remains challenging, particularly when a large number of realisations are required. In this talk I will present two novel contributions. First, we propose a new methodology based on Dirichlet-Neumann averaging (DNA) to generate GPs and GRFs with isotropic covariance on regularly spaced grids. The combination of discrete cosine and sine transforms in the DNA sampling approach allows for rapid evaluations without the need for modification or padding of the desired covariance function. While this introduces an error in the covariance, our numerical experiments show that this error is negligible for most relevant applications, representing a trade-off between efficiency and precision. We provide explicit error estimates for Matern covariances. The second contribution links our new methodology to the stochastic partial differential equation (SPDE) approach for sampling GRFs. We demonstrate that the concepts developed in our methodology can also guide the selection of boundary conditions in the SPDE framework. We prove that averaging specific GRFs sampled via the SPDE approach yields genuinely isotropic realisations without domain extension, with the error bounds established in the first part remaining valid. We finish by presenting an application within a parallel high-performance budgeted multi-level Monte Carlo method for estimates on the entire spatial domain of multi-PDE problems with random input data, demonstrating the advantages of the DNA approach in terms of ease of implementation, memory efficiency and parallel scalability. This is joint work with Robert Kutri and Niklas Baumgarten (Heidelberg).

## A High-Level Quantum Lattice Boltzmann Solver for Pure Advection with Subregion Local Measurement

Tomas Bezdek (1), Tobias Köppl (2) and Barbara Wohlmuth (1)

Department of Mathematics, TUM School of Computation, Information and Technology, Technical University of Munich, Boltzmannstraße 3, 85748 Garching b. München, Germany (1)

Fraunhofer Institute for Open Communication Systems, SQC, Kaiserin-Augusta-Allee 31, 10589 Berlin, Germany (2)

tomas.bezdek@tum.de, tobias.koeppl@fokus.fraunhofer.de, wohlmuth@cit.tum.de

### Abstract

The Lattice Boltzmann Method (LBM) is widely used for high-performance fluid simulation, yet its implementation on quantum architectures is still challenging due to complex circuit synthesis and the so-called read-out bottleneck [1]. In this talk, we present a complete quantum solver for the linear advection equation, highlighting two advances in quantum fluid dynamics (QFD). First, we implement the full LBM advection cycle in Qrisp [2], a high-level functional quantum programming language. Using automated uncomputation and structured variable management, we obtain a clear and modular mapping of the streaming step onto quantum registers. Unlike other algorithms that assume infinite domains, our approach incorporates physically relevant boundary conditions, including periodic, simple bounce-back, and slip setups, realized directly through high-level functional constructs. Second, we introduce a local measurement protocol based on algorithms for quantum amplitude estimation and amplification [3] to extract the macroscopic fluid density in selected sub-regions of the computational domain. By focusing on localized areas of interest, this strategy avoids the exponential cost of full state tomography. Simulations of pure advection demonstrate that the solver preserves sharp profiles without the numerical diffusion typical of classical discretizations. We further analyze circuit requirements and discuss how subregion-aware measurement supports scalable hybrid quantum–classical workflows, particularly when localized flow information is required for real-time analysis or boundary coupling.

### References

[1] M. A. Schalkers and M. Möller. Efficient and fail-safe quantum algorithm for the transport equation. *Journal of Computational Physics*, 502:112816, 2024.

[2] R. Seidel et al. Qrisp: A framework for compilable high-level programming of gate-based quantum computers. *arXiv preprint arXiv:2406.14792*, 2024.

[3] G. Brassard, P. Høyer, M. Mosca, and A. Tapp. Quantum amplitude amplification and estimation. *Quantum Computation and Information*, pages 53–74. American Mathematical Society, 2002.

## Mixed-precision Computing: High Accuracy with Low Precision

Erin Claire Carson

Charles University, Faculty of Mathematics and Physics, Prague, Czech Republic
carson@karlin.mff.cuni.cz

### Abstract

Mixed-precision algorithms have launched an era in which efficiency and accuracy are no longer mutually exclusive. Rather than rely entirely on high-precision formats like double (64-bit) precision, mixed-precision algorithms apply lower precisions such as single (32-bit) or half (16-bit) precision whenever possible, reserving higher precision only for critical steps. Doing so can drastically reduce memory requirements, improve performance, and lessen energy consumption on modern computer hardware without sacrificing accuracy or stability. In this talk, we discuss the challenges of using low/mixed precision, and present five cases, common in scientific applications, where using mixed precision makes sense.

### Hybrid Hierarchical Matrices with Adaptive Precision Storage

Ritesh Khan, Erin Carson

Charles University, Prague, Czech Republic
ritesh.khan@matfyz.cuni.cz

#### Abstract

Hierarchical matrices are multilevel block low-rank representations of matrices that are widely used for fast matrix computations. Hierarchical matrices are built using a tree data structure with low-rank blocks identified using various admissibility conditions, such as standard/strong admissibility and weak admissibility. In this talk, we introduce a novel hierarchical matrix framework, namely $\mathcal{H}_h$, based on a *hybrid* admissibility condition: we use strong admissibility at the coarser levels (larger blocks) and weak admissibility at the finer levels (smaller blocks). We carry out a rounding error analysis of $\mathcal{H}_h$-matrices and show that the low-rank blocks of $\mathcal{H}_h$-matrices can be represented in low precision without degrading the overall approximation quality. We provide an explicit rule for choosing the precision for a particular low-rank block. We propose an adaptive mixed precision algorithm for constructing and storing $\mathcal{H}_h$-matrices. Our numerical results illustrate that the proposed adaptive mixed precision $\mathcal{H}_h$-matrices can achieve significant storage reductions (up to $11\times$) compared to uniform double precision $\mathcal{H}$-matrices, without compromising accuracy.

## Parallel-in-Time Solver for the Runge–Kutta Discretization of the Heat Equation

Santolo Leveque

Charles University, Prague, Czech Republic
SANTOLO.LEVEQUE@MATFYZ.CUNI.CZ

### Abstract

Time-dependent PDEs arise quite often in many scientific areas, such as mechanics, biology, economics, or chemistry, just to name a few. Of late, researchers have devoted their effort in devising parallel-in-time methods for the numerical solution of time-dependent PDEs, adding a new dimension of parallelism and allowing to speed-up the solution process on modern supercomputers.

In this talk, we present a parallel-in-time preconditioner for the space-time discretization of the heat equation arising when employing a Runge–Kutta method in time. The resulting system is solved iteratively for the numerical solution and for the stages of the method. The proposed preconditioner results in a block-diagonal solve for the stages, and a Schur complement obtained by solving again systems for the stages. A range of numerical experiments validate the robustness of the preconditioner with respect to the discretization parameters and to the number of stages, showing the speed-up achieved on a parallel architecture.

### TNL: Numerical Library for Modern Parallel Architectures

Tomáš Oberhuber, Jakub Klinkovský, Tomáš Halada, and Radek Fučík

Czech Technical University in Prague, Prague, Czech Republic
TOMAS.OBERHUBER@FJFI.CVUT.CZ

#### Abstract

TNL (www.tnl-project.org) is a collection of building blocks that facilitate the development of efficient numerical solvers and HPC algorithms. It is implemented in C++ using modern programming paradigms in order to provide a flexible and user-friendly interface similar to, for example, the C++ Standard Template Library. TNL provides native support for modern hardware architectures such as multicore CPUs, GPUs, and distributed systems, which can be managed via a unified interface. In our presentation, we will demonstrate the main features of the library together with efficiency of the implemented algorithms and data structures.

#### References

[1] Klinkovský J., Oberhuber T., Fučík R., Žabka V., Configurable open-source data structure for distributed conforming unstructured homogeneous meshes with GPU support, ACM Transactions on Mathematical Software, vol. 48, no.3, article No. 30, pp. 1–30, 2022.

[2] Oberhuber T., Klinkovský J., Fučík R., TNL: Numerical library for modern parallel architectures, Acta Polytechnica, vol. 61, no. SI, pp. 122-134, 2020.

[3] Fučík R., Klinkovský J., Solovský J., Oberhuber T., Mikyška J., Multidimensional Mixed–Hybrid Finite Element Method for Compositional Two–Phase Flow in Heterogeneous Porous Media and its Parallel Implementation on GPU, Computer Physics Communications, vol. 238, pp.165-180, 2019.

## Reduced and Mixed Precision QR Decomposition Techniques

Eda Oktay

Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany
OKTAY@MPI-MAGDEBURG.MPG.DE

### Abstract

Modern high-performance computing (HPC) architectures increasingly rely on reduced-precision arithmetic to improve performance, reduce energy consumption, and more efficiently use limited memory bandwidth. These developments are particularly relevant for large-scale PDE solvers, where many algorithmic components ultimately rely on linear algebra operations. In particular, QR factorizations appear in least-squares formulations of PDE-constrained problems, in domain-decomposition methods that require orthogonalization, and in Krylov subspace methods where orthogonality and stability are essential. However, using uniformly low precision in these settings can introduce rounding errors that accumulate during orthogonalization, leading to numerical instability, loss of orthogonality, and slower or unreliable convergence of the overall PDE solver.

In this talk, we will cover several QR decomposition techniques, including Householder and (shifted) Cholesky factorizations, and study their behavior under reduced- and mixed-precision arithmetic. We show which parts of these algorithms can safely be executed in low precision and which require higher precision to maintain stability. We also discuss how combining multiple precisions within a single algorithm can balance efficiency and numerical reliability. Performance results on modern HPC architectures, together with numerical experiments, demonstrate the achievable speedups and clarify how precision choices influence stability, accuracy, and convergence in practice.

## Low-Rank Methods in Isogeometric Analysis

Tom-Christian Riemer and Martin Stoll

Technische Universität Chemnitz, Germany
tom-christian.riemer@math.tu-chemnitz.de

### Abstract

Isogeometric Analysis (IgA) bridges CAD and finite element analysis by employing spline technologies (typically B-splines or NURBS) for both geometry representation and field approximation, enabling exact geometrical descriptions and high-order smooth trial spaces. These advantages come with computational challenges: higher-order spline bases have supports spanning multiple knot spans, yielding dense element contributions in global system matrices and expensive multi-dimensional quadrature in tensor-product settings [1]. To address these bottlenecks, low-rank tensor techniques exploit the intrinsic tensor-product structure of IgA. The core idea is to separate variables in geometry-induced integral kernels and represent mass and stiffness tensors as sums of Kronecker products of univariate matrices, reducing assembly cost and memory footprint while maintaining accuracy control. Building on this, Bünger et al. introduced tensor-train (TT) approximations for coefficient tensors arising from kernel interpolation in tensor-product B-spline bases, computed via the Alternating Minimal Energy (AMEn) method [2]. The talk then presents two extensions of this TT-based paradigm from our recent work. First, we show how multi-patch IgA can be treated by transferring the IETI (isogeometric tearing and interconnecting) concept to the low-rank TT framework and enforcing interface continuity via dual Lagrange-multiplier constraints [3]. Second, we summarize a hierarchical refinement strategy based on (T)HB-splines, where separability is recovered on structured subregions to enable level-wise low-rank assembly and block-TT solution procedures on locally refined meshes. A remaining issue is that interpolating geometry-induced kernels in tensor-product spline spaces produces global coefficient tensors: such global parameterizations struggle to capture local features and cannot naturally represent discontinuities or singularities in complex geometries. As an outlook, we discuss how functional tensor trains may provide a route to local refinement and non-smooth phenomena without relying on a fixed tensorized discretization.

### References

[1] T. Hughes, J. Cottrell, Y. Bazilevs, Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement, Comp. Methods Appl. Mech. Eng. 194 (2005) 4135–4195.

[2] A. Bünger, S. Dolgov, M. Stoll, A low-rank tensor method for PDE-constrained optimization with isogeometric analysis, SIAM Journal on Scientific Computing 42 (1) (2020) A140–A161. doi:10.1137/18M1227238.

[3] T.-C. Riemer, A. Bünger, M. Stoll, IETI-based low-rank method for pde-constrained optimization, SIAM Journal on Scientific Computing 47 (3) (2025) A1758–A1782. doi:10.1137/24M1660024.

## Acceleration of a Nonoverlapping Domain Decomposition Solver by Using GPUs on Subdomains

Martin Hanek[1,2], Tomáš Oberhuber[1], and Jakub Šístek[2]

[1]Czech Technical University in Prague
[2]Institute of Mathematics of the Czech Academy of Sciences, Prague
martin.hanek@fs.cvut.cz, tomas.oberhuber@fjfi.cvut.cz, sistek@math.cas.cz

### Abstract

Balancing Domain Decomposition based on Constraints (BDDC) is a method for solving large sparse linear systems of equations arising from the discretization of partial differential equations (PDEs). It is based on using the conjugate gradient (CG) method preconditioned by a two-level preconditioner based on nonoverlapping domain decomposition. Since its introduction in 2003, BDDC has been successfully applied to a wide range of problems, including structural mechanics, fluid dynamics, and electromagnetics. We have implemented the multilevel extension of BDDC using adaptive selection of the coarse problem in the open-source library BDDCML[1].

In domain decomposition methods, the dominant cost is associated to the solution of local problems on subdomains, which are independent and can be solved in parallel. In BDDC, there are two local problems to be solved within each CG iteration: i) a local problem associated to the multiplication of the search direction vector by the Schur complement at the interface, and ii) a local subdomain correction problem within the preconditioner. In [1], an approach to accelerating these two dominant components of the solver by using graphics processing units (GPUs) was investigated. It is based on building dense matrices of the local Schur complements by the sparse direct solver on CPUs in the preconditioner setup. In the next step, the local matrices are copied to GPUs. Repeated multiplications of local vectors with the dense matrix of the Schur complement are performed for each subdomain on GPUs. In addition, factorizations and backsubstitutions with the dense saddle-point subdomain matrices of the subdomain correction problems are also performed on GPUs.

In this contribution, detailed times of the main components of the algorithm are measured on a benchmark Poisson problem on the LUMI supercomputer. The method is also applied to an unsteady problem of incompressible flow, where the Krylov subspace iterations are performed repeatedly in each time step. The results demonstrate the potential of the approach to speed up realistic simulations more than 10 times with a preference towards large subdomains.

### References

[1] J. Šístek, T. Oberhuber, Acceleration of a parallel BDDC solver by using graphics processing units on subdomains, The International Journal of High Performance Computing Applications 37 (2) (2023) 151–164.

[1]https://users.math.cas.cz/~sistek/software/bddcml.html

### Deep Symmetric Autoencoders: Error Estimates and Beyond

Simone Brivio†, Nicola R. Franco

MOX, Department of Mathematics, Politecnico di Milano, Milan, Italy
simone.brivio@polimi.it

**Abstract**

Within this talk, we aim to bridge deep autoencoders and proper orthogonal decomposition (POD) through deep symmetric autoencoders, which take the best of both worlds. Indeed, deep symmetric autoencoders retain the nonlinear nature of classical deep autoencoders, which is central to overcome the limitations of linear reduction methods when tackling problems with a slow decay of the Kolmogorov $n$-width. Furthermore, akin to linear reduction methods, deep symmetric autoencoders are endowed with a rich mathematical structure, which is the perfect ground to devise clear and natural error estimates [1], by exploiting the powerful Eckart-Young-Schmidt theorem [2] and the properties of the employed activation function [3]. Nonetheless, leveraging our theoretical framework and the proposed error estimates, we derive a data-driven initialization procedure for deep symmetric autoencoders, which entails the iterative application of POD at each layer. Finally, we discuss the practical aspects of our work through a series of numerical experiments, aimed to show the representation capacity of deep symmetric autoencoders and the effectiveness of our proposed initialization strategy.

#### References

[1] Brivio, S., & Franco, N. R. (2025). Deep Symmetric Autoencoders from the Eckart-Young-Schmidt Perspective. arXiv preprint arXiv:2506.11641.

[2] Eckart, C., & Young, G. (1936). The approximation of one matrix by another of lower rank. Psychometrika, 1(3), 211-218.

[3] Otto, S. E., Macchio, G. R., & Rowley, C. W. (2023). Learning nonlinear projections for reduced-order modeling of dynamical systems using constrained autoencoders. Chaos: An Interdisciplinary Journal of Nonlinear Science, 33(11).

## Preconditioning and Numerical Stability in Neural Network Training for Parametric PDEs

Chenguang Duan

Institut für Geometrie und Praktische Mathematik, Aachen, Germany
DUAN@IGPM.RWTH-AACHEN.DE

### Abstract

In the context of training neural network-based approximations of solutions of parameter-dependent PDEs, we investigate the effect of preconditioning via well-conditioned frame representations of operators and demonstrate a significant improvement on the performance of standard training methods. We also observe that standard representations of preconditioned matrices are insufficient for obtaining numerical stability and propose a generally applicable form of stable representations that enables computations with single- and half-precision floating point numbers without loss of precision.

## Learning the Continuous-Time Dynamics: from Trajectories to Velocities

Nicola Farenga†, Andrea Manzoni

MOX, Department of Mathematics, Politecnico di Milano, Milan, Italy.
NICOLA.FARENGA@POLIMI.IT

### Abstract

Learning nonlinear continuous-time dynamical systems is a central problem in many fields of science and engineering. Deep learning architectures characterized by a continuous-time inductive bias, such as Neural ODEs, have seen widespread adoption in this context, with applications ranging from low-dimensional dynamical systems modeling to data-driven order reduction for time-dependent PDEs by relying on suitable nonlinear dimensionality reduction strategies. Despite many advantages stemming from their continuous-time inductive bias, including mathematical interpretability and time super-resolution, they rely on a simulation-based training procedure, which, whether employed directly in state-space or in a latent space of reduced dimension, requires unrolling the predictions over multiple steps by means of numerical integration. Rollout-based training, while motivated by empirical evidence for providing stable predictions, involves high computational costs and memory requirements due to backpropagation through time, which are further compounded by higher-order numerical integration of the Neural ODE. In this talk, we first address the pitfalls of rollout-based training in the context of learning continuous-time dynamics, analyzing the bias introduced by the numerical solver when unrolling predictions in the infinite-horizon limit, thereby hindering proper identification of the underlying dynamics. Then, we discuss the advantages of a velocity-based training objective, by proposing the adoption of a stochastic objective that results in a higher-order approximation of the population risk, and is characterized by an implicit regularization effect. Numerical experiments, carried out in the context of dynamical systems and time-dependent PDEs, validate the efficiency of the proposed approach, highlighting faster convergence and improved generalization compared to a range of rollout-based training strategies.

### References

[1] N. Farenga, S. Fresca, S. Brivio, A. Manzoni, On latent dynamics learning in nonlinear reduced order modeling, Neural Networks, Volume 185, 2025, 107-146, ISSN 0893-6080.

[2] N. Farenga, A. Manzoni. Learning to evolve by matching velocities (In preparation).

## Discovering Adaptive Basis Representations for Parametrized PDEs with Deep Orthogonal Decomposition: Theory and Practice

Nicola Rares Franco[1,*], Andrea Manzoni[1], Paolo Zunino[1], Jan S. Hesthaven[2]

[1]MOX Laboratory, Department of Mathematics, Politecnico di Milano
[2]Karlsruhe Institute of Technology (KIT)
nicolarares.franco@polimi.it

### Abstract

We present Deep Orthogonal Decomposition (DOD) [1], a novel technique for dimensionality reduction and reduced order modeling of parametrized PDEs. The DOD consists in a deep neural network approximating the solution manifold through a continuously adaptive local basis. In contrast to global techniques such as Proper Orthogonal Decomposition (POD), the local adaptivity of the learned basis allows DOD to mitigate the Kolmogorov barrier, significantly broadening its applicability to challenging nonlinear problems. Additionally, thanks to the orthogonal structure of the latent space, DOD ensures a tight control on error propagation and enhanced interpretability, resulting in an appealing alternative to deep autoencoders. Beyond steady parametric settings, the proposed framework naturally extends to time-dependent PDEs by treating time as an additional input variable, allowing the learned basis to adapt continuously to the evolving system state. From this perspective, DOD can be interpreted as a data-driven tool for learning low-dimensional representations of nonlinear dynamical systems. The methodology is analyzed both theoretically and practically. On the one hand, we establish a connection between the truncation error of the DOD and a spectral gap condition related to the solution manifold [2], whereas on the other with assess the performances of the DOD on a set of numerical experiments involving nonlinear PDEs with parametrized geometries and high-dimensional parameter spaces [1].

### References

[1] N. R. Franco, A. Manzoni, P. Zunino, and J. S. Hesthaven, Deep orthogonal decomposition: a continuously adaptive neural network approach to model order reduction of parametrized partial differential equations. *Advances in Computational Mathematics*, accepted, 2026.

[2] N. R. Franco, Measurability and continuity of parametric low-rank approximation in Hilbert spaces: linear operators and random variables. *Revista Matemática Complutense*, pages 1–42, 2025.

### A Hybrid Approach for Surrogate Modeling of a Methanation Reactor

Luisa Peterson*, Ion Victor Gosea*, Peter Benner*, Kai Sundmacher*

* Max Planck Institute for Dynamics of Complex Technical Systems
Sandtorstr. 1, 39116, Magdeburg, Germany
gosea@mpi-magdeburg.mpg.de

**Abstract**

Operator inference (OpInf) [1] is an established scientific machine learning approach that enables the construction of reduced-order models by learning the governing dynamics directly from data, bypassing the need for intrusive access to explicit governing equations. A critical challenge in applying OpInf to experimental data is measurement noise, which degrades the finite-difference derivative estimates required for standard regression. To address this issue, we propose a hybrid learning strategy for experimental data-sets that bypasses direct dependence on noisy derivatives.

We treat the latent state as a continuous function approximated by a neural network. This approach is conceptually inspired by DeepMoD [2], which combines PINN and SINDy principles. Here, smoothing and matrix learning occur jointly: the network acts as a differentiable surrogate that filters noise, providing clean time derivatives via automatic differentiation. The OpInf model then serves as a physics-informed regularizer. To ensure robust convergence, we apply a three-stage learning strategy: (1) pre-training the network on data reconstruction (denoising); (2) initializing matrices; and (3) minimizing a composite MSE loss. This formulation decouples the derivative estimation from grid discretization artifacts. The targeted application in this work consists in developing reliable, predictive surrogates for a complex, nonlinear model of a catalytic carbon dioxide methanation reactor [3]. The learned OpInf model serves as the predictive core for optimization-based control. By embedding the reduced-order dynamics as constraints in a nonlinear optimization problem, we aim to maximize reactor performance while adhering to safety limits. We implement the continuous-time optimal control problem using the CasADi framework [4] and solve it via direct multiple shooting with an interior-point solver.

**References**

[1] B. Kramer, B. Peherstorfer, K. E. Willcox, Learning nonlinear reduced models from data with operator inference, Annual Review of Fluid Mechanics 56 (1) (2024) 521–548 https://doi.org/10.1146/annurev-fluid-121021-025220.

[2] G.-J. Both, S. Choudhury, P. Sens, R. Kusters, DeepMoD: Deep learning for model discovery in noisy data, Journal of Computational Physics 428 (2021) 109985 https://doi.org/10.1016/j.jcp.2020.109985.

[3] L. Peterson et al., Toward Digital Twins for Power-to-X Processes: Comparing Surrogate Models for a Catalytic $CO_2$ Methanation Reactor, IEEE Transactions on Automation Science and Engineering 23, (2026) 3469-3483 https://doi.org/10.1109/TASE.2025.3564637

[4] A. Wächter, L. T. Biegler, On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming, Mathematical Programming 106 (1) (2006) 25–57 https://doi.org/10.1007/s10107-004-0559-y.

**Convergence and Sketching-Based Efficient Computation of Neural Tangent Kernel Weights in Physics-Based Loss**

Max Hirsch[1] and Federico Pichi[2]

[1]Department of Mathematics, UC Berkeley, Berkeley, CA, USA
MHIRSCH@BERKELEY.EDU
[2]mathLab, Mathematics Area, SISSA, Trieste, Italy
FPICHI@SISSA.IT

**Abstract**

In multi-objective optimization, multiple loss terms are weighted and added together to form a single objective. These weights are chosen to properly balance the competing losses according to some meta-goal. For example, in physics-informed neural networks (PINNs), these weights are often adaptively chosen to improve the network's generalization error. A popular choice of adaptive weights is based on the neural tangent kernel (NTK) of the PINN, which describes the evolution of the network in predictor space during training [1]. The convergence of such an adaptive weighting algorithm is not clear *a priori*. Moreover, these NTK-based weights would be updated frequently during training, further increasing the computational burden of the learning process. In this talk, we will share results stating that under appropriate conditions, gradient descent enhanced with adaptive NTK-based weights is convergent in a suitable sense. Then addressing the problem of computational efficiency, we will present a randomized algorithm inspired by a predictor-corrector approach and matrix sketching which produces unbiased estimates of the NTK up to an arbitrarily small discretization error. Finally, we will show numerical experiments to support our theoretical findings and to exhibit the efficacy of our randomized algorithm.

**References**

[1] S. Wang, X. Yu, and P. Perdikaris. When and why PINNs fail to train: A neural tangent kernel perspective. *Journal of Computational Physics*, vol. 449, p. 110768, 2022. doi: 10.1016/j.jcp.2021.110768.

[2] Max Hirsch and Federico Pichi. Convergence and Sketching-Based Efficient Computation of Neural Tangent Kernel Weights in Physics-Based Loss. *arXiv preprint arXiv:2511.15530*, 2025. Available at: `https://arxiv.org/abs/2511.15530`.

## Low-Rank Surrogates for Parametric PDEs

Benno Huber, Robert Scheichl, Jakob Zech

Heidelberg University, Heidelberg, Germany
benno.huber@uni-heidelberg.de

### Abstract

Evaluating the forward solution operator for parametric partial differential equations (PDEs) typically involves solving large linear systems, making the process computationally expensive. Operator surrogates aim to provide a computationally efficient approximation to these parameter-to-solution operators. This is particularly beneficial in applications requiring many evaluations of the forward map, such as uncertainty quantification (UQ) or optimization.

Mathematically, operator surrogates provide approximations of mappings between infinite dimensional spaces. We analyze an encoder-decoder framework, where the input and output function spaces are parametrized using coefficient sequences of admissable representation systems. The goal then becomes to approximate a coefficient-to-coefficient map, which can be realized using various approximation tools. In the present work we focus on low-rank tensor representation using the tensor-train (TT) format. This format allows for efficient storage, evaluation and basic linear algebra as long as the tensor ranks remain low. Its structure also allows for efficient quadrature, useful e.g. in UQ applications. We show approximation rates with regards to the storage complexity of such TT surrogates for certain holomorphic maps by providing rank bounds. Additionally, we present benchmark results comparing the TT surrogate to a range of other operator surrogate methods applied to a parametric diffusion problem with varying input smoothness.

## Neural Network-Based Numerical Methods for One-Dimensional Conservation Laws

Imre Fekete[a,b], Ferenc Izsák[a], Vendel Péter Kupás[a]

[a] Department of Applied Analysis and Computational Mathematics,
ELTE Eötvös Loránd University, Budapest, Hungary
[b] Department of Network and Data Science, Central European University, Wien, Austria
kvendel@student.elte.hu, imre.fekete@ttk.elte.hu, ferenc.izsak@ttk.elte.hu

### Abstract

The numerical approximation of nonlinear conservation laws relies on structure preserving finite volume schemes whose accuracy and robustness depend on the choice of reconstruction procedures and other heuristic design components. In recent years, physics-informed neural networks [1] have provided fully data-driven solvers, while several studies incorporated machine learning into high-resolution schemes, for instance by improving WENO reconstructions [2, 3] or constructing neural network-based limiters [4].

In this work, we propose a neural network architecture that parametrizes a family of TVD slope limiters within a conservative finite volume framework for one dimensional hyperbolic conservation laws. Rather than replacing classical schemes, the network mimics a structured class of methods and selects the optimal member during the training process. The architecture is derived directly from the limiter formulation, yielding an explicit one-to-one correspondence between the trainable network parameters and the coefficients of the induced numerical method. In this sense, the approach provides a minimal and interpretable parametrization of the admissible method class while preserving consistency and conservation. Numerical experiments are presented for the Burgers equation, a traffic flow model, and the shallow water system in one space dimension. The results demonstrate improved accuracy compared to standard limiter choices while maintaining stability properties of the underlying finite volume scheme.

### References

[1] X. Jin, S. Cai, H. Li, G. E. Karniadakis, *NSFnets (Navier-Stokes flow nets): Physics-informed neural networks for the incompressible Navier-Stokes equations*, Journal of Computational Physics, 426 (2021), 109951.

[2] T. Kossaczka, A. D. Jagtap, M. Ehrhardt: *Deep smoothness WENO scheme for two-dimensional hyperbolic conservation laws: A deep learning approach for learning smoothness indicators*, Physics of Fluids, 36 (2024), 036603.

[3] X. Nogueira, J. Fernández-Fidalgo, L. Ramos, I. Couceiro, L. Ramírez, *Machine learning-based WENO5 scheme*, Computers & Mathematics with Applications, 168 (2024), pp. 84–99.

[4] D. Hillebrand, S.-C. Klein, P. Öffner: *Applications of Limiters, Neural Networks and Polynomial Annihilation in Higher-Order FD/FV Schemes*, Journal of Scientific Computing, 97 (2023), 13.

## Pricing American Real Options with Double Continuation Region under Heston Model

Íñigo Arregui, Alejandro López-Núñez and Carlos Vázquez

Universidade da Coruña, A Coruña, Spain and CITIC, A Coruña, Spain
ARREGUI@UDC.ES, ALEJANDRO.LOPEZN@UDC.ES, CARLOSV@UDC.ES

### Abstract

The classical assumption of positive interest rates in financial problems that involve taking decisions is not always realistic. In fact, endogenous negative interest rates are frequently present in this kind of problems, such as gold loans or capital investment options, that can be formulated as American options. In this framework, the presence of a double continuation region has been theoretically studied for American options with one stochastic factor.

In the present work, especially motivated by the gold loan problem, we propose a more realistic approach considering a stochastic volatility model. For American call options with negative rates, we pose the pricing problem in terms of partial differential equations (PDEs) under Heston model. Next, we propose numerical methods based on finite differences, Crank-Nicolson scheme and duality algorithms for complementarity problems, to solve American options with negative rates and one stochastic factor to illustrate the theoretical results. Using the same numerical methods, we obtain a double continuation region also in the case of stochastic volatility.

### References

[1] Í. Arregui, A. López-Núñez, and C. Vázquez, *Pricing American real options with double continuation region under Heston model*, Applied Mathematics and Computation, 513 (2026), Article Nr. 129810.

[2] A. Battauz, M. De Donno, and A. Sbuelz, *Real options and American derivatives: the double continuation region*, Management Science, 61 (2015), pp. 1094–1107.

[3] S. L. Heston, *A closed-form solution for options with stochastic volatility with applications to bond and currency options*, Review of Financial Studies, 1 (1993), pp. 327–343.

[4] C. W. Oosterlee and L. A. Grzelak, *Mathematical Modeling and Computation in Finance*, World Scientific, 2020.

# Jump-Diffusion Models for the Generation Rate in the Pricing of Renewable Energy Certificates (RECs)

M. Carmen Calvo-Garrido, Pablo Pérez Picos, and Carlos Vázquez

Universidade da Coruña, A Coruña, Spain and CITIC, A Coruña, Spain
carmen.calvo.garrido@udc.es, pablo.perez.picos@udc.es, carlosv@udc.es

## Abstract

Although international organizations and countries acknowledge the benefits of renewable energy, the deployment of the related technologies usually requires large investments. So, additional market tools have been created to foster their developments. In particular, Renewable Energy Certificates (RECs), also known as green bonds, which gratify the obtention of electricity from renewables, have been designed and are currently traded in energy markets.

In this work, we extend previous PDEs based models for the pricing of RECs to the case where the generation rate of the renewable follows a jump-diffusion model. More precisely, in [1] a nonlinear PDE model is obtained when the generation rate and number of RECs are considered as underlying factors. Moreover, suitable numerical methods heve been proposed in [1, 2] to solve this model. The inclusion of jumps in the stochastic dynamics of the generation rate leads to a nonlinear PIDE model, with an additional nonlocal (integral) term with respect to the PDE model. In the present work, we also propose suitable numerical methods for the treatment of this term based on [4]. Numerical examples with data taken from [3] illustrate the behaviour of the model and the numerical methods.

## References

[1] M. A. Baamonde-Seoane, M. C. Calvo-Garrido, M. Coulon, C. Vázquez, Numerical solution of a nonlinear PDE model for pricing Renewable Energy Certificates (RECs), Applied Mathematics and Computation, 404 (2021), 126199.

[2] M. A. Baamonde-Seoane, M. C. Calvo-Garrido, C. Vázquez, Pricing renewable energy certificates with a Crank-Nicolson Lagrange-Galerkin numerical method, Journal of Computacional and Applied Mathematics, 422 (2023), 114891.

[3] M. Coulon, J. Khazaei, W. B. Powell, SMART-SREC: A stochastic model of the New Jersey solar renewable energy certificate market, Journal of Environmental Economics and Management, 73 (2015), 13-31.

[4] V. Maurya, V.S. Yadav, A. Singh, M.K. Rajpoot, Efficient pricing of options in jump–diffusion models: Novel implicit–explicit methods for numerical valuation, Mathematics and Computers in Simulation, 217 (2024), 202-225.

## Matrix Riccati BSDEs with Singular Terminal Condition and Stochastic LQ Control with Linear Terminal Constraint

Julia Ackermann[1], Thomas Kruse[1], Petr Petrov[1], Alexandre Popier[2]

[1] Department of Mathematics & Informatics, University of Wuppertal, Germany
[2] Laboratoire Manceau de Mathématiques, Le Mans Université, France
PPETROV@UNI-WUPPERTAL.DE

### Abstract

We analyze a class of multidimensional linear-quadratic stochastic control problems with random coefficients, motivated by multi-asset optimal trade execution. The problems feature non-diffusive controlled state dynamics and a terminal constraint that restricts the terminal state to a prescribed random linear subspace. We derive the associated Riccati backward stochastic differential equation (BSDE) and identify a suitable formalization of its singular terminal condition. Via a penalization approach, we establish existence of a minimal supersolution of the Riccati BSDE and use it to characterize both the value function and the optimal control. We analyze the asymptotic behavior of the supersolution near terminal time and discuss special cases where closed-form solutions can be obtained.

## Partial Integro-Differential Equations and their Applications in Financial Modeling

Daniel Ševčovič

Comenius University in Bratislava, Slovakia
SEVCOVIC@FMPH.UNIBA.SK

### Abstract

In this presentation, we analyze solutions of a non-local nonlinear partial integro-differential equation (PIDE) in multidimensional spaces. Such a class of PIDE often arises in financial modeling. We employ the theory of abstract semilinear parabolic equations in order to prove the existence and uniqueness of solutions on the scale of Bessel potential spaces. We consider a wide class of Lévy measures satisfying suitable growth conditions near the origin and infinity. The novelty of the paper is the generalization of already known results in the one-space dimension to the multidimensional case. We consider Black-Scholes models for option pricing on underlying assets following a Lévy stochastic process with jumps. As an application to option pricing in the one-dimensional space, we consider a general shift function arising from nonlinear option pricing models taking into account a large trader stock-trading strategy. We prove the existence and uniqueness of a solution to the nonlinear PIDE in which the shift function may depend on a prescribed large investor stock-trading strategy function. This is a joint work with J. Cruz and C.I. Udeani based on papers [1], [2], [3] and [4].

### References

[1] Cruz J., Ševčovič, On solutions of a partial integro-differential Black-Scholes equation in Bessel potential spaces, Japan Journal of Industrial and Applied Mathematics 37, 691-721 (2020)

[2] Cruz J., Ševčovič D., Option Pricing in Illiquid Markets with Jumps, Applied Mathematical Finance, 25(4), 389-409 (2018)

[3] Ševčovič D., Udeani C. I., Application of maximal monotone operator method for solving Hamilton-Jacobi-Bellman equation arising from optimal portfolio selection problem. Japan Journal of Industrial and Applied Mathematics, 38(3), 693-713 (2021)

[4] Ševčovič D., Udeani, C. I., Multidimensional linear and nonlinear partial integro-differential equation in Bessel potential spaces with applications in option pricing, Mathematics, 9(13), 1463 (2020)

## Numerical Solution of Allen-Cahn Equation for Planar Curvature Flow with Constraints

Michal Beneš and Maneesh Narayanan

Department of Mathematics,
Faculty of Nuclear Sciences and Physical Engineering,
Czech Technical University, Prague, Czech Republic
MICHAL.BENES@FJFI.CVUT.CZ

### Abstract

In the contribution we discuss numerical solution of the planar curvature flow with the area constraint (see [1]) and its applications. The underlying motion law is captured by the phase-field method using the non-local Allen- Cahn equation (as in [3]), and compared to the parametric method (as in [4]) in selected cases. We summarize the theoretical background of the non-local Allen- Cahn equation with respect to the existence, uniqueness and matched asymptotics recovering the sharp-interface law. The numerical solution used for both approaches is based on the method of lines, with space discretization using the finite-difference method and time discretization using a higher-order time solver (see [2]). In our computational studies, we investigate numerical convergence and demonstrated how the selected examples apply to various problems in natural sciences, such as in the aerosol dynamics, recrystallization, droplet shapes and the cell motion.

### References

[1] J. Rubinstein and P. Sternberg. Nonlocal reaction-diffusion equations and nucleation. *IMA Journal of Applied Mathematics*, 48(3):249–264, 1992.

[2] M. Beneš, S. Yazaki, and M. Kimura. Computational studies of non-local anisotropic Allen-Cahn equation. *Mathematica Bohemica*, 136(4):429–437, 2011.

[3] M. Brassel and E. Bretin. A modified phase field approximation for mean curvature flow with conservation of the volume. *Mathematical Methods in the Applied Sciences*, 34(10):1157–1180, 2011.

[4] M. Beneš, M. Kolář, J.M. Sischka, and A. Voigt. Degenerate area preserving surface Allen-Cahn equation and its sharp interface limit. *International Journal of Numerical Analysis and Modeling*, 22(5):603–613, 2025.

## Stability and Instability in Anisotropic Multi-Phase Interface Evolution

Tokuhiro Eto, Harald Garcke and Robert Nürnberg

CNRS, ICJ (Villeurbanne, France),
Universität Regensburg (Regensburg, Germany),
Università di Trento (Trento, Italy)
eto@math.univ-lyon1.fr, harald.garcke@ur.de, robert.nurnberg@unitn.it

### Abstract

Interfacial motion driven by diffusion plays a fundamental role in materials science and phase separation phenomena. A prototypical example is the Mullins–Sekerka problem, describing the evolution of interfaces as a gradient flow of surface energy coupled to a quasi-static diffusion field. While the two-phase isotropic Mullins–Sekerka flow has been extensively studied, far fewer results are available for multi-phase configurations with junctions, and even fewer in anisotropic settings. Recent work introduced structure-preserving parametric finite element methods for isotropic two- and multi-phase Mullins–Sekerka problems, enabling robust numerical simulation of evolving interface networks with triple junctions [1, 2]. These schemes are unconditionally stable, exactly volume-conserving, and faithfully reproduce the underlying variational structure of the continuous model, providing a reliable computational foundation for diffusion-driven multi-phase dynamics.

Building on this approach, we develop a systematic computational framework for anisotropic multi-phase interfacial motion, focusing on the anisotropic multi-phase Mullins–Sekerka problem. The model can be viewed as a degenerate multi-phase Stefan-type problem incorporating anisotropic surface energies and mobilities, kinetic undercooling, and force balance conditions at triple junctions. We propose a fully discrete parametric finite element scheme extending the structure-preserving methodology of [1, 2] to the anisotropic setting [3]. The scheme satisfies a discrete energy dissipation property. Numerical experiments in two and three dimensions demonstrate robustness for complex evolving networks, including strongly anisotropic energies, nontrivial triple junction dynamics, and characteristic Mullins–Sekerka instabilities.

### References

[1] Eto, T., Garcke, H., Nürnberg, R., *A structure-preserving finite element method for the multi-phase Mullins–Sekerka problem with triple junctions*, Numerische Mathematik, Vol.156, pp.1479–1509, 2024.

[2] Eto, T., Garcke, H., Nürnberg, R., *A parametric finite element method for a degenerate multi-phase Stefan problem with triple junctions*, Computational Method and Applied Mathematics, Vol.26, no.1, pp.43–67, 2026.

[3] Eto, T., Garcke, H., Nürnberg, R., *A Parametric Finite Element Approach for an Anisotropic Multi-Phase Mullins–Sekerka Problem with Kinetic Undercooling*, arXiv:2602.18226, 2026.

### A Threshold-Type Algorithm for Fourth Order Geometric Motions

Katsuyuki Ishii

Kobe University, Kobe, Japan
ISHII@MARITIME.KOBE-U.AC.JP

**Abstract**

In this talk, we are concerned with an algorithm for the motion of hypersurfaces/curves $\{\Gamma(t)\}_{t\geq 0}$ given by the following equation:

$$V = \begin{cases} -\Delta_{\Gamma(t)} H - H\left(|A|^2 - \frac{1}{2}H^2\right), & (N=2,3), \\ -\Delta_{\Gamma(t)} H - H\left(|A|^2 - \frac{1}{2}H^2\right) - 2\sum_{1\leq i<j<k\leq N-1} \kappa_i\kappa_j\kappa_k, & (N\geq 4), \end{cases} \tag{10}$$

on $\Gamma(t)$, $t>0$. Here $V$ is the outward normal velocity of $\Gamma(t)$, $\kappa_i$ is the principal curvature of the hypersurface $\Gamma(t)$ with respect to the outer unit normal ($i=1,2,\ldots,N-1$), $H := \kappa_1+\kappa_2+\cdots+\kappa_{N-1}$, $|A|^2 := \kappa_1^2+\kappa_2^2+\cdots+\kappa_{N-1}^2$, and $\Delta_{\Gamma(t)}$ denotes the Laplace–Beltrami operator on $\Gamma(t)$.

In this talk, we introduce a threshold-type algorithm for the motion defined by (10). This type of algorithm was first proposed by Bence, Merriman, and Osher in 1991 to numerically compute mean curvature flows. Our algorithm is an application of theirs, but the main difference is that we use the fourth order heat equation although they use the second order heat equation.

The main purposes are to define our algorithm and to show the consistency result.

This work is based on my joint work with Professors Y. Kohsaka (Kobe U.), N. Miyake (Kyushu U.), and K. Sakakibara (Kanazawa U. & RIKEN).

## Unified Numerical Analysis of Moving Boundary Problems via the Stabilized SAV Approach

Koya Sakakibara

Institute of Science and Engineering, Kanazawa University, Kanazawa, Japan
ksakaki@se.kanazawa-u.ac.jp

### Abstract

This talk presents a unified mathematical framework for moving boundary problems described as gradient flows, using the stabilized Scalar Auxiliary Variable (SAV) method. The geometric evolution of interfaces presents significant numerical challenges due to severe stiffness and the deterioration of mesh quality during large deformations. To overcome these issues, we establish a fully discrete, linear, and unconditionally energy-stable scheme.

The main focus of this presentation will be on the Curve Shortening Flow (CSF) and the Area-Preserving Curve Shortening Flow (APCSF) as fundamental examples to establish our framework. Special attention is paid to the rigorous treatment of the tangential velocity field and the linearized area constraint, ensuring that the discrete energy dissipation structure is exactly preserved. We introduce a relaxation flow utilizing a weighted Laplacian, and carefully project this tangential velocity onto the unit tangent vector. Furthermore, our adaptive weighting strategies automatically concentrate vertices in high-curvature regions, effectively controlling mesh quality even for non-convex domains.

## Entropy Stable DGSEM for Multi-Fluid 5-Moment GLM-Maxwell Systems

Daniel Bach*, Gregor J. Gassner†

University of Cologne, Germany*,†
DANIEL.BACH@UNI-KOELN.DE*
GGASSNER@UNI-KOELN.DE†

**Abstract**

Multi-fluid 5-moment GLM-Maxwell systems [1] simulate plasma flows using a set of Euler equations for each plasma species, coupled via Lorentz force source terms to a full set of Maxwell's equations extended with the Generalized Lagrange Multiplier (GLM) [2] ansatz to reduce divergence errors occurring during a numerical simulation. Our goal is to derive and construct suitable high order approximations that are provably entropy-stable and in the future we aim to exactly satisfy $\text{div}B = 0$. In this talk we give an overview of the status quo of our research and demonstrate an entropy analysis for multi-fluid 5-moment systems with an arbitrary number of species. We show that modifications of the original system are necessary to obtain an entropy inequality. We mimic the continuous entropy analysis based on recent developments for the Discontinuous Galerkin Spectral Element Method (DGSEM) with the Summation-by-Parts (SBP) property [3]. A key ingredient is the derivation of an entropy-conserving numerical flux function for the system. We will demonstrate our theoretical findings with numerical simulations.

### References

[1] Entropy Stable Numerical Schemes for Two-Fluid Plasma Equations, H. Kumar and S. Mishra, Journal of Scientific Computing 52, 401–425 (2012).

[2] Divergence Correction Techniques for Maxwell Solvers Based on a Hyperbolic Model, C.-D. Munz and P. Omnes and R. Schneider and E. Sonnendrücker and U. Voß, Journal of Computational Physics, 2000

[3] An entropy stable nodal discontinuous Galerkin method for the resistive MHD equations. Part I: Theory and numerical verification, Marvin Bohm and Andrew R. Winters and Gregor J. Gassner and Dominik Derigs and Florian Hindenlang and Joachim Saur, Journal of Computational Physics, 2020

## Adaptive Domain Decomposition Preconditioners for Discontinuous Galerkin Discretization

Vít Dolejší[1], Jakub Šístek[2]

[1]Charles University, Faculty of Mathematics and Physics,
Sokolovská 83, 186 75 Praha, Czech Republic
[2]Institute of Mathematics of the Czech Academy of Sciences,
Žitná 25, 110 00 Praha, Czech Republic
VIT.DOLEJSI@MATFYZ.CUNI.CZ, SISTEK@MATH.CAS.CZ

### Abstract

We developed efficient adaptive strategy solving time-dependent partial differential equations with the aid of space-time discontinuous Galerkin (STDG) method in combination with anisotropic *hp*-mesh adaptation, cf. [4, 3]. To make the STDG approach competitive, efficient algebraic solvers are required. The iterative solution of nonlinear algebraic systems leads to sequences of linear systems that are frequently solved iteratively by Krylov methods with suitable preconditioners. A prominent role is played by the *domain decomposition* (DD) techniques, their combination with discontinuous Galerkin discretization leads to a very comfortable setting. Particularly, two-level Schwarz preconditioners, which accelerate the transfer of information through the whole system, can be constructed in a natural way, see e.g., [1, 2].

We consider additive and hybrid two-level Schwarz preconditioners, we study the convergence of the linear solver in dependence on the number of subdomains and the number of elements of the coarse grid. We propose a simplified cost model measuring the computational costs in terms of floating-point operations, the speed of computation, and the wall-clock time for communications among computer cores. Moreover, the cost model serves as a base of the presented adaptive domain decomposition method which chooses the number of subdomains and the number of elements of the coarse grid in order to minimize the computational costs. The efficiency of the proposed technique is demonstrated by two benchmark problems of compressible flow simulations.

### References

[1] Antonietti, P.F., Houston, P.: A class of domain decomposition preconditioners for *hp*-discontinuous Galerkin finite element methods. J. Sci. Comput. **46**(1), 124–149 (2011)

[2] Antonietti, P.F., Houston, P., Smears, I.: A note on optimal spectral bounds for nonoverlapping domain decomposition preconditioners for *hp*-version discontinuous Galerkin methods. Int. J. Numer. Anal. Model. **13**(4), 513–524 (2016)

[3] Dolejší, V.: Non-hydrostatic mesoscale atmospheric modeling by the anisotropic mesh adaptive discontinuous Galerkin method. Int. J. Comput. Fluids. Dyn. **38**(2-3), 81–101 (2024)

[4] Dolejší, V., May, G.: An anisotropic *hp*-mesh adaptation method for time-dependent problems based on interpolation error control. J. Sci. Comput. **95**(2) (2023)

# A Simple and General Framework for the Construction of Exactly Div-Curl-Grad Compatible Discontinuous Galerkin Finite Element Schemes on Unstructured Simplex Meshes

Michael Dumbser

University of Trento, Trento, Italy
MICHAEL.DUMBSER@UNITN.IT

## Abstract

We introduce a new family of discontinuous Galerkin (DG) finite element schemes for the discretization of first order systems of hyperbolic partial differential equations (PDE) on unstructured simplex meshes in two and three space dimensions that respect the two basic vector calculus identities exactly also at the discrete level, namely that the curl of the gradient is zero and that the divergence of the curl is zero. The key ingredient here is the construction of two compatible discrete nabla operators, a primary one and a dual one, both defined on general unstructured simplex meshes in multiple space dimensions. Our new schemes extend existing cell-centered finite volume methods based on corner fluxes to arbitrary high order of accuracy in space. An important feature of our new method is the fact that only two different discrete function spaces are needed to represent the numerical solution, and the choice of the appropriate function space for each variable is related to the origin and nature of the underlying PDE. The first class of variables is discretized at the aid of a discontinuous Galerkin approach, where the numerical solution is represented via piecewise polynomials of degree N and which are allowed to jump across element interfaces. This set of variables is related to those PDE which are mere consequences of the definitions, derived from some abstract scalar and vector potentials, and for which involutions like the divergence-free or the curl-free property must hold if satisfied by the initial data. The second class of variables is discretized via classical continuous Lagrange finite elements of approximation degree M=N+1 and is related to those PDE which can be derived as the Euler-Lagrange equations of an underlying variational principle. The primary nabla operator takes as input the data from the FEM space and returns data in the DG space, while the dual nabla operator takes as input the data from the DG space and produces output in the FEM space. The two discrete nabla operators satisfy a discrete Schwarz theorem on the symmetry of discrete second derivatives. From there, both discrete vector calculus identities follow automatically. We apply our new family of schemes to three hyperbolic systems with involutions: the system of linear acoustics, in which the velocity field must remain curl-free and the vacuum Maxwell equations, in which the divergence of the magnetic field and of the electric field must remain zero. In our approach, only the magnetic field will remain exactly divergence free. As a third model we study the Maxwell-GLM system of Munz *et al.*, which contains a unique mixture of curl-curl and div-grad operators and in which the magnetic field may be either curl-free or divergence-free, depending on the choice of the initial data. In all cases we prove that the proposed schemes are exactly total energy conservative and thus nonlinearly stable in the $L^2$ norm.

## $L^2$-Stability of Explicit Runge–Kutta Methods with SUPG Stabilization for Transient Transport Problems

Alexandre Ern

CERMICS, ENPC, IPP and INRIA Paris, 77455 Marne-la-Vallée Cedex 2, France
ALEXANDRE.ERN@ENPC.FR

### Abstract

We analyze the approximation of linear transport problems using continuous finite elements in space with SUPG stabilization and explicit Runge–Kutta (ERK) schemes in time. We establish stability for ERK3 and ERK4 schemes in the $L^2$-norm augmented by the mesh-dependent weighted advective derivative induced by the SUPG stabilization. We provide two proofs. The first proof hinges on the stability of Taylor polynomials and covers both ERK3 and ERK4 schemes [1, 2]. The second proof uses more classical energy arguments and covers only ERK3 schemes. This second proof is presented to provide a more direct connection to previous work in the literature [3, 4]. We present numerical experiments illustrating the theoretical findings, including a Fourier analysis in 1D and the approximation of smooth solutions and solutions in BV in 2D. This is joint work with Jean-Luc Guermond (TAMU).

### References

[1] Z. Sun and C.-W. Shu. Strong stability of explicit Runge-Kutta time discretizations. *SIAM J. Numer. Anal.*, 57(3):1158–1182, 2019.

[2] A. Ern and J.-L. Guermond. Convergence of explicit Runge-Kutta discontinuous Galerkin approximations of the first-order form of Maxwell's equations with low regularity solutions. *hal-05142005*, 2025.

[3] E. Burman, A. Ern, and M. A. Fernández. Explicit Runge-Kutta schemes and finite elements with symmetric stabilization for first-order linear PDE systems. *SIAM J. Numer. Anal.*, 48(6): 2019–2042, 2010.

[4] E. Burman. Consistent SUPG-method for transient transport problems: stability and convergence. *Comput. Methods Appl. Mech. Engrg.*, 199(17-20):1114–1123, 2010.

# DG = FEM + Flat Elements

Václav Kučera

Charles University, Prague, Czech Republic
KUCERA@KARLIN.MFF.CUNI.CZ

## Abstract

The *tempered finite element method* (TFEM), [1], is a simple, easy to implement modification of vanilla finite elements (FEM) that allows computation on meshes containing nearly or completely degenerate (flattened, or zero-measure) elements. In the original paper [1], we showed how jump-penalization mortaring can be obtained by considering degenerate elements in a FEM triangulation. Here we go one step further and show how a simple modification of the computational mesh (by flattening certain 'edge' elements) allows one to trivially implement discontinuous Galerkin (DG) methods in any off-the-shelf FEM code. We demonstrate this on diffusion, as well as nonlinear evolutionary convection problems. We theoretically analyze the resulting DG scheme and demonstrate its performance on problems ranging from Poisson's problem to compressible Euler equations. Thus the TFEM framework not only fixes FEM by making it more robust to mesh degeneracy, but one can actively use TFEM to easily implement mortaring and/or DG schemes in ordinary FEM codes simply by collapsing chosen elements.

## References

[1] Antoine Quiriny, Václav Kučera, Jonathan Lambrechts, Nicolas Moës and Jean-François Remacle. *The tempered finite element method.* Journal of Computational Physics, 549, 2026.

## Positivity-Preserving Stationarity Preserving PAMPA-DG Scheme with Global Flux Quadrature

Remi Abgrall†, Yongle Liu† and Mario Ricchiuto‡
† Institute of Mathematics, University of Zürich (remi.abgrall@math.uzh.ch, yongle.liu@math.uzh.ch)
‡ University of Bordeaux (mario.ricchiuto@inria.fr)

### Abstract

In this talk, we present a novel stationarity preserving Point-Average-Moment PolynomiAl-interpreted (PAMPA) method for solving the nonlinear hyperbolic problems. We begin with the one-dimensional setting and the mathematical model considered is the shallow water equations. The idea is based on a global flux quadrature formulation, in which the discretization of the source terms is obtained from the derivative of and additional flux function computed via high order quadrature of the source term. The reformulated system is quasi-conservative with global integral terms computed using Gauss–Lobatto quadrature nodes. The resulting method is capable of preserving a large family of smooth moving equilibria: supercritical and subcritical flows, in a super-convergent manner. We also show that, by an appropriate quadrature strategy for the source, we can exactly preserve the still water states. Moreover, to guarantee the positivity of water depth and eliminate the spurious oscillations near shocks, we blend the high-order PAMPA schemes with the first order local Lax–Friedrichs schemes using the method developed in [3]. The first-order schemes are designed to preserve the still water equilibria and positivity of water height, as well as to deal with wet-dry fronts. We then extend the method to the two-dimensional setting on Cartesian meshes using the PAMPA-DG framework [2]. Similar to the 1D, the div$\mathbf{f}$ will be represented using some global flux. The numerical scheme will be tested on both the 2D Euler equations and shallow water equations. Extensive numerical experiments are tested to validate the advantages and robustness of the proposed scheme.

### References

[1] Rémi Abgrall, Miaosen Jiao, Yongle Liu, and Kailiang Wu, "Bound-preserving Point-Average-Moment PolynomiAl-interpreted (PAMPA) scheme: One-dimensional Case", *Communications in Computational Physics*, 39, pp. 29–58, 2026.

[2] Rémi Abgrall and Yongle Liu, Robust PAMPA Scheme in the DG Formulation on Unstructured Triangular Meshes: Bound Preservation, Oscillation Elimination, and Boundary Conditions, in the revision of Journal of Computational Physics, https://doi.org/10.48550/arXiv.2511.16180

## Oscillation-Free Discontinuous Galerkin Method for Conservation Laws

Chi-Wang Shu

Division of Applied Mathematics, Brown University, Providence, RI 02912, USA
CHI-WANG_SHU@BROWN.EDU

### Abstract

We discuss a recently developed class of oscillation-free discontinuous Galerkin (OFDG) method for solving hyperbolic conservation laws. Discontinuous Galerkin methods are well-known for their nice properties for solving hyperbolic conservation laws, such as entropy stability, high order convergence, parallel efficiency and easiness for h-p adaptivity. However, for strong shocks, unmodulated discontinuous Galerkin methods still suffer from spurious oscillations. Traditional strategies to control such oscillations include artificial viscosity and various limiters. Recently, a class of oscillation-free discontinuous Galerkin method has been developed, which contains a carefully designed local damping term and ensures essentially oscillation-free performance. The OFDG method maintains the nice theoretical properties of discontinuous Galerkin methods such as entropy stability, optimal order of convergence, and superconvergence. The method has been developed for scalar equations and systems, including Euler equations and MHD equations. Numerical results demonstrate the nice performance of the method. This is a joint work with Yong Liu and Jianfang Lu.

### References

[1] J. Lu, Y. Liu and C.-W. Shu, *An oscillation-free discontinuous Galerkin method for scalar hyperbolic conservation laws*, SIAM Journal on Numerical Analysis, v59 (2021), pp.1299-1324.

[2] Y. Liu, J. Lu and C.-W. Shu, *An essentially oscillation-free discontinuous Galerkin method for hyperbolic systems*, SIAM Journal on Scientific Computing, v44 (2022), pp.A230-A259.

[3] Y. Liu, J. Lu and C.-W. Shu, *An entropy stable essentially oscillation-free discontinuous Galerkin method for hyperbolic conservation laws*, SIAM Journal on Scientific Computing, v46 (2024), pp.A1132-A1159.

[4] Y. Liu, J. Lu and C.-W. Shu, *An entropy stable essentially oscillation-free discontinuous Galerkin method for solving ideal magnetohydrodynamic equations*, Journal of Computational Physics, v530 (2025), 113911.

## Fluid-Structure Interaction Algorithm for an Elastic Structure with Large Deformations

Ondřej Bublík

NTIS – New Technologies for the Information Society
Faculty of Applied Sciences, University of West Bohemia
Univerzitní 8, 301 00 Plzeň, Czech Republic
OBUBLIK@KME.ZCU.CZ

### Abstract

This contribution presents a modular fluid-structure interaction (FSI) methodology with an elastic structure model. A partitioned approach with weak or strong coupling is used, allowing independent fluid and structure solvers and nonmatching meshes at the fluid-solid interface. The fluid flow is described by the compressible Navier-Stokes equations in the ALE formulation and solved using an implicit discontinuous Galerkin scheme with interior penalty for viscous fluxes. The elastic structure is modeled by nonlinear elastodynamics with large deformations and solved by an implicit finite-element method. Because the meshes do not conform at the interface, aerodynamic stresses and structural displacements are transferred using radial basis function interpolation. The complete FSI framework is validated on the well-known Turek-Hron benchmark.

## Numerical Approximation of Fluid-Structure Interaction: From Mathematical Modeling to Applications

Petr Sváček

Czech Technical University in Prague, Faculty of Mechanical Engineering, Czech Republic
PETR.SVACEK@FS.CVUT.CZ

### Abstract

This talk is devoted to the mathematical modeling and numerical approximation of fluid-structure interaction (FSI) problems. Mutual interactions between flowing fluids and vibrating structures are critical across a wide range of engineering and scientific disciplines. A prominent historical example of such phenomena is the collapse of the Tacoma Narrows suspension bridge due to aeroelastic instability. Today, FSI must be rigorously accounted for in the design of bridges, tall buildings, and lightweight membrane structures, as well as in mechanical engineering, where aircraft wings or turbomachinery blades may oscillate dangerously due to fluid-structure coupling. Furthermore, FSI is essential in biomechanics, notably in the study of blood flow through elastic arteries and the self-induced vibrations of vocal folds during human phonation. While traditional aeroelasticity and hydroelasticity often rely on linearized models limited to small deformations to determine stability regions, these approaches fail to capture nonlinear phenomena such as post-flutter behavior or limit cycle oscillations. Numerical simulations represent a practical tool for exploring these complex interactions.

Moving from modeling to numerical implementation, we provide a survey of several key aspects of the numerical approximation of FSI problems in the context of incompressible flows. To treat the domain motion, the Arbitrary Lagrangian-Eulerian (ALE) formulation is introduced, and the application of ALE techniques across different spatial discretization methods — including the finite volume and finite element methods — are examined. In particular, the effects of time discretization schemes on the ALE framework are explored, evaluating the overall stability and accuracy of the numerical solution. Coupling strategies, such as monolithic and partitioned approaches, are reviewed. To satisfy the divergence constraint, the choice of suitable finite element pairs is addressed in relation to stabilization methods for convection-dominated flows at high Reynolds numbers. Finally, numerical examples are presented, ranging from classical aeroelastic problems to the complex simulation of human phonation.

### References

[1] P. Sváček and J. Horáček. Numerical simulation of glottal flow in interaction with self oscillating vocal folds. *Communications in Computational Physics*, 12(3):789–806, 2012.

[2] Petr Sváček and Jaromír Horáček. Numerical simulation of aeroelastic response of an airfoil in flow with laminar–turbulence transition. *Applied Mathematics and Computation*, 267:28–41, 2015.

## Efficient Linear Finite Element Scheme for Fluid–Structure Interaction: Partitioned vs Monolithic Approach

S. Schwarzacher, B. She, T. Tian, Karel Tůma

Faculty of Mathematics and Physics, Charles University, Czech Republic
ktuma@karlin.mff.cuni.cz

We compare two efficient linear finite element schemes for non-linear fluid–structure interaction between an incompressible viscous fluid and an elastic plate or shell, formulated in the arbitrary Lagrangian–Eulerian (ALE) framework on a fixed reference domain. Both methods employ backward Euler time discretization and a low-order spatial discretization with P1-bubble/P1 elements for the fluid and P1 elements for the structure. The semi-implicit monolithic scheme from [1] advances the coupled system by a single linear solve per time step and dissipates the total energy at the discrete level. The partitioned scheme from [2] updates fluid and structure sequentially without sub-iterations while preserving a discrete energy estimate, even for large deformations with a convection term.

We establish optimal first order convergence with respect to the time step and mesh size and confirm it numerically in strongly deforming regimes. Figure 1 illustrates representative large-deformation snapshots together with the observed timestep convergence. Finally, we compare efficiency: the monolithic scheme reduces cost relative to a fully implicit solve [1], and the partitioned variant further lowers per-step cost while maintaining comparable accuracy. The approach naturally extends to fully non-linear bulk elasticity models, including compressible neo-Hookean and Saint Venant–Kirchhoff materials.

### References

[1] S. Schwarzacher, B. She, K. Tůma: *Stability and error estimates of a linear numerical scheme approximating nonlinear fluid-structure interactions*. Numerische Mathematik 157.3: 1023-1077, 2025.

[2] B. She, T. Tian, K. Tůma: *Stability and error estimates of a linear and partitioned finite element method approximating nonlinear fluid-structure interactions*, arXiv preprint arXiv:2601.02847, 2026.

## Fluid–Structure–Acoustic Interaction Simulation Motivated by Human Phonation

Jan Valášek[1,2] and Prokop Pučejdl[2] and Petr Sváček[2]

[1] Institute of Mathematics, Czech Academy of Sciences, Žitná 25, Praha 1, Czechia
[2] Faculty of Mechanical Engineering, CTU in Prague, Karlovo nám. 13, Praha 2, Czechia
jan.valasek@fs.cvut.cz

### Abstract

This presentation focuses on the fluid–structure–acoustic interaction (FSAI) problem with particular emphasis on its application to human phonation, [1]. The problem involves the coupling of three physical subsystems: a complex airflow, the vibration of an elastic structure represented by the vocal folds, and the surrounding acoustic field. The general mathematical model of the FSAI problem is introduced and possible simplifications of the fully coupled formulation are discussed.

The fluid–structure interaction (FSI) is governed by the incompressible Navier–Stokes equations coupled with a geometrically nonlinear elasticity model, which is suitable for large deformations and, more importantly, significant rotations of the vocal folds. The fluid formulation employs the Arbitrary Lagrangian–Eulerian (ALE) method to treat the time-dependent fluid domain, and SUPG stabilization is applied within the finite element discretization to ensure numerical stability in convection-dominated regimes. Special attention is devoted to the inlet boundary condition, which is imposed in a penalized form, [4]. This approach can be interpreted as a generalization of classical Dirichlet and Neumann boundary conditions and has proven to be more robust and physically appropriate for modelling inflow conditions in phonatory flows. The influence of structural prestressing—mimicking the prephonatory configuration of the vocal folds—is also investigated and shown to be significant, [2].

An aeroacoustic model based on a perturbation approach is briefly introduced as a reasonable approximation of the full compressible Navier–Stokes equations that retains the essential sound-generation mechanisms, [3]. Finally, numerical simulations of flow-induced vocal fold vibrations and the subsequent propagation of sound through the human vocal tract are presented.

#### References

[1] M. Döllinger, Z. Zhang, S. Schoder, P. Šidlof, B. Tur, and S. Kniesburges, *Overview on state-of-the-art numerical modeling of the phonation process*, 1–25.

[2] P. Pučejdl, J. Valášek, T. Marhan, and P. Sváček, *Modal analysis of prestressed vocal fold model*, Topical problems of fluid mechanics 2026, to appear in 2026.

[3] J. Valášek and P. Sváček, *Numerical simulation of fluid–structure–acoustic interaction motivated by healthy human phonation*, Journal of Computational and Applied Mathematics **472** (2026), 116771.

[4] J. Valášek, P. Sváček, and J. Horáček, *On suitable inlet boundary conditions for fluid-structure interaction problems in a channel*, Applications of Mathematics **64**:2 (2019), 225–251.

## Hölder Regularity for Degenerate First-Order Hamilton-Jacobi Equations under Controllability Conditions

Megan Griffin-Pickering

Universität Zürich, Zürich, Switzerland
MEGAN.GRIFFIN-PICKERING@MATH.UZH.CH

### Abstract

This is joint work with Alpár Mészáros (Durham University, Durham, England, United Kingdom).

Hamilton-Jacobi(-Bellman) equations are key elements in optimal control and mean field game theory. We study the regularity of solutions to first-order Hamilton-Jacobi equations. Despite the lack of parabolic structure, it is known that viscosity solutions of first-order Hamilton-Jacobi equations are Hölder continuous provided that the Hamiltonian is coercive with respect to the momentum variable with superlinear growth (e.g. [1, 2]). However, a large class of first-order HJB equations do not satisfy this hypothesis, even when the Lagrangian cost in the underlying optimal control problem has superlinear growth with respect to the control variable. This occurs if, in the controlled ODE, certain directions are forbidden to the controller. Concretely, consider a linear time-invariant system

$$\dot{x} = Ax + B\alpha, \qquad x, \alpha \in \mathbb{R}^n \tag{11}$$

where $A$ and $B$ are constant $n \times n$ matrices; if the rank of $B$ is strictly less than $n$, then the Hamiltonian in the HJB equation for an optimal control problem based on this system cannot be coercive in any direction $p$ that is orthogonal to the image of $B$.

Motivated by this type of control problem, we study viscosity solutions $u$ of first-order Hamilton-Jacobi equations of the form

$$\partial_t u + (Ax)^\top D_x u + H(t, x, D_x u) = 0,$$

where $H$ has *degenerate* superlinear growth of the form

$$|B^\top D_x u|^q - f(t,x) \lesssim H(t, x, D_x u) \lesssim |B^\top D_x u|^q + C$$

for some $q > 1$, constant $C \in \mathbb{R}$ and $f$ locally $L^r$ for $r$ sufficiently large. We show that, if the system (11) is *controllable*, then $u$ is Hölder continuous.

### References

[1] P. Cardaliaguet. A note on the regularity of solutions of Hamilton-Jacobi equations with superlinear growth in the gradient variable. *ESAIM Control Optim. Calc. Var.*, 15(2):367–376, 2009.

[2] Chan, C.H., Vasseur, A. De Giorgi Techniques Applied to the Hölder Regularity of Solutions to Hamilton–Jacobi Equations. *In: Gonçalves, P., Soares, A. (eds) From Particle Systems to Partial Differential Equations. PSPDE 2015.* Springer Proceedings in Mathematics & Statistics, vol 209. Springer, Cham, 2017.

## The Effect of Quadrature on the Convergence of Policy Iteration for Hamilton–Jacobi–Bellman Equations

Thomas Hall, Iain Smears, and Harry Wells

University College London, UK
tom.hall.21@ucl.ac.uk

### Abstract

Modern finite element libraries allow users to express partial differential equations directly in variational form, with the added convenience of automatic selection of numerical quadrature. In the context of Hamilton–Jacobi–Bellman equations, we show that the choice of quadrature plays a significant role in the observed performance of the policy iteration solver for the nonlinear problem, which is known from theory to converge locally superlinearly in the absence of quadrature error.

In particular, we show that automatic quadrature selection can result in inconsistent choices in the assembly of matrix and right-hand side, thereby causing stagnation and, in some cases, loss of convergence of the policy iteration. We also show that the simple remedy of enforcing consistent choices of quadrature for both matrix and right-hand side recovers the expected superlinear convergence. These results reveal a subtle but significant interaction between quadrature and nonlinear iteration in finite element implementations.

## Wellposedness and Quantitative Convergence for Distributed Equilibria of Displacement Monotone $N$-Player Games with Interaction Through Controls

Hei Jie Lam

Durham University, UK
hei.j.lam@durham.ac.uk

### Abstract

In this talk we will study the wellposedness of distributed equilibria of $N$-player games under displacement semi-monotonicity and convexity assumptions, in which the running cost of a player depends also on the controls used by other players. We consider running costs that are not necessarily separable, resulting in a set of consistency/fixed point relations on infinite dimensional spaces. We will also talk about quantitative convergence results (both for optimal trajectories/control and value functions) for the $N$-player games to the corresponding Mean Field Games of Controls (MFGC). Our approach works for both stochastic and deterministic cases but in this talk we will focus on the deterministic case (distributed equilibria coincides with open-loop equilibria in this case), where further quantitative convergence results can be proved for the gradients of value functions. This talk is based on joint work with Alpár Mészáros (Durham University).

### On Schauder Estimates for Nonlocal Viscous Hamilton Jacobi Equations

Robin Lien

Norwegian University of Science and Technology (NTNU), Trondheim, Norway
ROBIN.O.LIEN@NTNU.NO

#### Abstract

Viscous Hamilton–Jacobi equations arise as dynamic programming equations in optimal control and differential games, where the unknown function represents the value function of an optimally controlled process. We use a heat-kernel and semigroup approach to study the spatial regularity for such equations with a nonlocal diffusion term, specifically Lévy-type diffusion operators such as the fractional Laplacian. Using the Duhamel (mild) formulation together with quantitative smoothing estimates in Hölder spaces, we explain how the order of the diffusion determines the spatial smoothing effect: for any positive time the solution gains the optimal amount of Hölder regularity in space, and we can quantify how the relevant norms behave as time approaches zero. We focus on two canonical regimes: solutions with uniformly bounded gradients, and solutions whose gradients may blow up as time approaches zero. In addition to regularity, we discuss existence, uniqueness, and conditions under which solutions are classical. Our results are obtained under mild assumptions on the heat-kernel and the regularity of the data. Our motivation comes from Mean Field Games, and our goal is to extend our results to the full nonlocal Mean Field Game system with low-regularity data. The talk is based on our paper "On Schauder estimates for nonlocal and mixed local–nonlocal viscous Hamilton–Jacobi equations".

**Long Time Behaviour, Stabilisation and Turnpike for Displacement Monotone Mean Field Games**

Alpár R. Mészáros

Durham University, UK
ALPAR.R.MESZAROS@DURHAM.AC.UK

**Abstract**

In this talk we will study the long time behaviour of Nash equilibria in Mean Field Games within the framework of displacement monotonicity. We first show that any two equilibria defined on the time horizon $(0,T)$ must be close as $T \to +\infty$, in a suitable sense, independently of initial/terminal conditions. The way this stability property is made quantitative involves the $L^2$ distance between solutions of the associated Pontryagin system of FBSDEs that characterises the equilibria. Therefore, this implies in particular the stability in the 2-Wasserstein distance for the two flows of probability measures describing the agent population density and the $L^2$ distance between the co-states of agents, that are related to the optimal feedback controls. We then prove that the value function of a typical agent converges as $T \to +\infty$ and we describe this limit via an infinite horizon MFG system, involving an ergodic constant. All of our convergence results hold true in a unified way for deterministic and idiosyncratic noise driven Mean Field Games, in the case of strongly displacement monotone non-separable Hamiltonians. All these are quantitative at exponential rates. The talk is based on a recent joint work with Marco Cirant (Padova).

## Error Bounds for Finite Element Approximations of Stationary Second-Order Mean Field Game Systems

Yohance Osborne

Durham University, Durham, United Kingdom
YOHANCE.A.OSBORNE@DURHAM.AC.UK

### Abstract

We present error bounds for monotone stabilized finite element discretizations of stationary second-order mean field game (MFG) partial differential inclusion systems of the form

$$\begin{aligned} -\nu\Delta u + H(x,\nabla u) &= F[m](x) && \text{in } \Omega, \\ -\nu\Delta m - G(x) &\in \operatorname{div}(m\partial_p H(x,\nabla u)) && \text{in } \Omega, \end{aligned} \tag{12}$$

supplemented with homogeneous Dirichlet boundary conditions $u = 0$, $m = 0$ on $\partial\Omega$, where the Hamiltonian $H = H(x,p)$ is merely Lipschitz, convex, but (possibly) nondifferentiable with respect to $p$. Here, $\partial_p H$ denotes the partial subdifferential of $H$ with respect to its second argument, and $\Omega \subset \mathbb{R}^d$ denotes a general bounded polytopal Lipschitz domain. For the system (12) with strongly monotone running cost $F$ and non-negative source $G$, we prove an asymptotic rate of convergence in the $H^1$-norm for the approximations of the value function $u$ and in the $L^2$-norm for the approximations of the player density $m$, despite the density having minimal regularity *a priori* as a function in $H^1$ and there being no continuity of the set-valued partial subdifferential $\partial_p H$. The analysis we present jointly leverages recent *a priori* finite element error analysis for MFG systems with differentiable $C^{1,1}$ Hamiltonians [1], and new quantitative results on the approximation of MFG systems with merely $C^{0,1}$ Hamiltonians by MFG systems with regularized $C^{1,1}$ Hamiltonians as the regularization parameter vanishes [2]. This talk is based on joint work with Iain Smears in [3].

### References

[1] Y. A. P. Osborne and I. Smears, *Near and full quasi-optimality of finite element approximations of stationary second-order mean field games*, Math. Comp., (2025), https://doi.org/10.1090/mcom/4080.

[2] Y. A. P. Osborne and I. Smears, *Regularization of stationary second-order mean field game partial differential inclusions*, SIAM J. Math. Anal., 57 (2025), pp. 5189–5215, https://doi.org/10.1137/24M1686401.

[3] Y. A. P. Osborne and I. Smears, *Rates of convergence of finite element approximations of second-order mean field games with nondifferentiable Hamiltonians*, arXiv preprint arXiv:2506.03039, (2025), https://arxiv.org/abs/2506.03039. (To appear in *Mathematics of Computation*)

## Fully Nonlinear Mean Field Games with Nondifferentiable Hamiltonians

Thomas Sales

University of Sussex, Brighton, United Kingdom
T.P.SALES@SUSSEX.AC.UK

### Abstract

Mean field games [1] are systems of partial differential equations modelling the Nash equilibria of dynamic differential games for large populations of players. In their full generality this system is fully nonlinear and may involve a nondifferentiable Hamiltonian — in which case not only is the analysis more involved than the usual quasilinear case (with a differentiable Hamiltonian), but even the statement of the problem is not so obvious. In this talk, we discuss a novel approach to studying a model problem of the form

$$\begin{aligned} H(x, D^2u(x)) &= F[m](x) \quad \text{in } \Omega, \\ D^2 : \left( m \frac{\partial H}{\partial M}(x, D^2u(x)) \right) &= G(x) \qquad \text{in } \Omega, \\ u = 0, \qquad m = 0 & \qquad\qquad \text{on } \partial\Omega \end{aligned}$$

by introducing a non-standard variational inequality formulation. Here $H : \Omega \times \mathbb{R}^{d\times d}_{\text{sym}} \to \mathbb{R}$ is a Hamiltonian of the form

$$H(x, M) := \sup_{\alpha\in\mathcal{A}} \{-a(x, \alpha) : M - f(x, \alpha)\}.$$

Under reasonable assumptions, such as assuming the Hamiltonian satisfies a uniform Cordes condition, we show that this variational inequality formulation admits a solution and moreover this solution is unique under the usual monotonicity assumptions. This talk is based on joint work [2] with Iain Smears (UCL).

### References

[1] Lasry, J. M. and Lions, P. L., 2007. Mean field games. Japanese journal of mathematics, 2(1), pp. 229-260.

[2] Sales, T. and Smears, I., 2025. Fully nonlinear second-order mean field games with nondifferentiable Hamiltonians. arXiv preprint arXiv:2511.12780.

### A Posteriori Error Bounds for Finite Element Approximations of Mean Field Games

Harry Wells & Iain Smears

University College London, United Kingdom
HARRY.WELLS@UCL.AC.UK

#### Abstract

A posteriori analysis is a powerful tool in the finite element method (FEM) approximation of partial differential equations (PDE). The analysis provides computable error estimators which can be used to reliably evaluate the accuracy of a numerical approximation without prior knowledge of the underlying analytical solution.

In this talk we present recent works on the a posteriori analysis of stabilised FEM approximations to mean field games (MFG). A posteriori analysis for MFG presents several interesting challenges due to the lack of coercivity of the MFG system, the nonlinear coupling terms, the requirement of a discrete maximum principle, and the loss of Galerkin orthogonality due to stabilisation. To overcome this we prove an equivalence result between approximation error and the dual norm of the corresponding PDE residuals of the MFG system. Then we derive computable error estimators and we prove their reliability and efficiency with respect to the error between exact and numerical solutions.

**Unconditionally Stable Finite Element Scheme for Fluid-Structure Interaction Problems**

Daniele Boffi

King Abdullah University of Science and Technology (KAUST), Saudi Arabia
and University of Pavia, Italy
DANIELE.BOFFI@KAUST.EDU.SA

**Abstract**

In this talk I will present a model for the numerical approximation of fluid-structure interaction problems based on a fictitious domain approach and on the use of a Lagrange multiplier. I will follow the formulation of [1] which is a follow up of a method of the family of Immersed Boundary schemes such as [2, 3].

One of the main features of our approach is the unconditional stability in time also when a semi-explicit time marching scheme is used [4]. As opposed to other unfitted approach, the presence of small cut cells doesn't affect the stability and the conditioning of the numerical scheme [5].

**References**

[1] D. Boffi and L. Gastaldi. A fictitious domain approach with distributed Lagrange multiplier for fluid-structure interactions. Numer. Math., 135(3) (2017) 711-732.

[2] D. Boffi, N. Cavallini, and L. Gastaldi. Finite element approach to immersed boundary method with different fluid and solid densities. Mathematical Models and Methods in Applied Sciences, 21(12) 2011, pp. 2523-2550.

[3] D. Boffi. The immersed boundary method for fluid-structure interactions: mathematical formulation and numerical approximation. Bollettino U.M.I., (9) V (2012) pp. 711-724.

[4] D. Boffi, N. Cavallini, and L. Gastaldi. The Finite Element Immersed Boundary Method with Distributed Lagrange multiplier. SIAM Journal on Numerical Analysis, 53(6), (2015), 2584-2604.

[5] D. Boffi, F. Credali, and L. Gastaldi, On the stability and conditioning of a fictitious domain formulation for fluid-structure interaction problems. SIAM Journal on Scientific Computing, to appear.

## Hierarchical Surrogate Modeling for Droplet-Laden Stokes Flows

Davide Pradovera*, Thomas Frachon†, and Sara Zahedi†

*Stockholm University, Stockholm, Sweden
†KTH Royal Institute of Technology, Stockholm, Sweden
DAVIDE.PRADOVERA@MATH.SU.SE

### Abstract

We present a surrogate modeling strategy for Stokes flows where deformable liquid droplets are suspended in a carrier fluid [1]. Our approach is based on a multi-fidelity framework: at the lowest fidelity, droplets are treated as passive tracers, neglecting their influence on the ambient flow field. Building on this approximation, we derive a PDE whose solution represents the current modeling error. This error equation is then solved approximately to correct the flow field, and the procedure is iterated.

Two fidelities are employed in an alternating fashion: Stokes flow in the absence of droplets and flow around a single droplet in free space. By systematically combining these models, the method can capture droplet-flow, droplet-boundary, and droplet-droplet interactions. If the droplets are geometrically similar, we further develop an efficient strategy that exploits this structure through an offline–online decomposition, enabling the reuse of precomputed single-droplet solutions. Numerical experiments demonstrate the accuracy and efficiency of the proposed surrogate across a range of tests, including scenarios with up to $10^5$ droplets. Notably, we show that the proposed surrogate achieves significantly reduced computational cost compared to fully resolved multi-fluid simulations with state-of-the-art software.

### References

[1] D. Pradovera, T. Frachon, and S. Zahedi, "Hierarchical surrogate modeling for droplet-laden Stokes flows", in preparation.

### Structure-Preserving Fully Discrete Methods for Magnetoelastic Materials

Michele Ruggeri

Department of Mathematics, University of Bologna, Bologna, Italy
M.RUGGERI@UNIBO.IT

**Abstract**

Magnetoelastic materials are smart materials with a strong interplay between their mechanical and magnetic properties, and as such, they provide an example for a multiphysics system. In this talk, we present structure-preserving fully discrete numerical schemes [1, 2] for approximating solutions to a nonlinear system of evolutionary partial differential equations modeling the dynamics of magnetization and displacement in magnetoelastic materials in the small strain regime. For each method, we discuss well-posedness, energetic behavior, accuracy, (unconditional) stability and convergence, and illustrate its performance by means of numerical experiments. This is joint work together with Martin Kružík (Czech Academy of Sciences) and Hywel Normington (University of Strathclyde).

#### References

[1] M. Kružík, H. Normington, M. Ruggeri. *A decoupled, stable and second-order integrator for the Landau–Lifshitz–Gilbert equation with magnetoelastic effects*, arXiv:2512.01741 (2025).

[2] H. Normington, M. Ruggeri. *A decoupled, convergent and fully linear algorithm for the Landau–Lifshitz–Gilbert equation with magnetoelastic effects.* Comput. Math. Appl. 187 (2025), 1–29.

## Domain Decomposition with Neural Model Order Reduction for Multiscale Mixed-Dimensional Problems

Francesca Arceci, Nunzio Dimola, Piermario Vitullo, Paolo Zunino

MOX, Department of Mathematics, Politecnico di Milano, Milan, Italy
PAOLO.ZUNINO@POLIMI.IT

### Abstract

Multiscale and multiphysics problems involving heterogeneous media and coupled processes across different spatial dimensions arise in a wide range of scientific and engineering applications, including transport phenomena in biological tissues, porous media, and vascularized systems [1, 2]. The numerical approximation of such problems is particularly challenging due to geometric complexity, strong scale separation, and the high computational cost associated with repeated simulations in parametrized or data-rich settings. In this work, we present a computational framework that combines non-overlapping domain decomposition methods with non-intrusive, data-driven reduced order models to efficiently approximate multiscale mixed-dimensional partial differential equations, with a particular focus on coupled 3D–1D formulations.

The proposed approach relies on a localization strategy in which the global problem is decomposed into parametrized local subproblems, coupled through optimized Schwarz-type transmission conditions. Reduced order models, constructed from localized high-fidelity data and implemented using neural-network-based surrogates [2], are embedded within domain decomposition iterations to accelerate the solution of the most computationally demanding subdomain problems. We discuss the mathematical structure of the method, including the notion of local representability and the impact of surrogate-induced perturbations on convergence and stability. Numerical results on representative mixed-dimensional diffusion and transport problems illustrate the effectiveness of the proposed domain decomposition–neural reduced order model paradigm in achieving significant computational savings while preserving accuracy and global coherence. The presented framework provides a flexible foundation for scalable, learning-augmented solvers for complex multiphysics systems.

### References

[1] E. Vidotto, T. Koch, T. Köppl, R. Helmig, and B. Wohlmuth, Hybrid models for simulating blood flow in microvascular networks, *Multiscale Modeling and Simulation*, 17(3) (2019), pp. 1076–1102.

[2] P. Vitullo, A. Colombo, N. R. Franco, A. Manzoni, and P. Zunino, Nonlinear model order reduction for problems with microstructure using mesh informed neural networks, *Finite Elements in Analysis and Design*, 229 (2024), 104068.

## Block-Jacobi Preconditioners for Advection Problems

Ivo Dravins

KTH Royal Institute of Technology, Stockholm, Sweden
DRAVINS@KTH.SE

### Abstract

We present an a class of block-Jacobi preconditioners for advection-dominated problems. We discuss its construction and application for various discretization schemes including high-order discontinuous Galerkin discretizations on hexahedral meshes with tensor product basis functions. We form the element-wise inverses based on the fast diagonalization method for an approximation of the cellwise problem by constant coefficients on a Cartesian mesh. In order to overcome the degradation of the cellwise inverse when the transport speed is highly variable, we analyze cellwise iterative solvers.

This talk is based on joint work with Martin Kronbichler and Katharina Kormann of the Ruhr University Bochum, Germany.

### Determining the Space Dependent Coefficients in Space-Time Fractional Diffusion Equations via Krylov Preconditioning

Muhammad Faisal Khan

Charles University Prague, Czech Republic
faisalkhanniazi758@gmail.com

**Abstract**

We consider a time-space fractional diffusion equation with a variable coefficient and investigate the inverse problem of reconstructing the source term, after regularizing the problem with the quasi-boundary value method to mitigate the ill-posedness. The equation involves a Caputo fractional derivative in the space variable and a tempered fractional derivative in the time variable, both of order in $(0, 1)$. A finite difference approximation leads to a two-by-two block linear system of large dimensions. We conduct a spectral analysis of the associated matrix sequences, employing tools from Generalized Locally Toeplitz (GLT) theory, and construct the preconditioner guided by the GLT analysis. Numerical experiments are reported and commented, followed by concluding remarks.

## $\mathcal{H}$-LU Preconditioners for the RBF-FD Discretized Oseen Equations

Michael Koch, Sabine Le Borne

Hamburg University of Technology, Hamburg, Germany
MICHAEL.KOCH@TUHH.DE

### Abstract

The possibility of using $\mathcal{H}$-matrices as preconditioners for the Oseen equations has been known for a while. They have been shown to be robust for finite elements as well as meshless discretizations [1, 2]. However, despite having an asymptotic runtime complexity of $\mathcal{O}(N\log(N)^2)$, they may in practice be quite slow when compared to competing methods due to large constants hidden in the $\mathcal{O}$-notation.

We present some recent improvements to reduce the runtime for setting up these preconditioners. These improvements are due to a recently introduced arithmetic for $\mathcal{H}$-matrices as well as clusterings that are adapted to the structure of the Oseen problem. These changes can be applied regardless of the underlying discretization.

Lastly, we present one more improvement that is possible if we discretize the Oseen equations using the meshless Radial Basis Function - Finite Difference method (RBF-FD) [3] by utilizing the flexibility of this method in the construction of the preconditioner.

### References

[1] Le Borne, Sabine and Oliveira, Suely and Yang, Fang (2008) *$\mathcal{H}$-matrix preconditioners for symmetric saddle-point systems from meshfree discretization*, Numerical Linear Algebra with Applications

[2] Le Borne, Sabine (2008) *Hierarchical matrix preconditioners for the Oseen equations*, Computing and Visualization in Science

[3] Koch, Michael and Le Borne, Sabine (2025) *RBF-FD discretization of the Oseen equations*, Journal of Computational Physics

## On Direct Multilevel Solvers Induced by AMG

Conor McCoid, José Pablo Lucero Lorca and Michal Outrata

Charles University, Prague, Czechia
OUTRATA@KARLIN.MFF.CUNI.CZ

### Abstract

We consider an algebraic multigrid (AMG) scheme for the direct solution of complex-valued square linear systems based on a recursive $2 \times 2$ block partitioning of the coefficient matrix and study the optimal choices of its components. In particular, we complement existing results that characterize optimal choices for nonsymmetric-cycle methods [1, 2] by analyzing the spectral behavior of the symmetrized variant. We analyze the error propagation operator of the two-level symmetric-cycle method by calculating its invariant subspaces and its nonzero eigenvalues that govern the behavior of the error after a single cycle. We show that the error propagation operator can be studied separately for pairs of modes, working as bases of the invariant subspaces. The main result is an explicit choice of smoothing parameters that makes all the pairs of modes respond identically, forcing the nontrivial eigenvalues of the error propagation operator to collapse to a single, a priori known value, i.e., we give an explicit choice for the smoothing parameters so that the nonzero part of the spectrum of the error propagation operator collapses to a single, a priori known point on the real line. As a consequence, we give a closed-form formula for the inverse of a general square matrix that, to the best of our knowledge, is new in the literature. Additionally, the framework provides a clear, self-contained description of an ideal AMG W-cycle, offering a concrete target for the design of related schemes. We illustrate the theory with direct applications to general matrices and with analyses of representative matrices arising in numerical methods.

### References

[1] S. MacLachlan, T. Manteuffel, and S. F. McCormick. Adaptive reduction-based AMG. *Numerical Linear Algebra with Applications*, volume 13(8): 599–620, 2006.

[2] J. P. Lucero Lorca. Towards a multigrid preconditioner interpretation of Hierarchical Poincaré-Steklov solvers. *arXiv:2511.00735*, 2026.

### Exploiting Low-rank Structures for the Solution of PDEs

Davide Palitta

Alma Mater Studiorum, Università di Bologna, Bologna, Italy
DAVIDE.PALITTA@UNIBO.IT

### Abstract

Exploiting low-rank structures for the solution of partial differential equations (PDEs) has emerged as a powerful technique in the field of numerical simulation and scientific computing. Many real-world systems exhibit inherent low-dimensional patterns among solution variables, allowing for more efficient computation. By efficiently approximating complex PDE solutions with fewer degrees of freedom, low-rank methods facilitate scalable algorithms and offer enhanced interpretability, making them invaluable tools for tackling high-dimensional problems in various scientific and engineering domains.

In this talk we are going to focus on the solution of evolutionary PDEs. In particular, we will show how the space-time formulation of the discrete problem can be cast in terms of a single Sylvester matrix equation, in place of a sequence of shifted linear systems as costumary in time-marching schemes. This reformulation allows to design ad-hoc numerical strategies tackling the space and time components of the algebraic equation separately. In particular, a novel solution strategy that combines projection techniques – for the space component – with the full exploitation of the entry-wise structure of the discrete time operator is proposed. The resulting scheme is able to efficiently solve problems with a tremendous number of degrees of freedom while maintaining a low storage demand as illustrated by several numerical results.

### References

[1] D. Palitta, *Matrix Equation Techniques for Certain Evolutionary Partial Differential Equations*, J Sci Comput 87, 99 (2021)

## Iterative Methods for Fractional Differential Equations via the ⋆-Product

Fabio Durastante, Pierre-Louis Giscard, Stefano Pozza

Charles University, Prague, Czech Republic.
POZZA@KARLIN.MFF.CUNI.CZ

### Abstract

We address the numerical solution of non-autonomous linear fractional differential equations of the form

$$D_t^\alpha u(t) = \widehat{A}(t)\, u(t), \qquad u(0) = v,$$

where $0 < \alpha < 1$ and $\widehat{A}(t)$ is analytic in time. These problems arise in applications involving memory effects and nonlocal dynamics. Our new method expands the solution on a suitable polynomial basis and reformulates the problem as a structured matrix equation that links the fractional dynamics across time; see [1]. The approach is related to the ⋆-product formalism, a generalization of the Volterra convolution that enables a tractable discretization of the fractional operator and, in certain cases, even yields closed-form expressions.

We also present preliminary results on the extension of this framework to partial differential equations, showing that similar structured systems appear and can be exploited for numerical computations.

### References

[1] Fabio Durastante, Pierre-Louis Giscard, Stefano Pozza, *A ⋆-Product Approach for Analytical and Numerical Solutions of Nonautonomous Linear Fractional Differential Equations*, arXiv:2507.16652 [math.NA], 2025.

## Block Schwarz Methods and Preconditioning Strategies Using Generalized Locally Toeplitz Tools: Proposals, Analysis, and Numerical Validation

Abdessadek Rifqui

University Mohammed VI Polytechnic, Palaiseau, France
abdessadek.rifqui@um6p.ma

### Abstract

In the current work we present a spectral analysis of the additive and multiplicative Schwarz methods within the framework of domain decomposition techniques, by investigating the spectral properties of these classical Schwarz preconditioning matrix sequences, with emphasis on their convergence behavior and the effect of transmission operators. In particular, we focus on restricted variants of the Schwarz methods aimed at improving parallel efficiency, while preserving their convergence features. In order to rigorously describe and analyze the convergence behavior, we employ the theory of generalized locally Toeplitz (GLT) sequences, which provides a robust framework for studying the asymptotic spectral distribution of the discretized operators arising from Schwarz iterations. By associating each operator sequence with the appropriate GLT symbol, we derive explicit expressions for the GLT symbols of the convergence factors, for both additive and multiplicative Schwarz methods. The GLT-based spectral approach offers a unified and systematic understanding of how the spectrum evolves with mesh refinement and overlap size (in the algebraic case). Our analysis not only deepens the theoretical understanding of classical Schwarz methods, but also establishes a foundation for examining future restricted or hybrid Schwarz variants using symbolic spectral tools. These results enable the prediction of the remarkable efficiency of block Jacobi/Gauss–Seidel and block additive/multiplicative Schwarz preconditioners for GLT sequences, as further illustrated through numerical experiments.

**Reliable Waveform Relaxation Convergence Estimates for Atmosphere-Ice-Ocean Coupling**

Valentina Schüller, Philipp Birken

Lund University, Lund, Sweden
VALENTINA.SCHULLER@MATH.LU.SE

**Abstract**

Climate models are large, coupled codes solving nonlinear systems of partial differential equations. We focus on the atmosphere, ocean, and sea ice components, which are coupled at the sea surface via boundary conditions. Coupling algorithms used in practice correspond to a single step of iterative Schwarz waveform relaxation (WR) methods. Not iterating is computationally cheap but introduces numerical coupling errors which can be substantial [1]. Coupling error reduction is directly linked to the convergence factor of the WR iteration. We are therefore interested in reliably estimating the latter.

To this end, we recently contributed with the WR convergence analysis of a simplified model for thermodynamical atmosphere-ice-ocean coupling: in essence, coupled heat equations with discontinuous material parameters and nonstandard interface boundary conditions [2]. In this talk, we compare the results from this analysis to WR convergence results we obtain with a coupled single column climate model: the EC-Earth AOSCM, which uses the same coupling setup as its host model, EC-Earth. Our analysis in [2] explains observed fast WR convergence in ice-free simulations. When sea ice is present, WR in the EC-Earth AOSCM converges much faster than the analysis predicts. This discrepancy can be reduced by modifying the sea ice component in the simplified model. We also show how discontinuities in the EC-Earth code can slow down numerically observed WR convergence. Our work illustrates that simplified models and established WR analysis techniques are useful tools to understand WR convergence for state-of-the-art atmosphere-ice-ocean codes.

**References**

[1] Valentina Schüller et al. "Quantifying coupling errors in atmosphere-ocean-sea ice models: A study of iterative and non-iterative approaches in the EC-Earth AOSCM". In: *Geosci. Model Dev.* 18.22 (Nov. 2025), pp. 9167–9187. ISSN: 1991-9603.

[2] Valentina Schüller, Philipp Birken, and Hanna Kjellson. "Analysis of bulk interface conditions for atmosphere-ice-ocean coupling". In: *DD29 Proceedings*. Dec. 2025. DOI: 10.5194/gmd-18-9167-2025.

**The Hopf Bifurcation for Neutral Delay Differential Equations with State-Dependent Delay**

Ahlam Alhadbani

Department of Mathematics, Universty of Exeter,Exeter, United Kingdom
AA1366@EXETER.AC.UK

**Abstract**

We consider neutral delay differential equations (NDDEs) with state-dependent delays of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}[x(t) + g(x_t, a)] = f(x_t, a),$$

where $x_t(s) = x(t+s) \in \mathbb{R}^n$, and $a \in \mathbb{R}$ is a parameter. The functionals $f$ and $g$ do not satisfy the classical smoothness properties that are assumed for DDEs with constant delay. Consequently, the Hopf bifurcation theorem cannot be applied directly. In this research, we study periodic boundary-value problems for NDDEs using a weaker differentiability concept (mild differentiability) and prove the Hopf bifurcation theorem (existence of a smooth branch of periodic orbits near an equilibrium $x_{\mathrm{eq}}(a)$ changing stability at $x_{\mathrm{eq}}(a_{\mathrm{H}})$) under standard conditions with the following additional restriction on the neutral term:

$$|\partial_x g(x, a)z| \leq \kappa \|z\|_0 \quad \text{for some } \kappa < 1$$

and all $a$ and continuously differentiable $x$ near $(a_{\mathrm{H}}, x_{\mathrm{eq}}(a_{\mathrm{H}}))$.

### Functional Runge-Kutta Methods for Stochastic Delay Differential Equations

Dimitri Breda

Department of Mathematics, Computer Science and Physics – University of Udine, Udine, Italy
DIMITRI.BREDA@UNIUD.IT

#### Abstract

Stochastic Delay Differential Equations (SDDEs) hold significant importance in the advanced modeling of problems in the natural sciences, economics and engineering, e.g., those related to supply chains and sustainable innovation in general. While time integration is essential for simulating and understanding their behavior and properties, it poses nontrivial challenges due to the presence of both randomness and memory effects. In this talk we present an extension to SDDEs of the so-called *functional* Runge-Kutta methods, introduced for DDEs in [2] and further analyzed in [1], which provide an approximation to the whole history of the system at each time step. This allows for greater freedom in the choice of the integration mesh beyond the necessary breaking points independently of the delay structure or the underlying Brownian motion, thus potentially overcoming the limitation of traditional methods, which typically require commensurable delays and step sizes.

This is a joint work with Alessia Andò, Davide Liessi and Faraz William from the University of Udine and Stefano Maset from the University of Trieste, and it is partially supported by the Italian Ministry of University and Research (MUR) through the PRIN 2022 project (No. 20229P2HEA) "Stochastic numerical modelling for sustainable innovation", Unit of Udine (CUP G53C24000710006).

#### References

[1] S. Maset, *Numerical methods for Delay Differential Equations*, in "Delays and Structures in Dynamical Systems: Modelling, Analysis and Numerical Methods", D. Breda, R. Vermiglio, J. Wu eds, CISM Lecture notes, Springer, submitted

[2] S. Maset, L. Torelli and R. Vermiglio, *Runge-Kutta methods for retarded functional differential equations*, Math. Models Methods Appl. Sci. 15(8):1203-1251, 2005

## Extracting Tent Maps from the Mackey-Glass Attractor

Davide Di Lenarda

Department of Mathematics, Computer Science and Physics – University of Udine, Udine, Italy
DILENARDA.DAVIDE@SPES.UNIUD.IT

### Abstract

Since its appearance in the celebrated paper [5], the Mackey–Glass equation has become an essential reference in the field of delay differential equations (DDEs). Its importance goes well beyond being a prototype model for investigating dynamical diseases in biology and medicine or for investigating the effects of delayed feedback in control and relevant applications. Even from a strictly mathematical point of view, it represents a cornerstone benchmark to understand emerging analytical techniques or to test advanced computational methods. Despite the great attention it has received in almost half a century, a rigorous mathematical proof of chaos in the Mackey-Glass equation is still an open problem. However, several numerical simulations highlight the chaotic nature of the Mackey-Glass system.

Inspired by the pioneering work of Lorenz [2], in this talk we present several computational results showing the presence of tent-like scalar maps in the dynamics of the Mackey-Glass equation. Up to our knowledge, an in-depth study on tent maps in the dynamics of this system has not been provided yet. The numerical evidence of the presence of such maps can be a strong indicator of chaotic behavior, since their iteration gives rise to chaotic dynamics. We provide an interpretation of the adopted tools on the basis of the arguments proposed by Siegberg in [4] to reduce the dynamics of simpler DDEs to tent maps in order to apply standard results on chaotic maps [1]. The emerging key mechanism is that of a double reduction: first from continuous to discrete time, second from infinite to finite dimensional state stapce.

This is a joint work with Alessia Andò and Dimitri Breda (University of Udine), supported by FSE+ grant: "Computational and datadriven models for advanced sustainable processes", CUP: G23C25000620008.

### References

[1] T. Y. Li, J. A. Yorke, *Period Three Implies Chaos*, The American Mathematical Monthly, Dec., 1975, Vol. 82, No. 10 (Dec., 1975), pp. 985-992. https://doi.org/10.2307/2318254

[2] E. N. Lorenz, *Deterministic Nonperiodic Flow*, Journal of the atmospheric sciences, Massachussetts Insistute of Technology, 1963.

[3] M. C. Mackey, L. Glass, *Oscillation and Chaos in Physiological Control Systems*, Science, New Series, Volume 197, Issue 4300 (Jul. 15, 1977), 287-289.

[4] H. W. Siegberg, *Chaotic Behavior of a Class of Nonlinear Differential Delay Equations*, Bremen, West Germany.

### Towards a Proof of Chaotic Dynamics in the Mackey–Glass Equation

Olivier Hénot

National Taiwan University, Taipei, Taiwan
olivierhenot@ntu.edu.tw

#### Abstract

In this talk, we present a constructive strategy to prove the existence of a transverse intersection between the stable and unstable manifolds of a periodic orbit in the Mackey–Glass equation

$$\frac{\mathrm{d}}{\mathrm{d}t}u(t) = -au(t) + b\frac{u(t-\tau)}{1+u(t-\tau)^{\rho}}, \qquad a, b, \rho \geq 0,\ \tau > 0.$$

Such a phenomenon is generally referred to as the Poincaré scenario, and constitutes a classical geometric mechanism for the existence of chaotic dynamics via a Smale horseshoe.

The approach begins with reformulating the delay differential equation as a discrete dynamical system, defined a function space, through the time-$\tau$ map. The phase space is discretized using functions admitting convergent Chebyshev expansions, whose coefficients are modeled in a weighted $\ell^1$ Banach space. The transverse homoclinic orbit is then characterized as the zero of a nonlinear operator $F(x) = 0$, arising from a projected shooting method on the invariant stable and unstable manifolds. The strategy requires ($i$) computing a (non-degenerate) periodic orbit, ($ii$) determining its Morse index, and ($iii$) retrieving a local graph of its (infinite-dimensional) stable manifold. With this in place, we verify the existence of a transverse intersection via a computer-assisted proof based on the contraction of a quasi-Newton operator.

## Compressible Active Nematic Liquid Crystals: Existence of Dissipative Solutions

Kuntal Bhandari

Institute of Mathematics, Czech Academy of Sciences, Prague, Czech Republic
BHANDARI@MATH.CAS.CZ

**Abstract**

We discuss the hydrodynamics of compressible active nematic liquid crystals in a three-dimensional and bounded domain, with a nonlinear viscosity tensor and nonhomogeneous boundary data, in a Landau-de Gennes framework. We prove the existence of *dissipative solutions* within a Beris-Edwards type model for active nematodynamics, which are weak solutions satisfying the underlying equations modulo a defect measure. The proof follows from a three level approximation scheme – the Galerkin approximation, the classical parabolic regularization of the continuity equation, and the convex regularization of the potential generating the viscous stress. This is a joint work with Apala Majumdar (Manchester) and Šárka Nečasová (Prague).

### References

[1] K. Bhandari, A. Majumdar, Š. Nečasová. Dissipative solutions to a Beris-Edwards type model for compressible active nematic liquid crystals. *https://arxiv.org/pdf/2601.02051*, 2026.

## A Free-Discontinuity and Free-Boundary Problem for Smectic A Liquid Crystals

Giacomo Canevari

Università degli Studi di Verona, Verona, Italy
GIACOMO.CANEVARI@UNIVR.IT

### Abstract

Smectic liquid crystals are a phase of matter in which the constituent molecules tend to align locally parallel to one another and to arrange themselves in layers. Experimental evidence shows that the configuration of the layers in smectic films may be rather complex, possibly with defects — that is, localised regions of sharp change in the orientation of the layers. Defects may occur at isolated points, along lines or surfaces. In this talk, we discuss a free-discontinuity and free-boundary variational problem for smectic A liquid crystals in two dimensions. We focus on a specific form of the energy functional, which is lower semicontinuous and penalises dislocations of the layers along the defects. Our results provide existence of minimisers and some regularity properties (away from the free boundary). The talk is based on joint work with JOHN M. BALL (Heriot-Watt University, Edinburgh and Hong Kong Institute of Advanced Studies) and BIANCA STROFFOLINI (Università Federico II, Napoli).

## A Relative Energy Inequality for Electrolytes with Liquid-Crystal-Like Solvent: Weak-Strong Uniqueness and a Posteriori Error Estimates

Luisa Plato

Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany
plato@wias-berlin.de

### Abstract

We study an anisotropic Navier–Stokes–Nernst–Planck–Poisson (NSNPP) system whose constant in time anisotropy is motivated by continuum models for liquid crystals. Building on the concept of suitable weak solutions [1], we derive a relative energy inequality for the NSNPP system [2]. This inequality yields a weak–strong uniqueness principle and provides a robust analytical tool for a posteriori error estimation. We further investigate the high-viscosity (low-Reynolds number) regime, which leads to an anisotropic Stokes–Nernst–Planck–Poisson (SNPP) limit system. Using the relative energy framework, we obtain quantitative error estimates measuring the distance between solutions of the NSNPP and SNPP models in natural energy and dissipation norms.

### References

[1] D. Hömberg, R. Lasarzik and L. Plato. Existence of suitable weak solutions to an anisotropic electrokinetic flow model. *J. Differ. Equations, 428:511-584, 2025*

[2] R. Lasarzik and L. Plato. A relative energy inequality for an anisotropic Navier–Stokes–Nernst–Planck–Poisson system — Weak-strong uniqueness and *a posteriori* error estimates. *WIAS Preprint No. 3208 Berlin, 2025.* 37 pages.

## Variational Approximation of the Heat flow of Harmonic Maps

Antonio Segatti

University of Pavia, Italy
ANTONIO.SEGATTI@UNIPV.IT

### Abstract

Harmonic maps play a central role in both differential geometry and the analysis of partial differential equations. In particular, harmonic maps and their associated heat flow naturally arise in the Oseen–Frank model for nematic liquid crystals, where they describe the local orientation of the molecules.

In this talk, I will present a variational approximation of the harmonic map heat flow. The approach is based on the Weighted Energy–Dissipation (WED) scheme, which relies on a functional defined over entire trajectories and depending on a small parameter $\varepsilon$. As $\varepsilon \searrow 0$, minimizers of this functional are shown to converge to weak solutions of the heat flow.

For smooth target manifolds with nonpositive sectional curvature, we recover—through a purely variational argument—the classical theorem of Eells and Sampson.

Moreover, the variational structure of the scheme allows us to extend the analysis to CAT(0) targets. In this more general setting, we prove that solutions to the flow are Lipschitz continuous in the space variable, thereby settling a long-standing open problem in the area. This is a joint work with Fang-Hua Lin, Yannick Sire and Changyou Wang.

## High and Low-Mach Number Regimes for Capillary Fluids

Matteo Caggio

Institute of Mathematics, CAS, Prague, Czech Republic
caggio@math.cas.cz

**Abstract**

The talk will be devoted to the high and low-Mach number limit for compressible capillary fluids with density dependent viscosity. We will present recent results concerning high-Mach number flows, weak-strong uniqueness property and dispersion of the acoustic waves in the low-Mach number regime. This is a joint work with Donatella Donatelli (DISIM, L'Aquila) and Lars Eric Hientzsch (KIT, Karlsruhe).

### References

[1] Caggio, M.; Donatelli, D., *High Mach number limit for Korteweg fluids with density dependent viscosity,* J. Differential Equations 277 (2021), 1-37.

[2] Caggio, M.; Donatelli, D., *Relative entropy inequality for capillary fluids with density dependent viscosity and applications.,* Mathematische Annalen 390 (2024), no. 2 , 2897-2929.

[3] Caggio, M.; Donatelli, D.; Hientzsch, L. E., *Inviscid incompressible limit for capillary fluids with density dependent viscosity,* J. Evol. Equ. 26 (2026), no. 1, Paper No. 43.

## Anelastic Approximation for the Degenerate Compressible Navier-Stokes Equations

Nilasis Chaudhuri

University of Warsaw, Warsaw, Poland
NCHAUDHURI@MIMUW.EDU.PL

### Abstract

In this talk, we revisit the low Mach and low Froude number limits for the compressible Navier–Stokes equations with degenerate, density-dependent viscosity. Using the relative entropy inequality based on the concept of $\kappa$-entropy, we rigorously justify convergence to the generalized anelastic approximation in the three-dimensional periodic domain for well-prepared initial data. For general ill-prepared initial data, we also establish a similar convergence result in the whole space, relying essentially on dispersive estimates for acoustic waves. This is a joint work with F. Fanelli (Bilbao), Y. Li (Anhui) and E. Zatorska (Warwick).

### Recent Progresses in the Asymptotic Dynamics of Fast Rotating Fluids

Francesco Fanelli

Basque Center for Applied Mathematics & Ikerbasque – Bilbao, Spain
FFANELLI@BCAMATH.ORG

#### Abstract

In this talk, we will review some recent results describing the asymptotic dynamics of fast rotating fluids. We will see how to derive the celebrated Taylor-Proudman theorem and the Ekman pumping effect, in the context of incompressible flows either moving in a domain with topography, or in presence of non-trivial variations of the density.

These are joint works with M. Bravin (Universidad de Cantabria), J.-Y. Chemin (Université Claude Bernard Lyon 1) and I. Gallagher (Université Paris Cité).

### Continuous Data Assimilation for Compressible Temperature Driven Fluids

Eduard Feireisl

Institute of Mathematics, Czech Academy of Sciences, Praha, Czech Republic
feireisl@math.cas.cz

#### Abstract

We consider the Navier-Stokes-Fourier system describing the evolution of a compressible temperature driven rotating fluid arising in meteorology. We show convergence of a continuous data assimilation (CDA) method in the regime of low Mach/high Rossby numbers. This is the first result on convergence of CDA method for a system that is not (known to be) well posed.

**Derivation of the Vlasov-Stokes Equation – the Monokinetic and the Small-Inertia Case**

Richard M. Höfer, Amina Mecherbet, Richard Schubert

University of Regensburg, Germany
RICHARD.HOEFER@UR.DE

**Abstract**

We consider the sedimentation of $N$ spherical particles with identical radii $R$ in a Stokes flow in $\mathbb{R}^3$. The particles satisfy a no-slip boundary condition and are subject to constant gravity. The dynamics of the particles is modeled by Newton's law. In a mean-field scaling with $N \to \infty$ $NR \sim 1$, one expects that the evolution of the $N$-particle system is well approximated by the Vlasov-Stokes equation. The non-binary and singular nature of the implicit particle interaction through the fluid poses a major challenge for its rigorous derivation; as for the Vlasov-Poisson equation, it is still open whether particle clustering might prevent the validity of the mean-field limit.

We show that clustering does not occur and derive the Vlasov-Stokes equation in the following two cases. Firstly, in the monokinetic case, where we show a short time result. Secondly, in the case where the particle inertia is vanishing as $N \to \infty$. In this case, the microscopic system is to leading order well-approximated by the transport-Stokes equation. However, the Vlasov-Stokes equation takes into account the (small) inertia and therefore provides a better error estimate. The proof is based on stability estimates for the Vlasov-Stokes equation in the 2-Wasserstein distance and on an adaptation of Hauray's work for mean-field limits to 2-Wasserstein distances.

## Hydrodynamic Limit of the Kinetic Cucker-Smale Model Toward the Incompressible Euler-Alignment Model

David Poyato

University of Granada, Granada, Spain
davidpoyato@ugr.es

**Abstract**

We present the rigorous hydrodynamic limit of the kinetic Cucker-Smale model in the d-dimensional torus including collisions described by the nonlinear Fokker-Planck operator. We focus on the regime where collisions have large frequency and also large mean thermal velocity. The limiting system is characterized by the incompressible Euler-alignment model with weakly singular influence function. Contrarily to previous literature, where all the resulting hydrodynamic limits led to (both pressureless and pressured) compressible versions of the Euler-alignment system, we obtain incompressible alignment models for the first time in the literature. We develop a holistic method combining techniques from hydrodynamic limits for kinetic systems based on relative entropy methods for the macroscopic quantities, together with tools from incompressible limits of Euler-type systems via compensated compactness arguments. We also discuss some well-posedness results of the involved systems, with particular emphasis on the new target system.

This is based on a recent joint work with Francesco Fanelli (Basque Center for Applied Mathematics) and Gabriele Sbaiz (Università degli studi di Trieste).

### References

[1] F. Fanelli, D. Poyato, G. Sbaiz, From the kinetic Cucker-Smale model to the incompressible Euler-alignment system with weakly singular couplings. *In preparation.*

**Dynamics of a Large System of Heavy Particles in a Newtonian Fluid**

Arnab Roy

Basque Center for Applied Mathematics (BCAM), Bilbao, Spain
aroy@bcamath.org

**Abstract**

We consider the motion of a large number of heavy particles in a Newtonian fluid occupying a bounded spatial domain. When we say "heavy", we mean a particle with a mass density that approaches infinity at an appropriate rate as its radius vanishes. We show that the collective effect of heavy particles on the fluid motion is similar to the Brinkman perturbation of the Navier–Stokes system identified in the homogenization process.

## Incompressible Limits at Large Mach Number for a Reduced Compressible MHD System

Aneta Wróblewska-Kamińska

Institute of Mathematics of the Polish Academy of Sciences, Warsaw, Poland
AWROB@IMPAN.PL

### Abstract

This paper studies a singular limit problem for a reduced model for compressible non-resistive MHD. This system can also be related to a certain class of two-fluid models. By a suitable rescaling of the magnetic pressure in terms of some parameter $\varepsilon > 0$, by letting $\varepsilon \to 0$ we perform the incompressible limit while keeping the Mach number of order $O(1)$. The study is conducted in the framework of global in time finite energy weak solutions and for ill-prepared initial data. We also consider a similar problem in presence of a strong Coriolis term. The key ingredient of the proof, based on a compensated compactness argument, is the use of the transport equation (well-known in the context of two-fluid models) underlying the dynamics. Thanks to it, and differently from previous studies about the incompressible limit, we are able to identify the asymptotics of the terms of order $O(\varepsilon)$ and to characterise their dynamics; such an information is in fact crucial to obtain a closed system in the limit. This is recent joint work with Francesco Fanelli and Young-Sam Kwon.

### References

[1] Francesco Fanelli, Young-Sam Kwon, Aneta Wróblewska-Kamińska. Incompressible limits at large Mach number for a reduced compressible MHD system. arXiv:2512.18078

### On the Singular Set of Fractional Harmonic Maps

Alessandro Audrito

Politecnico di Torino, Torino, Italy
alessandro.audrito@polito.it

**Abstract**

In this talk I will present an Allard-type regularity theorem for the singular set of $s$-harmonic maps into $\mathbb{S}^1$, with $s \in (1/2, 1)$. The main result shows that the singular points in the top stratum attaining the fundamental energy level are an $(n-2)$-dimensional manifold of class $C^{1,\alpha}$. Then, I will discuss how to interpret such result as a regularity theory for codimension-two nonlocal minimal surfaces.

The talk is based on some joint work with María Medina de la Torre (UAM) and Joaquim Serra (ETHZ).

## Non-Local Constrained Curvature Flows within Non-Flat Ambients

Esther Cabezas-Rivas

Universitat de València, Spain
esther.cabezas-rivas@uv.es

### Abstract

The mean curvature flow is the most natural way to deform a hypersurface according to its curvature, since it evolves the parametrization by means of the heat equation. Huisken in 1987 introduced a variant that keeps the enclosed volume constant while the area decreases. For this modification, a global term is added to the speed of the original flow, which makes the usual methods in geometric flows (e.g. maximum principles) either fail or become more intricate. Moreover, the resulting evolution is not equivalent to the unconstrained flow because, for instance, an initially embedded curve may develop self-intersections. Accordingly, constrained mean curvature type flows are more challenging; indeed, there was no extension of Huisken's result to a non-Euclidean ambient space until 2007 [2]. In this talk, we will give a sketchy portrait of the development of the theory from the early stages to the recent applications, including the quermassintegral preserving mean curvature flows in the sphere [3] and constrained flows of curves within pinched Hadadard surfaces [1].

### References

[1] S. Albert Niclòs and E. Cabezas-Rivas, Constrained curvature flows on pinched Hadamard sufaces. *In preparation.*

[2] E. Cabezas-Rivas and V. Miquel Molina, Volume preserving mean curvature flow in the hyperbolic space, *Indiana Univ. Math. J.* **56** (2007), no. 5, 2061–2086.

[3] E. Cabezas-Rivas and J. Scheuer, The quermassintegral-preserving mean curvature flow in the sphere, *Anal. PDE* **17** (2024), no. 10, 3589–3621.

## Backwards Uniqueness and Rates of Singularity Formation in Mean Curvature Flow

Joshua Daniels-Holgate

Queen Mary University of London, London, United Kingdom
J.DANIELSHOLGATE@QMUL.AC.UK

### Abstract

The question of backwards uniqueness for parabolic PDE asks whether the same final state can be reached via the flow starting from two different sets of initial data. Backwards uniqueness is a far more subtle property compared to standard uniqueness but gives insight into the fine structure of solutions and singularity formation. I will discuss two recent results concerning the comparative rate of singularity formation in closed Mean Curvature Flows:

- For flows with a compact singularity, we show that two flows cannot approach each other faster than polynomially, extending a result of Martin-Hagemayer–Sesum.
- For isolated conical singularities, we show backwards uniqueness by first proving a backwards uniqueness theorem assuming a rate of convergence and then show that this rate is always saturated by two flows that agree on the singular time slice.

This talk is based on joint work with Or Hershkovits.

## Phase Transitions with Bounded Index: Parallels to De Giorgi's Conjecture

Enric Florit-Simon

ETH Zurich (Switzerland)
ENRIC.FLORIT@MATH.ETHZ.CH

### Abstract

We will discuss recent progress on the regularity for stable [1] and finite Morse index [2] solutions to the Allen–Cahn equation. The focus will be on the connections (or, perhaps more surprisingly, the lack thereof) with minimal surface theory and two influential conjectures of De Giorgi and Yau.

### References

[1] E. Florit-Simon and J. Serra, *On stable solutions to the Allen–Cahn equation with bounded energy density in* $\mathbb{R}^4$, arXiv:2509.02739.

[2] E. Florit-Simon, *Phase transitions with bounded index: Parallels to De Giorgi's conjecture*, arXiv:2602.03136.

## Non-Uniqueness of Locally-Minimizing Clusters

Anna Skorobogatova

ETH Zürich, Switzerland
anna.skorobogatova@eth-its.ethz.ch

### Abstract

Optimal bubble cluster problems concern the study of partitions of the $\mathbb{R}^n$ into a finite collection of chambers, some with finite volume and some with infinite volume. One looks for local minimizers of interfacial area subject to volume constraints on the finite-volume chambers. The case of one infinite-volume chamber is the classical multiple bubble problem and has received much attention in recent decades, with a well-known conjecture of Sullivan predicting the existence of a unique minimizing configuration when there are not too many chambers.

We study a variant of the multiple bubble problem in $\mathbb{R}^n$ with more than one infinite-volume chamber, with particular focus on the simplest case of 1 finite-volume chamber and 2 infinite-volume chambers. Here, Bronsard & Novack showed that uniqueness of local minimizers also holds in low dimensions $n \leq 7$. In stark contrast, we show that uniqueness fails in a large number of dimensions $n \geq 8$, and we provide some particular surprising phenomena that local minimizers can exhibit in higher dimensions. This is joint work with Lia Bronsard, Robin Neumayer and Michael Novack.

## Brakke's Mean Curvature Flows: an Overview

Salvatore Stuvard

University of Milan, Italy
SALVATORE.STUVARD@UNIMI.IT

### Abstract

The aim of this talk is to provide an overview of the state of the art concerning the notion of weak solutions to Mean Curvature Flow known as Brakke Flow. The focus will be on recent advances on both existence and regularity.

## Stationary Points of Conformally Invariant Polyconvex Energies

Riccardo Tione

Dipartimento di Matematica, Università degli Studi di Torino,
Via Carlo Alberto 10, 10123 Torino, Italy
riccardo.tione@unito.it

### Abstract

In this seminar we present recent work, in collaboration with A. Guerra, concerning the regularity of stationary points for a class of planar polyconvex functionals that are conformally invariant, a natural assumption in view of geometric applications. After a brief overview of the literature, we will introduce a partial regularity result for this class of energies. Furthermore, if time allows, we will show how full $C^1$ regularity can be established for orientation-preserving stationary points.

## Global Minimality of the Hopf Map in the Faddeev-Skyrme Model with Large Coupling Constant

Konstantinos Zemas

Hausdorff Center for Mathematics, University of Bonn, Bonn, Germany
zemas@iam.uni-bonn.de

### Abstract

The Faddeev-Skyrme model is a by now classical nonlinear $O(3)$-sigma model which has proved very successful in quantum field theory, in particular in detecting topological solitons. A prototype mathematical model for fields $u : \mathbb{S}^3 \to \mathbb{S}^2$ consists of the harmonic map energy together with its symplectic variant acting as a singular perturbation. We prove that, modulo rigid motions, the Hopf map is the unique minimizer of the Faddeev–Skyrme energy in its homotopy class, for an explicit sufficiently large range of the coupling constant. This is joint work with A. Guerra (University of Cambridge) and X. Lamy (University of Toulouse).

### References

[1] A. Guerra, X. Lamy, K. Zemas. *Global minimality of the Hopf map in the Faddeev-Skyrme model with large coupling constant* (2025), arXiv preprint, 2507.10686.

## Linearization in Elastodynamics

Barbora Benešová

Charles University, Faculty of Mathematics and Physics, Prague, Czech Republic
BENESOVA@KARLIN.MFF.CUNI.CZ

### Abstract

Nonlinear viscoelastodynamics predicts the deformation and internal stresses of a solid body under the action of applied forces. The state of the specimen is described by a *deformation* $y : [0,T] \times \Omega \to \mathbb{R}^d$ such that $y(t,\cdot) : \Omega \to \mathbb{R}^d$ is one-to-one and continuous. Denoting by $\varrho : \Omega \to \mathbb{R}$ the density of the body, the deformation fulfills the following force balance.

$$\varrho \partial_{tt} y - \operatorname{div} \mathcal{T} = \tilde{f}, \tag{13}$$

in the time-space cylinder $[0,T] \times \Omega$; here $\mathcal{T}$ is a stress tensor and $\tilde{f}$ is the volume density of applied body forces.

The stress tensor $\mathcal{T}$ in (13) must be defined constitutively. We will consider *generalized standard materials* for which it is assumed that the stress tensor can be determined by taking an appropriate functional derivative of two underlying functionals: The so-called elastic energy functional $y \mapsto \mathcal{E}(y)$ and the dissipation functional $y \mapsto \mathcal{R}(y, \partial_t y)$. Written abstractly, the stress tensor is assumed to be obtained via the additive rule

$$-\mathrm{div}\mathcal{T} = D\mathcal{E}(y) + D_2\mathcal{R}(y, \partial_t y), \tag{14}$$

which is also referred to as the *Kelvin-Voigt rheology.*

It has been shown recently [1] can actually be approximated by suitable quasistatic and static equations; that is by

$$-\operatorname{div} \mathcal{T} = \tilde{f}, \quad \text{and} \quad D\mathcal{E}(y) = \tilde{f}, \tag{15}$$

with appropriately changed $\mathcal{E}$ and $\mathcal{R}$.

Problem (15) has been extensively studied in the past years in the context of model reduction, i.e. passing to a simpler model in solid mechanics. A prominent example is linearization [2] when a model reduction is sought for very small strains.

On the example of linearization we show how these results can be extended to the dynamic setting.

*This is a joint work with Malte Kampschulte (Charles University, Prague) and Martin Kružík (ÚTIA, Prague).*

### References

[1] B. Benešová, M. Kampschulte, and S. Schwarzacher, A variational approach to hyperbolic evolutions and fluid-structure interactions, *Journal of the European Mathematical Society*, 26(12):4615–4697, 2024.

[2] G. Dal Maso, M. Negri, and D. Percivale, Linearized elasticity as Γ-limit of finite elasticity, *Set-Valued Analysis*, 10(2–3):165–183, 2002.

### Core-Radius Approximation of Singular Minimizers in Nonlinear Elasticity

Marco Bresciani

Johannes Kepler University Linz, Austria
marco.bresciani@jku.at

**Abstract**

We study a variational model in nonlinear elasticity allowing for cavitation which penalizes both the volume and the perimeter of the cavities. Specifically, we investigate the approximation of the energy (in the sense of $\Gamma$-convergence) by means of functionals defined on perforated domains. Perforations are introduced at flaw points where singularities are expected and, hence, the corresponding deformations do not exhibit cavitation. Notably, those points are not prescribed but rather selected by the variational principle. Our analysis is motivated by the numerical simulation of cavitation and extends previous results on models which solely accounted for elastic energy without contributions related to the formation of cavities.

The talk is based on joint work with Manuel Friedrich (JKU Linz)

#### References

[1] M. Bresciani, M. Friedrich. *Core-radius approximation of singular minimizers in nonlinear elasticity.* Applied Mathematics & Optimization (2026) 93:21.

## Nonlinear Models in Thermo-Viscoelasticity

Manuel Friedrich

JKU Linz, Austria
MANUEL.FRIEDRICH@JKU.AT

### Abstract

In this talk, I present a quasistatic nonlinear model in thermo-viscoelasticity at finite strains within the Kelvin–Voigt rheology, where both the elastic and the viscous stress tensors satisfy the principle of frame indifference. Weak solutions are obtained by means of a staggered time-discretization scheme in which the deformation and the temperature are updated alternately. In particular, we discuss the relevance of frame-indifferent approximation schemes.

## Problems with Inertia and Their Variational Limits

Malte Kampschulte

Department of Mathematical Analysis, Faculty of Mathematics and Physics,
Charles University Prague
kampschulte@karlin.mff.cuni.cz

### Abstract

The aim of this talk is to present some recent results on the variational characterisation of problems involving inertia in terms of energy and dissipation. Not only can we directly show existence of weak solutions from this characterisation in a straighforward approach, but also we can consider variational limits of a sequences of such problems, far extending the well-known idea of $\Gamma$-convergence.

This will be illustrated at classical examples, such as the limit from non-linear to linearized elasticity, highlighting its usefulness for continuum mechanics.

### An Effective Damage and Fracture Model Based on Micro-Structure

Dorothee Knees

Institute for Mathematics, University of Kassel, Germany
DKNEES@UNI-KASSEL.DE

#### Abstract

Macroscopic damage and failure phenomena of solids typically are the result of the accumulation of small cracks or defects on a microscopic scale. In engineering literature, various multi-scale or homogenized models are proposed in order to describe time-dependent damage phenomena originating from fine structure on the micro-scale. In this lecture, we discuss these approaches in the framework of homogenization and evolutionary $\Gamma$-convergence. The lecture relies on joint work with Hauke Hanke (formerly WIAS Berlin).

### Existence and Selection in the Solutions in the Energy-Variational Framework

Robert Lasarzik

Weierstrass Institute for Applied Analysis and Stochastic, Germany
ROBERT.LASARZIK@WIAS-BERLIN.DE

#### Abstract

We introduce the concept of energy-variational solutions for a broad class of nonlinear evolutionary partial differential equations (PDEs), with a particular emphasis on low-regularity settings [1]. This framework, based on a variational inequality, provides a robust solution theory applicable to diverse problems—from fluid dynamics and elasticity to geometric flows [3]. A key advantage of this approach lies in its ability to handle nonlinear and singular evolutionary PDEs, where classical solution concepts may fail [2]. We establish existence of solutions via a minimizing movements scheme and systematically analyze their properties. Finally, we discuss selection criteria within this framework to ensure uniqueness and physical relevance of solutions [4].

#### References

[1] A. Agosti, R. Lasarzik, and E. Rocca. Energy-variational solutions for viscoelastic fluids. *Adv. Nonlinear Anal.*, 13, 2024. https://doi.org/10.1515/anona-2024-0056 `doi:10.1515/anona-2024-0056`.

[2] T. Eiter and R. Lasarzik. Existence of energy-variational solutions to hyperbolic conservation laws. *Calc. Var.* 63:103, 2024. https://doi.org/10.1007/s00526-024-02713-9 `doi:10.1007/s00526-024-02713-9`.

[3] R. Lasarzik. Energy-variational structure in evolution equations. *arXiv* 2503.11438, 2025. https://doi.org/10.48550/arXiv.2503.11438 `doi.org/10.48550/arXiv.2503.11438`.

[4] T. Eiter, R. Lasarzik, and M. Śliwiński. Existence and selection of solutions in the energy-variational framework with applications in fluid dynamics. *iarXiv* 2601.20455, 2025. https://doi.org/10.48550/arXiv.2601.20455 `doi.org/10.48550/arXiv.2601.20455`.

**Derivation of Membrane Models in Nonlocal Hyperelasticity**

Anastasia Molchanova

Institute of Analysis and Scientific Computing, TU Wien, Vienna, Austria
anastasia.molchanova@tuwien.ac.at

**Abstract**

Motivated by the analysis of thin structures, we study the variational dimension reduction of hyperelastic energies involving nonlocal gradients to an effective membrane model. When rescaling the thin domain, initially isotropic interaction ranges naturally become anisotropic, which leads us to develop a theory of anisotropic nonlocal gradients with direction-dependent interaction ranges. In contrast to existing nonlocal derivatives with finite horizon, which are typically defined via interaction kernels supported on balls of positive radius, our formulation employs ellipsoidal interaction regions whose principal radii may vanish independently. This yields a unified framework interpolating between fully nonlocal, partially nonlocal, and purely local models. Using these tools, we establish a $\Gamma$-convergence analysis for the associated nonlocal thin-film energies. The limit functional retains the structural form of the classical membrane energy, and the classical local model is recovered exactly when all interaction radii vanish.

This talk is based on joint work with Dominik Engl and Hidde Schönberger.

### A Two-Dimensional Variational Approach to the Modelling of Ferronematic Thin Films

Anja Schlömerkemper

University of Würzburg, Germany
ANJA.SCHLOEMERKEMPER@UNI-WUERZBURG.DE

#### Abstract

Colloidal suspensions of magnetic particles in liquid crystals are complex fluids that show a coupling between the magnetic particles and the surrounding liquid crystal, which we assume to be in the nematic phase. The material is then referred to as ferronematics. The coupling results in special effective material properties and potential applications in display technologies or multifluidics devices. In the joint work with Shilpa Dutta, James Dalby and Apala Majumdar [1], we are particularly interested in thin films and consider a variational approach that combines the Landau-de Gennes approach for nematic liquid crystals with the theory of micromagnetics. The talk summarizes our analytical findings and shows our numerical results obtained as stable ferronematic equilibria by solving the related gradient flow equations. We observe the influence of the stray field on interior nematic defects and magnetic vortices.

#### References

[1] S. Dutta, J. Dalby, A. Majumdar, A. Schlömerkemper. *A study of ferronematic thin films including a stray field energy*, arXiv:2509.10442

## Directed Hypernetworks: Heteroclinic Dynamics and Reluctant Synchrony Breaking

Sören von der Gracht

Paderborn University, Germany
SOEREN.VON.DER.GRACHT@UNI-PADERBORN.DE

### Abstract

Interconnected real-world systems frequently involve non-pairwise interactions among agents – known as higher-order interactions. Their role is increasingly recognized as crucial in shaping collective dynamics. The higher order interaction structure can be naturally represented using hypergraphs or hypernetworks. This talk explores the dynamics of such hypernetworks, revealing that simple model equations exhibit obstructions to constructing certain heteroclinic structures in phase space when interactions are undirected, while their directed counterparts do not impose such restrictions. Motivated by this, we introduce a general class of directed hypernetworks and define corresponding admissible maps that preserve the underlying interaction structure. For this class, we establish a complete classification of all robust patterns of (cluster) synchrony supported by a given hypernetwork. Remarkably, these robust synchrony patterns are determined by higher-degree polynomial admissible maps. This contrasts sharply with classical networks, where cluster synchronization arises linearly; here, it emerges as a higher-order, nonlinear phenomenon. This nonlinear nature gives rise to a novel type of "reluctant" synchrony-breaking bifurcation, in which a high-order tangency of the solution branch to a non-robust synchrony space causes previously synchronous nodes to separate exceptionally slowly.

### References

[1] Sören von der Gracht, Eddie Nijholt, Bob Rink (2023). Hypernetworks: cluster synchronisation is a higher-order effect. SIAM Journal on Applied Mathematics, 83(6), 2329–2353.

[2] Sören von der Gracht, Eddie Nijholt, Bob Rink (2024). Higher order interactions lead to 'reluctant' synchrony breaking. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, Volume 480, Issue 2301, 20230945.

[3] Christian Bick, Sören von der Gracht (2024). Heteroclinic Dynamics in Network Dynamical Systems with Higher-Order Interactions. Journal of Complex Networks, Volume 12, Issue 2, April.

## Emergent Spatiotemporal Dynamics in Large-Scale Brain Networks with Next Generation Neural Mass Models

Gemma Huguet[1,2], Rosa Maria Delicado[3] and Pau Clusella[1]

(1) Universitat Politècnica de Catalunya, Barcelona, Spain
(2) Centre de Recerca Matemàtica, Barcelona, Spain
(3) Universitat de les Illes Balears, Mallorca, Spain
GEMMA.HUGUET@UPC.EDU

### Abstract

Macroscopic brain activity is characterized by prominent oscillatory phenomena that support a wide range of cognitive processes. Understanding how these collective dynamics arise from large-scale anatomical connectivity remains a central challenge in computational neuroscience. Here we study a whole-brain model comprising 90 anatomically connected regions, each represented by a next-generation neural mass model (NG-NMM) that explicitly accounts for interacting excitatory and inhibitory populations. We characterize homogeneous steady and oscillatory solutions of the network and examine their stability under both global and spatially structured perturbations. To address heterogeneous perturbations, we project them onto an appropriate eigenbasis of the connectivity and derive a dispersion relation, known as the Master Stability Function in the case of periodic orbits, linking network structure to perturbation growth rates. Analytical predictions are supported by numerical simulations and Lyapunov exponent calculations, which reveal that instabilities of homogeneous states give rise to complex spatiotemporal activity, including traveling waves and chaotic patterns. NG-NMMs exhibit a substantially rich dynamical landscape, both near homogeneous regimes and in symmetry-broken states. These results suggest that next-generation neural mass models offer a powerful and biologically grounded framework for investigating the emergence of complex dynamics in large-scale brain networks.

## Switching States: Heteroclinic Cycles as Organizing Centers of Neuronal Dynamics

Kateryna Nechyporenko

Living Systems Institute, University of Exeter, Exeter EX4 4QD, UK
K.NECHYPORENKO@EXETER.AC.UK

### Abstract

Neuronal networks exhibit spontaneous and evoked transitions between high- and low-activity regimes, referred to as up and down states. These states, along with the network's rhythmic patterns, are critical for perception, memory consolidation, and sensory processing. Due to their importance, identifying the fundamental dynamical principles that organize these large-scale state transitions is a major open question. Here, we propose necessary conditions for the existence of a novel bifurcation structure as a universal organizing center governing these transitions. Using bifurcation analysis and simulations of two-dimensional canonical mean-field network models we show that this bifurcation structure emerges robustly in this setting. An extension of the Wilson-Cowan model incorporating astrocyte dynamics and the Jansen-Rit model, enable us to demonstrate the persistence of the organizing center in higher dimensions. Our results reveal that the dynamic interplay between external inputs and network (synaptic) connectivity converges onto this shared mechanism, providing a fundamental principle for understanding how distinct brain states arise and are regulated across neuronal systems. Extending beyond phenomenological mean-field network models, we further show that it is the shared mathematical structure of the nonlinear input–output relationships rather than model-specific details that preserves the organizing center across modeling frameworks, thus revealing a fundamental and general mechanism underlying dynamic state transitions.

### Heteroclinic Networks in Coupled Cell Systems

Liliana Sofia Garrido da Silva

Universidade Católica Portuguesa, Católica Porto Business School, CEGE & CMUP, Portugal
LSSILVA@UCP.PT

**Abstract**

A coupled cell system is an ODE system associated with a coupled cell network, where the dimension is determined by the number of cells. A heteroclinic connection is a set of solution trajectories between two equilibria of an ODE system. A realization of a heteroclinic network is an ODE system that exhibits equilibria corresponding to the nodes and heteroclinic connections between them according to the heteroclinic network. This paper investigates the realization of heteroclinic networks within coupled cell systems, focusing on embedding heteroclinic connections in 2D and 3D invariant subspaces. We adapt Field's method of embedding each heteroclinic connection in distinct 2D synchrony subspaces to support multiple connections within the same subspace. Using the concept of book embedding from graph theory, we demonstrate that any heteroclinic network can be realized using a coupled cell system with a number of cells proportional to the network's book-thickness. Additionally, we extend our analysis to 3D synchrony subspaces, allowing for more complex realizations. In this case, the number of cells necessary for a realization is proportional to the number of nodes in the heteroclinic network.

For a deeper discussion of the subject we refer the reader to [1].

### References

[1] Garrido-da-Silva, L., and Soares, P. (2026). Heteroclinic networks in coupled cell systems. *arXiv preprint arXiv:2601.03370.*

**Phase Synchronisation in Coupled Oscillator Chains with Endpoint Heterogeneity**

Nicolas Verschueren van Rees, Daniel Galvis, Kyle C.A. Wedgwood

Department of Mathematics and Statistics, University of Exeter, Exeter,UK
N.VERSCHUEREN-VAN-REES@EXETER.AC.UK

**Abstract**

The emergence of collective dynamics within heterogeneous networks is a key feature of many biological networks. Heterogeneity of excitability, for example, has become a focus in the study of how pancreatic islets coordinate insulin secretion. This has raised the question of how highly excitable nodes might coordinate collective dynamics through networks where a large percentage of the population is intrinsically quiescent. To study this, we consider a discrete version of the Complex Ginzburg–Landau equation, parameterised such that in the absence of coupling, the endpoints exhibit globally attracting limit cycle behaviour and the interior nodes exhibit globally attracting trivial fixed point dynamics. Through model simulation and numerical continuation, we interrogate the relationship between model parameters and the stability of several phase-locked solutions of the system, focussing on two key solution types, the *chevron* and *anti-phase chevron* solutions, in which the exterior nodes exhibit a phase difference of 0 and $\pi$, respectively. We find that the anti-phase chevron solution stabilises as the excitability of the interior nodes decreases, or as the shearing effect of non-zero, coupling-induced amplitude perturbations from the natural limit cycle increases. Moreover, we find multiple regions of bistability with solutions with different phase synchronisation properties, highlighting that solutions observed in such networks may depend sensitively on initial conditions. Overall, our work highlights that chains with distributed heterogeneity exhibit a multitude of phase synchronised solutions, which are likely to be relevant in a range of real world networks.

## Computing Phase Dynamics in Delay–Coupled Oscillators

Babette de Wolff

Universität Hamburg, Germany
BABETTE.DE.WOLFF@UNI-HAMBURG.DE

### Abstract

Network interactions between coupled oscillator units are often subject to time delays, since signals travel at finite speed from one node to another. Motivated by this, I will discuss a higher-order *phase reduction* method – i.e. a method to derive equations for the phases of coupled oscillators – for networks systems with delayed coupling. The resulting phase equations are finite dimensional, which makes them in particular suitable for a further (numerical) analysis.

In the talk, I will first discuss the dynamical systems approach behind the phase reduction method; next, I will discuss how one can compute such a phase reduction numerically, thus making it a starting point of further numerical analysis of synchronisation and bifurcation behaviour. This is based upon joint work with Christian Bick and Bob Rink [1] and upon currently ongoing work with Kyle Wedgwood.

### References

[1] Christian Bick, Bob W. Rink and Babette A. J. de Wolff, *Higher-order phase reduction for delay-coupled oscillators beyond the phase-shift approximation*, https://arxiv.org/abs/2510.27524 arxiv preprint

# Stability of Phase-Locked States in Signed Kuramoto Networks: Structure versus Adaptation

Jaeyoung Yoon

Technical University of Munich, Garching bei München, Germany
wodud1516@gmail.com

**Abstract**

Adaptive Kuramoto models admit a variety of nontrivial phase-locked configurations, including antipodal and rotating-wave states. A central open question is whether the observed persistence of such configurations can be attributed to intrinsic properties of the associated signed interaction networks, or whether it relies essentially on adaptive coupling dynamics.

In this talk, we address this question by studying the stability of antipodal and rotating-wave phase configurations on fixed signed networks that preserve the same phase symmetries but are not generated by adaptive dynamics. We will show that for two canonical classes of static signed networks, stability is highly constrained, with unstable modes persisting under parameter variations generically. Furthermore, we characterize how adaptive coupling influences invariant sets and basins of attraction for the configurations where stability is permitted.

Taken together, these results demonstrate that while static network structure imposes severe constraints on the stability of phase-locked configurations, adaptive coupling dynamics organize and delineate their robustness when stability is permitted.

## References

[1] J.Yoon and C. Kuehn, Stability of Phase-Locked States in Signed Kuramoto Networks: Structure versus Adaptation. *arXiv preprint* arXiv:2602.11981.

[2] R. Berner, E. Schöll and S. Yanchuk, Multiclusters in Networks of Adaptively Coupled Phase Oscillators. SIAM Journal on Applied Dynamical Systems 18.4 (2019): 2227-2266.

### Self-Attention as a Multi-Agent System: a Dynamical Perspective on Transformers

Jan Peszek

Institute of Applied Mathematics and Mechanics, University of Warsaw, Warszawa, Poland
J.PESZEK@UW.EDU.PL

#### Abstract

Transformers [1], the architecture underlying modern large language models, (specifically its self-attention component) was recast in [2] as the multi-agent system

$$\dot{x}_k(t) = \mathrm{P}^{\perp}_{x_k(t)} \left( \sum_{j=1}^{n} \exp\left(\langle Q x_k(t), K x_j(t) \rangle\right) V x_j(t) \right), \quad k = 1, \dots, n, \tag{16}$$

where $x_k(t) \in \mathbb{S}^{d-1}$ represents the state of the $k$-th token after the $t$-th attention layer and $Q, K, V \in \mathbb{R}^{d \times d}$ are the trained query, key and value matrices and $\mathrm{P}^{\perp}_{x_k(t)}$ is the projection onto $\mathbb{S}^{d-1}$. The output of the model is determined by the long-time state of the system, making its asymptotic behavior a central object of study. The goal of this talk is to illustrate how methods from collective dynamics can help analyze large language models.

In this talk I present a dynamical systems perspective on a 2D linearized transformer architecture, where the evolution of tokens across attention layers can be reformulated as a Kuramoto-type interacting particle system. Using the Ott–Antonsen ansatz, the high-dimensional dynamics admits a low-dimensional reduction describing the evolution in a reduced vocabulary regime. I will show that this reduced system exhibits a rich range of behaviors depending on the trained parameters, including gradient-flow regimes leading to steady states and Hamiltonian regimes generating oscillatory dynamics. I will also discuss how these phenomena persist beyond the Ott–Antonsen manifold using the Watanabe–Strogatz transformation to analyze the stability of the full particle system.

#### References

[1] A. Vaswani, N. Shazeer, N. Parmar, J. Uszkoreit, L. Jones, A. N. Gomez, Ł. Kaiser, and I. Polosukhin, Attention is all you need, in *Advances in Neural Information Processing Systems 30 (NeurIPS)*, 2017.

[2] B. Geshkovski, C. Letrouit, Y. Polyanskiy, and P. Rigollet, A mathematical perspective on transformers, *arXiv preprint arXiv:2312.10794*, 2023.

## Spectral Analysis of Bochner-Positive Alignment Models and Application to Relaxation

Roman Shvydkoy

University of Illinois at Chicago
shvydkoy@uic.edu

### Abstract

We will identify a class of alignment models where the spectral gap condition – necessary for the analysis of long time behavior – can be associated with the positive-definiteness of the averaging protocol. We show that if the communication kernel factors through a feature space with organized underlying structure, such as a connected abelian group, then the spectral gap can be computed in terms of thickness of the flock. This leads to vast generalizations of the previously established relaxation results for kinetic Fokker-Planck-alignment models as well as regularity of solutions.

## Global Solutions to Cross-Diffusion Systems with Independent Advections in One Dimension

Jakub Skrzeczkowski

St John's College & Mathematical Institute, University of Oxford, Oxford, United Kingdom
JAKUB.SKRZECZKOWSKI@MATHS.OX.AC.UK

### Abstract

We consider cross-diffusion systems describing evolution of two species $u$ and $v$ moving according to the Darcy's law with the pressure law $p(s) = \frac{1}{\alpha-1} s^{\alpha-1}$ where $s = u+v$. One of the most challenging questions in the field is the construction of solutions to the problem in the presence of additional advection fields, without imposing any artificial structure (for instance, preventing the species from mixing). Although advection arises naturally in these models, it breaks the symmetry of the system and prevents application of techniques developed in recent years. We solve the problem in one space dimension in a unified way for all pressure exponents $\alpha \in (0, \infty)$ and arbitrary initial data (segregated, mixed, or partially mixed). In the regime $\alpha \in (0, 1]$ this extends existing results to general initial conditions, while for $\alpha > 1$, it provides the first existence result without structural assumptions. We construct the solutions as a limit of a vanishing viscosity approximation $(u_\varepsilon, v_\varepsilon)$. The main challenge is to identify the limit of the product $u_\varepsilon\, \partial_x p(s_\varepsilon)$ with $s_\varepsilon = u_\varepsilon + v_\varepsilon$ and the key new insight is that possible oscillations of $u_\varepsilon$ and $\partial_x p(s_\varepsilon)$ are correlated, which simplifies the analysis of the associated Young measures in the compensated compactness argument and yields strong convergence of both quantities. Quite surprisingly, the argument relies on only three entropies, which is especially helpful for $\alpha > 2$, where it is unclear whether additional entropies are available.

# Existence and Weak-Strong Uniqueness of Measure Solutions to Euler-Alignment/Aw–Rascle-Zhang Model of Collective Behaviour

Ewelina Zatorska

University of Warwick, Coventry, United Kingdom
EWELINA.ZATORSKA@WARWICK.AC.UK

## Abstract

I will talk about the multi-dimensional Euler-alignment system with a matrix-valued communication kernel, motivated by models of anticipation dynamics in collective behaviour. A key feature of this system is its formal equivalence to a nonlocal variant of the Aw–Rascle–Zhang (ARZ) traffic model, in which the desired velocity is modified by a nonlocal gradient interaction. We prove the global-in-time existence of measure solutions to both formulations, obtained via a single degenerate pressureless Navier–Stokes approximation. Furthermore, we establish a weak-strong uniqueness principle adapted to the pressureless setting and to nonlocal alignment forces. As a consequence, we rigorously justify the formal correspondence between the nonlocal ARZ and Euler-alignment models: they arise from the same inviscid limit, and the weak-strong uniqueness property ensures that, whenever a classical solution exists, both formulations coincide with it. The talk is based on the article [1].

## References

[1] J. Woznicki, E. Zatorska: Existence and weak-strong uniqueness of measure solutions to Euler-alignment/Aw–Rascle–Zhang model of collective behaviour. arXiv:2511.05326

### Heterogeneity in Reaction–Diffusion Systems: A Feature, Not a Perturbation

Martina Chirilus-Bruckner

Leiden University, Leiden, the Netherlands
m.chirilus-bruckner@math.leidenuniv.nl

#### Abstract

Spatial heterogeneities are frequently regarded as an inconvenience: they break translation invariance, make spectral analysis more intricate, and can obscure the clarity of bifurcation scenarios familiar from homogeneous media. In this talk, we contend that heterogeneity is not merely a complication, but also a valuable tool offering a controlled mechanism to generate, choose, and stabilize coherent structures. We develop an analytical framework addressing existence, stability, and bifurcations of solutions in reaction–diffusion systems with spatially dependent coefficients, with particular emphasis on front and wave train dynamics. The presentation is structured around two representative case studies. First, we examine heterogeneous front solutions in a FitzHugh–Nagumo equation, where spatial variability produces fronts that propagate at non-constant speeds through stationary heterogeneous background states. Second, we investigate wave trains governed by a Ginzburg–Landau amplitude equation arising as slow modulations within a Swift–Hohenberg model, highlighting how spatially non-uniform coefficients influence wave-number selection. A common thread throughout is the use of perturbation techniques involving a small parameter - but importantly, this parameter does not quantify the magnitude of the spatial heterogeneity. Rather, it reflects a scale separation or nearness to a critical regime, which permits order-one heterogeneities while preserving analytical tractability. The key novelty is an extension of classical perturbative approaches to systematically include non-autonomous terms, leading to solvability conditions and reduced modulation equations in spatially varying environments.

### Subcritical Transition to Turbulence via Large-Scale Coherent Structures

Sébastien Gomé

Department of Physics, Technion - Israel Institue of Technology, Haifa, Israel
SGOME@CAMPUS.TECHNION.AC.IL

### Abstract

Subcritical wall-bounded flows (like pipe flow, plane Couette flow or plane channel flow) undergo a transition to turbulence through the nonlinear dynamics of coexisting laminar and turbulent regions. When generated, turbulent motions at sufficiently low Reynolds number spontaneously take the form of large-scale localized structures, known as puffs in pipe flow or bands in planar flows, which coexist with laminar flow. In contrast to large Reynolds number regimes where turbulence always contaminates the flow via expanding fronts, these low-Reynolds-number structures either decay or propagate by self-replicating.

I will review some of the progress made in modelling these self-sustained nonlinear structures with reduced-order PDEs[1,2]. Localized turbulent structures are sustained by spatial energy fluxes from laminar to turbulent regions, driven by emergent large-scale mean flows, which, in planar setups, cause regular patterns to form[4,5]. I will then discuss configurations where this large-scale mean flow is absent or suppressed, focusing on a controlled numerical simulation of plane Couette flow[6]. In this setup, transition to turbulence is not mediated by permanent localized structures, but rather occurs via the propagation of fluctuating fronts at all Reynolds numbers. This simpler transition scenario, also present in curved or body-forced pipes, can be modeled by a noise-driven metastable one-variable system. I will discuss how the noise level in this model dictates whether the transition to turbulence is discontinuous or continuous.

### References

[1] Barkley, D. (2016). Theoretical perspective on the route to turbulence in a pipe. Journal of Fluid Mechanics, 803, P1.

[2] Benavides, Santiago J., and Dwight Barkley. "Model for transitional turbulence in a planar shear flow." Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 481.2323 (2025).

[3] Gomé, S., Tuckerman, L. S., & Barkley, D. (2023). Patterns in transitional shear turbulence. Part 1. Energy transfer and mean-flow interaction. Journal of Fluid Mechanics, 964, A16.

[4] Gomé, Sébastien, Laurette S. Tuckerman, and Dwight Barkley. "Patterns in transitional shear turbulence. Part 2. Emergence and optimal wavelength." Journal of Fluid Mechanics 964 (2023): A17.

[5] Gomé, S., Rivière, A., Tuckerman, L. S., & Barkley, D. (2024). Phase transition to turbulence via moving fronts. Physical Review Letters, 132(26), 264002.

### Think Global, Act Local: Inducing Fully Localised Patterns via Spatial Heterogeneity

Dan J. Hill

University of Oxford, Oxford, UK
DAN.HILL@MATHS.OX.AC.UK

#### Abstract

The existence of localised two-dimensional patterns has been observed and studied in numerous experiments and simulations: ranging from optical solitons, to patches of desert vegetation, to fluid convection. And yet, our mathematical understanding of these emerging structures remains extremely limited beyond one-dimensional examples.

In this talk I will discuss how adding a compact region of spatial heterogeneity to a PDE model can not only induce the emergence of fully localised 2D patterns, but also allows us to rigorously prove and characterise their bifurcation [1]. The idea is inspired by experimental and numerical studies of magnetic fluids and tornados, where our compact heterogeneity corresponds to a local spike in the magnetic field and temperature gradient, respectively. In particular, we obtain local bifurcation results for fully localised patterns both with and without radial or dihedral symmetry, and rigorously continue these solutions to large amplitude. Notably, the initial bifurcating solution (which can be stable at bifurcation) varies between a radially-symmetric spot and a 'dipole' solution as the width of the spatial heterogeneity increases. This work is in collaboration with David J.B. Lloyd and Matthew R. Turner (both University of Surrey).

#### References

[1] D.J. Hill, D.L.B. Lloyd & M.R. Turner, *Global bifurcation of localised 2D patterns emerging from spatial heterogeneity* (2025), arXiv:2510.02867

## Dynamics of a Rimming Flow in a Cylinder of Finite Length

Juri Joussen

University of Stuttgart, Stuttgart, Germany
JURI.JOUSSEN@IADM.UNI-STUTTGART.DE

### Abstract

A rimming flow is the flow of a fluid partially filling a rotating cylinder. We assume that the fluid forms a capillary-driven thin film coating the inner wall. The film height $h$ of the surface above the cylinder can be modelled by the following fourth-order degenerate-parabolic evolution equation [1]:

$$h_t + h_\theta + \gamma \operatorname{div}\left(h^3 \nabla(\Delta h + h)\right) = \delta(h^3 \cos(\theta))_\theta$$

with time $t > 0$, cylindrical coordinates $(\theta, z) \in \mathbb{T} \times (0, L)$ and Neumann-type boundary conditions on the cylinder covers. Competing effects relevant to this equation are the cylinder rotation, surface tension $\gamma > 0$, viscosity and gravity $\delta \geq 0$. We investigate the stability of positive steady states. For small gravity $0 < \delta \ll 1$ and short cylinders ($L < \pi$) we show that certain steady states are stable and while they are unstable for long cylinders ($L > \pi$). For cylinders of critical length $L = \pi$ we investigate solutions of the form

$$h(t, \theta, z) = m + a_1(\delta^2 t)e^{i(\theta - t)} + a_{-1}(\delta^2 t)e^{-i(\theta - t)} + b(\delta^2 t)\cos(z)$$

and formally derive an ODE for the (slow) coefficients $a_1, a_{-1} \in \mathbb{C}, b \in \mathbb{R}$, thus describing the dynamics of such solutions on the slow time scale $\tau = \delta^2 t$.

This talk is based on joint work with Janne Laudien (Stuttgart), Christina Lienstromberg (Stuttgart) and Juan J. L. Velázquez (Bonn).

### References

[1] Pukhnachov, V. V. "Capillary/gravity film flows on the surface of a rotating cylinder." Journal of Mathematical Sciences 130.4 (2005): 4871-4883.

## Beyond the Marginal Stability Criterion: Travelling Waves and Model Discrimination in Reaction–Diffusion Systems

Václav Klika

Dept. of Mathematics, FNSPE, Czech Technical University in Prague, Prague, Czech Republic
VACLAV.KLIKA@CVUT.CZ

### Abstract

This work investigates the transient dynamics of pattern formation in reaction–diffusion systems in supercritical regime, with particular emphasis on waves emerging from localised perturbations in Turing-type models. While classical studies of diffusion-driven instability have largely focused on stationary patterns, we instead examine pattern formation as an inherently spatio-temporal process. Two complementary analytical approaches are employed: the marginal stability criterion and weakly nonlinear multiple-scales analysis via envelope equations. The former is first critically revisited, highlighting its assumptions and limitations. In contrast, the weakly nonlinear framework provides access not only to the wave speed but also to the profile and amplitude of the propagating front. By comparing the two approaches in the vicinity of the bifurcation point, we show that they yield consistent predictions for the wave speed, while also allowing us to delineate the regime of validity of the marginal stability criterion.

Our findings establish that Turing systems possess a unique characteristic propagation speed that depends on both diffusion and reaction parameters, offering a dynamic counterpart to the traditionally studied pattern wavelength. The analytical predictions are validated against numerical simulations for prototypical reaction kinetics, including Schnakenberg and CDIMA systems. Beyond their theoretical relevance, the results address an increasingly important issue in the application of reaction–diffusion models to real-world systems: model discrimination. Given the flexibility in choosing reaction terms, spatial heterogeneities, domain geometries, and boundary conditions, it is often possible to reproduce observed patterns without uniquely identifying the underlying mechanism, raising concerns about covariance versus causality. In this context, the propagation speed of travelling waves—available here in explicit analytical form—provides a complementary and potentially more discriminating observable alongside the classical pattern wavelength. More broadly, the study offers a systematic framework for analysing front propagation and transient dynamics, contributing to a more rigorous and testable application of reaction–diffusion models in experimental settings.

### References

[1] Klika, V., Gaffney, E. A., & Maini, P. K., On the speed of propagation in Turing patterns for reaction–diffusion systems, *Physica D: Nonlinear Phenomena*, 467 (2024), 134268. doi:10.1016/j.physd.2024.134268

## Some Remarks About an Effective Description of High-Frequency Non-Polarized Wave-Packet Propagation

Anna Logioti

Institut für Analysis, Dynamik und Modellierung, Universität Stuttgart, Germany
anna.logioti@mathematik.uni-stuttgart.de

### Abstract

We consider systems of the form

$$\partial_\tau \mathcal{U} + \mathcal{A}(\partial_\xi)\mathcal{U} + \frac{1}{\varepsilon}\mathcal{E}\mathcal{U} = \mathcal{T}_2(\mathcal{U},\mathcal{U}) + \varepsilon\mathcal{T}_3(\mathcal{U},\mathcal{U},\mathcal{U}),$$

with $0 < \varepsilon \ll 1$ a small perturbation parameter. We are interested in an effective description of high-frequency wave-packet propagation associated to highly oscillatory initial conditions

$$\mathcal{U}(\xi,0) = \mathcal{U}_*(\xi)e^{ik_0\xi/\varepsilon} + c.c..$$

For polarized initial conditions classical perturbation analysis yields NLS approximations up to an arbitrary order. For non-polarized initial conditions, in lowest order a system of decoupled NLS equations can be derived as an approximate description of the associated solutions. Assuming non-resonance conditions, we prove estimates that rigorously control the error between these formal approximations and true solutions of the original system. The estimates imply that solutions to non-polarized initial conditions split into polarized wave packets. The interaction of these wave packets results in phase and envelope shifts, for which explicit formulas are provided. The result improves results from the existing literature in at least two directions, firstly, the handling of higher order approximations in case of quadratic nonlinearities $\mathcal{T}_2(\mathcal{U},\mathcal{U})$ and secondly, the handling of non-polarized initial conditions which is the main novelty of the present work.

This is a joint work with X. Meng and G. Schneider.

## Traveling Bore Wave Solutions to the Free Boundary Incompressible Navier-Stokes Equations

Ian Tice

Carnegie Mellon University, Pittsburgh, PA, USA
IANTICE@ANDREW.CMU.EDU

### Abstract

In this talk we will discuss the construction of two-dimensional traveling bore wave solutions to the free boundary incompressible Navier-Stokes equations for a single finite depth layer of constant density fluid. Our construction is based on a rigorous justification of the formal shallow water limit, which postulates that in a certain scaling regime the full free boundary traveling Navier-Stokes system of PDEs reduces to a governing system of ODEs. We find heteroclinic orbits solving these ODEs and, through a delicate fixed point argument employing the Stokes problem in thin domains and a nonautonomous orbital perturbation theory, use these ODE solutions as the germs from which we build bore PDE solutions for sufficiently shallow layers. This is joint work with Noah Stevenson.

## Rolls, Runaway Modes and Energetic Consistency in Spectral Truncations of the Rayleigh Bénard Problem

Roland Welter

University of Hamburg, Hamburg, Germany
ROLAND.WELTER@UNI-HAMBURG.DE

### Abstract

The Rayleigh Bénard convection problem from fluid dynamics is well-known for its rich dynamics, nonlinear waves and roll state spatial patterns. In this talk we present here our recent results on the issue of energetic consistency in Fourier-Galerkin truncations of this problem [1, 2]. We give mode selection criteria which are necessary and sufficient for a truncated model to satisfy the potential energy balance associated with the PDE. We prove also that the potential energy balance is sufficient for the truncated system to admit a compact attractor, and that there exist unbounded trajectories (so called "runaway modes") in a large class of energetically inconsistent models. Finally, we examine the cascade of bifurcations of roll states, their stability and their heat transport via a hierarchy of energetic consistent models.

### References

[1] Roland Welter (2025) *Rotating Rayleigh–Bénard Convection: Attractors, Bifurcations, and Heat Transport via a Galerkin Hierarchy*, SIAM Journal on Applied Dynamical Systems, Vol. 24

[2] Jens D.M. Rademacher and Roland Welter (2026) *Energetic consistency and heat transport in Fourier-Galerkin truncations of free slip 3D rotating convection*, arXiv preprint: *https://arxiv.org/abs/2505.07678*

## Homogenization of Elasto-Plastic Plate Equations with Vanishing Hardening

Marin Bužančić, Igor Velčić, Josip Žubrinić

University of Zagreb Faculty of Electrical Engineering and Computing, Zagreb, Croatia
MARIN.BUZANCIC@FER.UNIZG.HR

### Abstract

In this work, we consider a lower dimensional homogenized thin plate model within the framework of linearized elasto-plasticity. Building on recent work analyzing the interaction between homogenization and dimension reduction in perfectly plastic materials, we focus on the extreme regime where the plate thickness vanishes much faster than the period of oscillation of the heterogeneous material. Our investigation is devoted to the derivation of the existence of a quasistatic evolution for a multiphase elasto-plastic composite without imposing any kind of ordering constraints on admissible yield surfaces of the various phases, generalizing in that way the result of [1].

The analysis is carried out in two steps: we first derive a heterogeneous plate model with hardening via dimension reduction using evolutionary $\Gamma$–convergence techniques and generalizing the result of [2], and then perform two-scale homogenization while simultaneously letting the hardening parameters tend to zero by using a stress-strain approach. This approach allows us to reconcile the Kirchhoff-Love structure of limiting displacements with the expected requirement that the effective dissipation potential at each point of the interface between two phases needs to be the pointwise-in-space inf-convolution of that in either phase.

### References

[1] M. Bužančić, E. Davoli and I. Velčić. "Effective quasistatic evolution models for perfectly plastic plates with periodic microstructure: The limiting regimes" *Advances in Calculus of Variations* **17**, no. 4, 1399–1444 (2024). https://doi.org/10.1515/acv-2023-0020

[2] M. Liero, and T. Roche. "Rigorous derivation of a plate theory in linear elastoplasticity via $\Gamma$-convergence", *Nonlinear Differential Equations and Applications NoDEA*, **19**, no. 4, 437—457 (2012).

### Homogenization in Magnetoelasticity Under Small Elastic Response

Lorenza D'Elia

TU Wien, Vienna, Austria
LORENZA.DELIA@TUWIEN.AC.AT

#### Abstract

In this talk, we perform a simultaneous homogenization and linearization analysis for a magnetoelastic energy functional featuring a mixed Eulerian-Lagrangian structure. Neglecting Zeeman and anisotropic contributions, we characterize the asymptotic behavior in the sense of $\Gamma$-convergence for the sum of a nonlinear magnetoelastic energy, a symmetric exchange term defined on the actual configuration, and for the associated magnetostatic self-energy.
This is a joint work with M. Cherdantsev (Cardiff University), E. Davoli (TU Wien) and S. Riccò (TU Wien).

### Variational Derivation of the Flamant Solution for a Nonlinear Elastic Wedge

Dominik Engl

KU Eichstätt-Ingolstadt, Eichstätt, Germany
DOMINIK.ENGL@KU.DE

#### Abstract

When a planar kirigami metamaterial is deformed by slowly modulating a mechanism motion across the pattern, its individual elastic panels still rotate predominantly, but they must also stretch slightly to accommodate the modulation. The resulting strain is not confined to a small neighborhood of the hinges, nor does it spread uniformly over the whole panel. Instead, it spreads at leading order in a manner consistent with the Flamant solution, which is a classical singular solution to linear elasticity arising from concentrated forces acting at the tip of a wedge.

In this talk, we provide a rigorous variational derivation of this behavior, starting from a generic family of hyperelastic energy functionals, by studying energy minimization problems on wedge-shaped domains truncated at a small radius and under small loading, with both parameters tending to zero simultaneously. We address displacement-controlled boundary conditions as well as pure-traction problems. By introducing logarithmic polar coordinates, we map the varying domains to a fixed reference domain and obtain a rescaling consistent with the total linear elastic energy of the Flamant solution; moreover, these variables induce an anisotropic scaling of derivatives akin to those arising in thin structure limits. In this framework, we prove convergence of minimal energies and strong convergence of the associated low-energy strains, including an explicit quantitative rate depending on the scaling of cut-off and loading. A key ingredient is a geometric rigidity estimate with a constant uniform across the family of truncated wedge domains. This is joint work with Paul Plucinsky and Ian Tobasco.

## Variational Models for Material Voids: Dimension Reduction and Homogenization

Manuel Friedrich

JKU Linz, Austria
MANUEL.FRIEDRICH@JKU.AT

### Abstract

In this talk, I present variational models for materials with voids in both geometrically linear and nonlinear elasticity. Specifically, we consider functionals featuring elastic bulk terms and a surface contribution proportional to the surface area of the voids within the material. We first discuss a relaxation result that leads to an additional surface term related to the phenomenon of collapsing voids. We then present applications to the derivation of plate theories via dimension reduction, as well as to effective homogenization limits for heterogeneous materials.

## Rigorous Derivation of a Biot-Plate-System for a Thin Poroelastic Layer

Markus Gahn

Institute of Mathematics, University of Augsburg, Augsburg, Germany
MARKUS.GAHN@UNI-A.DE

### Abstract

We consider incompressible fluid flow in a thin poroelastic layer and rigorously derive a macroscopic model when the thickness of the layer tends to zero. Within the layer we assume a periodic structure and both, the periodicity and the thickness of the layer, are of order $\varepsilon$. The fluid flow is described by quasistatic Stokes-equations and for the elastic solid we consider linear elasticity equations, and both are coupled via continuity of the velocities and the normal stresses. For $\varepsilon \to 0$ the thin layer reduces to a lower dimensional manifold. Via a simultaneous homogenization (structure within the layer) and dimension reduction (thickness of the layer) we derive a limit model given by a coupled Biot-Plate-system consisting of a generalized Darcy-law coupled to a Kirchhoff-Love-type plate equation including the Darcy pressure.

### The Effects of Pressure Loads in the Dimension Reduction of Elasticity Models

Martin Kružík

Institute of Information Theory and Automation, Czech Academy of Sciences,
Pod vodárenskou veží 4, CZ-182 00, Prague 8, Czechia
Faculty of Civil Engineering, Czech Technical University,
Thákurova 7, 166 29 Prague 6, Czechia
kruzik@utia.cas.cz

#### Abstract

We study the dimensional reduction from three to two dimensions in hyperelastic materials subject to a live load, modeled as a constant pressure force. Our results demonstrate that this loading has a significant impact in higher-order scaling regimes, namely those associated with von Kármán-type theories, where a nontrivial interplay between the elastic energy and the pressure term arises. In contrast, we rigorously show that in lower-order bending regimes, as described by Kirchhoff-type theories, the pressure load does not influence the minimizers. Finally, after identifying the corresponding $\Gamma$-limit, we conjecture that a similar independence from the pressure term persists in the most flexible membrane regimes. This is a joint work with F. Riva (Prague).

#### References

[1] M. Kružík, F. Riva: The effects of pressure loads in the dimension reduction of elasticity models. arXiv:2504.15864v2

## Simultaneous Dimensional Reduction and Homogenization in Manifold Valued Functions Spaces

Elvira Zappale

Department of Basic and Applied Science, Sapienze -University of Rome, Rome, Italy
elvira.zappale@uniroma1.it

### Abstract

Both dimensional reduction and homogenization is rigorously obtained for integral functionals defined on the manifold constrained Orlicz-Sobolev and BV spaces. The achieved results find application in the study of equilibria for liquid crystals, in ferromagnetism or for magnetostrictive materials where the physical quantities take their values into a given manifold.

### References

[1] M. Eleuteri, L. Lussardi, A. Torricelli, E. Zappale, Homogenization and 3D-2D dimension reduction of a functional on manifold valued Sobolev spaces, Nonlinear Analysis: Real World Applications, **91**, 104579,(2026), https://doi.org/10.1016/j.nonrwa.2025.104579

[2] L. Lussardi, A. Torricelli, E. Zappale, Homogenization and 3D-2D dimension reduction of a functional on manifold valued BV space, submitted.

[3] J. Dongo, J. Fotso Tachago, F. Tchinda, E. Zappale in preparation

## Poroelastic Plate Model Obtained by Simultaneous Homogenization and Dimension Reduction

Josip Žubrinić

University of Zagreb, Faculty of Electrical Engineering and Computing, Croatia
josip.zubrinic@fer.hr

### Abstract

In this talk, we consider a coupled system of linear elasticity and the Stokes equations as the starting point of our analysis. The model depends on two small parameters: the thickness $h$ of the plate and the pore scale $\varepsilon$, which in general depends on $h$. We focus on the regime in which the pore size is small compared to the plate thickness.

Our main goal is to derive an effective poroelastic plate model from the three-dimensional problem in the limit as $h \to 0$, by combining simultaneous homogenization and dimension reduction techniques. The resulting model extends the poroelastic plate model derived by Mikelić et al. [1] from the three-dimensional Biot system by dimension reduction. In particular, it also includes the case of contact between poroelastic and (poro)elastic plates, as well as the corresponding evolution equation with an inertial term.

### References

[1] A. Mikelić, M. F. Wheeler, and T. Wick, Derivation of a poroelastic plate model from 3D Biot's equations, *Arch. Ration. Mech. Anal.* **215** (2015), 1035–1062.

## Improving Accuracy in Eddy Viscosity Turbulence Models Through Filtering and Approximate Deconvolution

Argus-Adrian Dunca

Department of Mathematics and Informatics, Faculty of Applied Sciences,
University Politehnica of Bucharest, Bucharest, Romania
argus_adrian.dunca@upb.ro

### Abstract

The talk will present a class of Eddy Viscosity turbulence models whose derivation is based on filtering and approximate deconvolution. The resulting EV models are mathematically well posed and physically sound, leading to improved accuracy. Recent models in this class also allow simple numerical schemes for finite element computations. These properties, as well as some numerical results and comparison with other EV models, will be discussed.

### Reduced Basis Smagorinsky Turbulence Models

Tomás Chacón Rebollo

Dept. EDAN & IMUS, Universidad de Sevilla, Spain
chacon@us.es

#### Abstract

Large Eddy Simulation (LES) turbulence models provide accurate space-time predictions of turbulent flows. However, these models are little used in engineering applications due to very large computing times. Reduced Basis (RB) modeling aims at building very fast solvers for parametric PDEs, with "certified"error levels (that is, meeting pre-set error levels), typically with computing speed-up ratios ranging from tens to thousands.

We address in this talk the building of RB Smagorinsky turbulence models, a LES turbulence model for which the mathematical and numerical analysis is well established [3]. We shall use the so-called "Greedy"algorithms, based upon mathematical a-priori error estimates, what allows to obtain certified RB models [2]. We shall also use purely physics-based error estimation procedures in the Greedy algorithm, that have the advantage of being model-independent [1].

#### References

[1] Bandera Moreno, A.; Caravaca García, C.; Chacón Rebollo, T.; Delgado Ávila, E.; Gómez Mármol. M.: *Reduced Basis modelling of turbulence with well-developed inertial range.* Computer Methods in Applied Mechanics and Engineering, Vol. 419. DOI 10.1016/j.cma.2023.116683 (2024).

[2] Chacón Rebollo, T.; Delgado Ávila, E.; Gómez Mármol. M.: *On a certified VMS-Smagorinsky reduced basis model with LPS pressure stabilisation.* Appl. Numer. Math. 185 (2023), 365–385. DOI 10.1016/j.apnum.2022.12.003

[3] Chacón Rebollo, T., Lewandowski, R.: *Mathematical and Numerical Foundations of Turbulence Models and Applications.* Birkhauser, 472 pages (2014). http://www.springer.com/birkhauser/mathematics/book/978-1-4939-0454-9

## Existence of Solutions for Motions of Cosserat–Bingham Fluids

Michael Růžička

Department of Applied Mathematics, University of Freiburg
Ernst-Zermelo-Straße 1, D-79104 Freiburg, Germany
ROSE@MATHEMATIK.UNI-FREIBURG.DE

### Abstract

In the talk we present an existence result for global in time weak solutions for large data for the equations describing the motion of incompressible homogeneous Cosserat–Bingham fluids. Such fluids combine the behavior of micro-polar fluids and visco-plastic fluids. The existence proof uses the Lipschitz truncation technique and the theory of Bochner pseudomonotone operators.

### References

[1] L. Diening, M. Růžička, J. Wolf (2010). *Existence of weak solutions for unsteady motions of generalized Newtonian fluids*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. V, **IX**:, 1–46.

[2] A. Kaltenbach, M. Růžička (2021). *Note on the existence theory for pseudo-monotone evolution problems*, J. Evol. Equ., **21**:, 247–276.

[3] S. Andres, M. Růžička (2026). *Existence of weak solutions for unsteady motions of Cosserat-Bingham fluids*, in preparation.

### Semigroup Solutions for a Biot-Poroelastic Plate-Stokes System

George Avalos

University of Nebraska-Lincoln, USA
gavalos2@unl.edu

#### Abstract

In this talk, we discuss our recent work concerning a coupled system of partial differential equations which describe a multilayered filtration system. This PDE model describes the interaction of a viscous incompressible flow with bulk poroelasticity via a poroelastic interface. The geometry in which the PDE components evolve consists of two 3D toroidal subdomains connected via a plate interface, which permits elastic deformation and perfusive fluid dynamics. The governing dynamics comprise Stokes equations in the bulk fluid region, Biot's equations in the bulk poroelastic region, and the recent poroplate of Mikelić at the interface. Coupling occurs on the top and lower surfaces of the plate, and involves conservation of mass, stress balance, and a certain slip condition for the fluid free-flow. For this multilayered PDE system, we obtain strongly continuous semigroup generation, in large part via a resolvent analysis which employs a nonstandard mixed variational formulation that captures the complex, multi-physics coupling at the interface. Subsequently, we discuss how the availability of the Biot-poroelastic plate-Stokes semigroup straightforwardly admits structural (plate) nonlinearities into the dynamics, as well as allow for stability and regularity analyses. This is joint work with Galen Richard and Justin Webster.

## Mathematical Models for Hemorheology and Hemolysis

Tomáš Bodnár

Institute of Mathematics, Czech Academy of Sciences,
Žitná 25, 115 67 Prague 1, Czech Republic

BODNAR@MATH.CAS.CZ

### Abstract

Blood is a suspension of blood cells in blood plasma. It can be seen as a complex fluid with non-Newtonian rheology. Under specific flow conditions, such as in Ventricular Assist Devices, blood is exposed to high mechanical stress, which may lead to damage of Red Blood Cells, the hemolysis. Mathematical modeling of both, the visoelastic rheology of blood and the hemolysis requires solution of tensorial equations related to microstructure of the fluid. The aim of the present talk is to provide comparative analysis of specific tensorial models of hemorheology and hemolysis, pointing out their formal similarity based on shared physical concepts.

All models are formulated in a unified manner, enabling a systematic comparison. The analysis is complemented by numerical simulations of blood flow in a multiply corrugated tube, where both flow characteristics and hemolysis indicators are evaluated. The main contributions of this work are the development of a unified framework for strain- (stretch-) based models of rheology and hemolysis, the formulation of a combined model integrating the approaches of Rajagopal[1, 3] and Arora[2, 4], and the introduction of a new stretch-tensor invariant-based indicator of hemolysis.

*This is a part of an ongoing joint work with Anna Lancmanová, Nico Dirkes, Alberto Girelli, Adelia Sequeira and Giulia Giantesio.*

### References

[1] M. Anand and K. Rajagopal. A shear-thinning viscoelastic fluid model for describing the flow of blood. *International Journal of Cardiovascular Medicine and Science*, 4(2):59–68, 2004.

[2] D. Arora, M. Behr, and M. Pasquali. A tensor-based measure for estimating blood damage. *Artificial Organs*, 28(11):1002–1015, 2004.

[3] T. Bodnár, K. Rajagopal, and A. Sequeira. Simulation of the three-dimensional flow of blood using a shear-thinning viscoelastic fluid model. *Mathematical Modelling of Natural Phenomena*, 6(5):1–24, 2011.

[4] N. Dirkes, F. Key, and M. Behr. Eulerian formulation of the tensor-based morphology equations for strain-based blood damage modeling. *Computer Methods in Applied Mechanics and Engineering*, 426:116979, 2024.

## On the Compactness of Artificial Compressibility Approximations of Weak Solutions for Fluid Problems in a General Moving Geometry

Anna Hundertmark, Stephan Schmitz

RPTU University Kaiserslautern-Landau, Germany
a.hundertmark@rptu.de

### Abstract

In this talk a, fluid flow problem on a time dependent deforming domain for a Newtonian fluid in two and three space dimensions with artificial compressibility approximation is studied. The equicontinuity in time estimate of the weak solutions under suitable domain regularity assumptions is proven, which serves as an alternative compactness argument for approximate weak solution sequences with respect to artificial compressibility, bypassing the application of the Lions-Aubin Lemma and its generalizations for solenoidal solutions on moving domains.

The corresponding estimate, which is uniform with respect to compressibility parameter is obtained by remapping the problem onto a fixed reference domain and using appropriate divergence-free test functions involving the difference of two solutions at different points in time, thus on different domains. The solenoidal property of the test functions is guaranteed by an appropriate divergence-preserving transformation between the two domains. The presented result is a generalization of compactness arguments used in author's previous works on fluid-structure interaction in special 2D domains, see e.g., [1].

### References

[1] A. Hundertmark-Zaušková, On the convergence of fixed point iterations for the moving geometry in a fluid-structure interaction problem, J. Differ. Equ. 267(12) (2019), 7002-7046.

### Can One Control Large Sustained Oscillations in Fow-Structure Interactions?

Irena Lasiecka

University of Memphis, Memphis,TN, and University of Virginia, Charlottesvillle,VA,USA
LASIECKA@MEMPHIS.EDU

#### Abstract

Flow-structure interactions are ubiquitous in nature and in everyday life. Flow or fluid by interacting with structural elements can lead to oscillations, hence impacting stability or even safety. Thus problems such as attenuation of turbulence or flutter in an oscillating structure such flutter in tall buildings, fluid flows in flexible pipes, in nuclear engineering flows about fuel elements and heat exchanger vanes are just few prime examples of relevant applications which place themselves at the frontier of interests in applied mathematics. In this lecture we shall describe mathematical models describing the phenomena, These are represented by a 3 D Euler Equation coupled to a **nonlinear** dynamic elasticity on a 2 D manifold. Strong boundary-type coupling at the interface between the two media is at the center of the analysis. This provides for a rich mathematical structure, opening the door to several unresolved problems in the area of nonlinear PDE's, dynamical systems, related harmonic analysis, microlocal analysis and differential geometry. This talk aims at providing a brief overview of recent developments in the area along with a presentation of some new methodology addressing the issues of asymptotic control to coherent structure and overall stability of the relevant dynamics.
Part of this work performed while the author was a member of the MSRI program "Mathematical problem in fluid dynamics" at the University of California Berkeley and it is partially based on the developments reported in [1, 2, 3, 4].

#### References

[1] I.Lasiecka and J.Webster *Flutter stabilization for an unstable, hyperbolic flow-plate interaction* Adv. Math.Fluid Mech. Fluids under Control. ED: T. Bodnar, P. Galdi and S. Necasova. Birkhauser/Springer 2024, pp 157-258.

[2] A. Balakrishna , I. Lasiecka, J. Webster *Elastic stabilization of an intrinsically unstable hyperbolic flow-structure interaction on the 3D half-space.* Math. Models Methods Appl. Sci. 33 (2023), no. 3, 505–545.

[3] D. Bonheure, F. Gazzola, I.Lasiecka, J.Webster. *Long-time dynamics of a hinged-free plate driven by a nonconservative force*, Ann. Inst. H. Poincaré C Anal. Non Linéaire 39 (2022), no. 2, 457–500.

[4] I. Lasiecka, J. Rodriguez, M.Roy. *Attractors for second order in time non-conservative dynamics with nonlinear damping.* Journal Differential Equations, 451, 2026.

## Regularized Interface Methods for Geometrically Nonlinear Navier-Stokes-Biot Interaction

Jeffrey Kuan, Boris Muha, and Sunčica Čanić

Department of Mathematics, Faculty of Science, University of Zagreb, Zagreb, Croatia
BORISM@MATH.HR

### Abstract

We present recent advances in the existence theory for fluid-poroelastic structure interaction (FPSI) with nonlinear geometric coupling. The model couples incompressible Navier-Stokes flow in a moving domain to Biot's system for a poro(visco)elastic medium, with kinematic and dynamic transmission conditions on the a priori unknown interface. In a first step we study a multilayered configuration in which the free fluid and the Biot medium are separated by a reticular plate that carries inertia and elastic energy while remaining permeable to the fluid. A constructive Lie-splitting scheme yields finite-energy (Leray-Hopf) weak solutions to a regularized problem and a weak-classical consistency result as the regularization parameter vanishes.

We then introduce a regularized interface method for FPSI problems with low-regularity interfaces and direct fluid-poroelastic contact. The Biot displacement is smoothed by spatial convolution and we temporarily insert a thin elastic plate of thickness $h > 0$ at the interface. Uniform-in-$h$ energy estimates together with compactness on time-dependent domains (Aubin-Lions type) allow passage to the limit $h \to 0$, producing weak solutions to the regularized interface problem with fully vector-valued interface motion.

### References

[1] J. Kuan, S. Čanić, and B. Muha. *Fluid-poroviscoelastic structure interaction problem with nonlinear geometric coupling*. *Journal de mathématiques pures et appliquées*, 188 (2024), 345–445. DOI: 10.1016/j.matpur.2024.06.004.

[2] J. Kuan, S. Čanić, and B. Muha. *A Regularized Interface Method for Fluid-Poroelastic Structure Interaction Problems with Nonlinear Geometric Coupling*. arXiv:2508.18065, 2025.

### Generalized Newtonian Flows with Directional Do-Nothing Boundary Conditions

Ana Leonor Silvestre

Instituto Superior Técnico, Universidade de Lisboa, Portugal
ANA.SILVESTRE@MATH.TECNICO.ULISBOA.PT

**Abstract**

Generalized Newtonian models have attracted considerable attention over the past few decades, as they provide an accurate description of many complex fluids, including blood, food products such as ketchup, yogurt, and sauces, polymeric liquids, suspensions, and slurries; see, for example, [5] for a mathematical approach to such problems.

In the context of Computational Fluid Dynamics, artificial boundaries, particularly outflow boundaries, are introduced in the mathematical model to truncate the physical domain and make numerical simulations feasible.

Here, we consider a mixed boundary value problem for the equations governing incompressible generalized Newtonian fluids. On the Neumann part of the boundary, we prescribe a Directional Do-Nothing (DDN) boundary condition, as in [1, 4, 3].

In line with recent studies on the Navier-Stokes equations [2, 4, 3], we investigate the well-posedness of the generalized Newtonian problem with DDN boundary conditions and complement our analysis with numerical experiments comparing the classical do-nothing and DDN boundary conditions for a family of power-law fluids. This is joint work with Pedro Nogueira (CEMAT, Instituto Superior Técnico, Universidade de Lisboa).

#### References

[1] M. Braack and P.B. Mucha, Directional do-nothing condition for the Navier-Stokes equations, *J. Comput. Math.*, **32**, 5 (2014), 507-521.

[2] T. Neustupa, Existence of a steady flow through a rotating radial turbine with an arbitrarily large inflow and an artificial boundary condition on the outflow, *ZAMM* **103**, 10 (2023), 1-15.

[3] P. Nogueira, A.L. Silvestre. Regularized directional do-nothing boundary conditions for the Navier-Stokes equations: analytical and numerical study, *Appl Math Comput.*, **499** (2025), 129398.

[4] P. Nogueira, A.L. Silvestre. Navier-Stokes equations with regularized directional boundary condition. In: Da Veiga, H.B., et al. Nonlinear Differential Equations and Applications. PICNDEA 2022, 197-220. CIM Series in Mathematical Sciences. Springer, Cham. (2024).

[5] M. Růžička, Analysis of Generalized Newtonian Fluids. In: Topics in Mathematical Fluid Mechanics. Lecture Notes in Mathematics, vol 2073, Springer. Berlin, Heidelberg, 199–238 (2013).

## Contact in Fluid-Plate Interaction: Formation and Detachment

Srđan Trifunović

Faculty of Sciences, University of Novi Sad, Serbia
SRDJAN.TRIFUNOVIC@DMI.UNS.AC.RS

### Abstract

In this talk, I will present a recent result on contact problem for the interaction between an elastic plate and a compressible viscous fluid located between the plate and a rigid bottom z = 0. First, by utilizing the vertical fluid dissipation, a new estimate is obtained $\ln \eta(t) \in L^1$ for any $t > 0$ provided that $\ln \eta_0 \in L^1$, ensuring that additional contact can form only on a set of a measure zero. Then, by utilizing the expanding capability of compressible fluid pressure, it is shown that all contact has to detach in finite time provided that the source force is not pushing down too much. Finally, it is shown that contact at any point can be detached in any given time with a strong enough source force localized around that point which is pulling the plate up. This is the first result where detachment of contact is proven.

## Uniform Stabilization of the Boussinesq System in Besov Spaces with Boundary-Based, Localized, Finite Dimensional, Fluid and Thermal Feedback Controllers

Roberto Triggiani

University of Memphis, Memphis,USA
RTRGGANI@MEMPHIS.EDU

### Abstract

The Boussinesq system models heat transfer in a viscous incompressible heat conducting fluid. In an open, connected, bounded domain Ω, dimension d =2, 3, we consider an uncontrolled Boussinesq system which we assume to be unstable. We then seek boundary-based, localized, finite dimensional, feedback controllers that stabilize the Boussinesq system in the vicinity of an unstable solution. For d=3, the setting is a suitable Besov space by necessity, as the Sobolev setting is not adequate; even in the case of the Navier-Stokes equations (d=3), a component of the Boussinesq system. This was recently shown by the authors. The proof is constructive. It has no assumptions. The number of controls is minimal; ie equal to the largest geometric multiplicity of the unstable eigenvalues. The corresponding linear feedback problem defines a strongly continuous, uniformly stable, analytic semigroup, in fact of maximal regularity up to $T = \infty$. This latter property is critically used in the analysis of well-posedness and uniform stabilization of the non-linear feedback problem in the vicinity of the unstable equilibrium. A key effort is in showing controllability (Kalman algebraic conditions) for the finite dimensional projection on the unstable subspace. For this, the "ignition key" is based on two Unique Continuation Properties of suitable overdetermined adjoint eigenproblems, one of which was recently proved by Carleman-type estimates. This paper is joint work with Irena Lasiecka and Buddhika Priyasad.

### References

[1] Uniform Stabilization of 3D Navier-Stokes Equations in Low Regularity Besov Spaces with Finite Dimensional, Tangential -Like Boundary, Localized Feedback Controllers(I.Lasiecka and B. Priyasad, R. Triggiani) , *Archive of Rational Mechanics and Analysis* 241,1575-1654 (2021)

[2] Uniques Continuation Properties of static fluid equations critical for the uniform stabilization of dynamic fluid equations with boundary-based finite dimentional feedback controls (R. Triggiani), *Fluids under Control", Chapter 2*,invited paper, 75 pages , Birkhauser, to appear in 2023

## Numerical Construction of Multiple Positive Solutions of an Elliptic Problem with Concave and Convex Nonlinearity

Jiří Benedikt

Západočeská univerzita v Plzni, Plzeň, Czechia
BENEDIKT@KMA.ZCU.CZ

### Abstract

We are interested in positive solutions of the semilinear elliptic Dirichlet problem

$$\begin{aligned} -\Delta u &= \lambda u(x)^{q(x)-1} && \text{for } x \in \Omega, \\ u &= 0 && \text{on } \partial\Omega \end{aligned}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with a $C^{1,\alpha}$ boundary, $\lambda \in \mathbb{R}$ is a spectral parameter and $q(x) > 1$. The convexity/concavity of the reaction term depends on the spatial variable $x$, i.e., it is convex in $\Omega_+ \stackrel{\text{def}}{=} \{x \in \Omega : q(x) > 2\} \neq \emptyset$ and concave in $\Omega_- \stackrel{\text{def}}{=} \{x \in \Omega : q(x) < 2\} \neq \emptyset$.

In the case of one spatial dimension $N = 1$, namely $\Omega = (-1, 1)$, we numerically construct multiple (two or more) positive solutions, provided that $\lambda \in (0, \lambda^*)$ for a certain $\lambda^* > 0$. One of them (the minimal one) bifurcates from zero, whereas the others bifurcate from infinity as $\lambda \to 0+$.

### References

[1] BENEDIKT, J., GIRG, P., KOTRLA, L., AND TAKÁČ, P. *On the Existence of Two Positive Solutions for Spatially Nonhomogeneous Elliptic Problems.* To appear in Mathematica Slovaca (2026).

## On the Multiplicity of Positive Solutions for Spatially Nonhomogeneous Elliptic Problems

Petr Girg

University of West Bohemia, Plzeň, Czech Republic
pgirg@kma.zcu.cz

### Abstract

We consider the semilinear Dirichlet problem

$$-\Delta u = \lambda f(x, u(x)) \quad \text{for } x \in \Omega; \quad u = 0 \quad \text{on } \partial\Omega$$

with a nonlinear reaction term $f(x, \cdot) : s \longmapsto |s|^{q(x)-2}s$. For $s \geq 0$, this term is convex and concave in the nonempty open subsets

$$\Omega_+ \stackrel{\text{def}}{=} \{x \in \Omega : q(x) > 2\}, \qquad \Omega_- \stackrel{\text{def}}{=} \{x \in \Omega : q(x) < 2\}$$

of a bounded domain $\Omega \subset \mathbb{R}^N$ with a sufficiently smooth boundary, respectively. Here, $\lambda \in \mathbb{R}$ is a nonnegative spectral parameter that determines the existence and multiplicity of positive weak solutions (at least two).

For $0 < \lambda \leq \lambda^*$, we obtain a $C^1$-solution $u \equiv u_\lambda$ by monotone iterations. For $0 < \lambda < \lambda^*$, we obtain another $C^1$-solution, $v$, by the Leray-Schauder degree theory, which satisfies $v(x) > u_\lambda(x)$ for all $x \in \Omega$. Our a priori estimates are obtained by Young's inequality. Our main contribution is a method for handling the interplay between convex and concave nonlinearities in two disjoint, nonempty open subsets of $\Omega$. For $\Omega = (-1, 1)$, we will discuss higher multiplicity of positive solutions bifurcating from infinity as $\lambda \to 0+$ by means of the rescaling method. These multiplicity results were motivated by extensive numerical experiments performed by J. Benedikt. We will also mention mathematical models from combustion theory and explain the role of the spectral parameter $\lambda$ and the variable exponent $q(x)$ in these models.

### References

[1] Benedikt, J., Girg, P., Kotrla, L., and Takáč, P. *On the Existence of Two Positive Solutions for Spatially Nonhomogeneous Elliptic Problems.* Mathematica Slovaca, to appear.

## On Time-Dependent Schrödinger Equations Constrained by Measurements

Jochen Merker

HTWK Leipzig, Germany
JOCHEN.MERKER@HTWK-LEIPZIG.DE

### Abstract

If Schrödinger equation $\mathrm{i}\frac{du}{dt} = Hu$ is constrained to states satisfying $P(t_j)^\perp u(t_j) = 0$ for orthogonal projections $P(t_j)$ at discrete times $t_j$ due to ideal measurements, then time evolution is governed by the non-autonomous Hamiltonian equation

$$\mathrm{i}\, du = Hu\, dt - \mathrm{i}P^\perp u_-\, \mu\,, \tag{17}$$

which could be considered as a generalized time-dependent Schrödinger equation incorporating free evolution as well as evolution under measurements into one equation. Hereby, $\mu$ denotes counting measure, the complementary orthogonal projection $P^\perp(t_j) = \mathrm{Id} - P(t_j)$ acts at time $t_j$ on the left limit $u_-(t_j) := \lim_{t \nearrow t_j} u(t)$, and the non-autonomous part $P^\perp u_-\, \mu$ may be considered as Lagrangian multiplier determined to satisfy the constraints $P^\perp u = 0$ imposed on the system due to measurement. Generalizing the space-time formulation in [1], by applying Nečas theorem [2] we show that for every initial value $u_0$ in complex Hilbert space there exists a unique càdlàg function $u$, i.e. $u$ is continuous from the right and has limits from the left in time, solving (17) in an ultra-weak sense.

We further indicate how classical Schrödinger equations modeling non-ideal measurements, where evolution is governed by a particular interaction between measured system and measuring device, tend to generalized Schrödinger equations (17) for the measured system when time of interaction concentrates at an instant. In this way, the measurement problem of quantum mechanics can be solved: Quantum mechanical systems are still modeled by Hamiltonian equations on complex Hilbert spaces to quadratic Hamiltonian functions $\frac{1}{2}\langle Hu, u\rangle$ (inducing vector fields preserving Kähler structure in case of non-constrained free evolution), but in presence of constraints, e.g. due to ideal measurements, dynamics can differ from those governed by classical Schrödinger equation, and non-ideal measurements modeled by such equations converge for the measured system to (17) when time of interaction concentrates at an instant so that the measurement becomes ideal.

### References

[1] S. Hain, K. Urban, *An ultra-weak space-time variational formulation for the Schrödinger equation* Journal of Complexity **85** (2024), 101868.

[2] J. Nečas, *Sur une méthode pour résoudre les équations aux dérivées partielles du type elliptique, voisine de la variation-nelle* Ann. Sc. Norm. Super. Pisa, Cl. Sci. **3**(16) (1962), 305–326.

# An Elliptic Spectral Problem with Concave and Convex Nonlinearity: Uniqueness and Multiplicity of Positive Solutions

Peter Takáč

Universität Rostock, D–18055 Rostock, Germany
PETER.TAKAC@UNI-ROSTOCK.DE

## Abstract

We will discuss the question of ***existence*** and ***multiplicity*** of *positive solutions* to the semilinear elliptic Dirichlet problem

$$-\Delta u = \lambda\, u(x)^{q(x)-1} + f(x,u(x)) \quad \text{for } x\in\Omega\,; \qquad u=0 \quad \text{on } \partial\Omega\,, \tag{18}$$

where $\Omega\subset\mathbb{R}^N$ is a bounded domain with the boundary of class $C^{1,\alpha}$, $\lambda\in\mathbb{R}$ a spectral parameter, and $f(x,u)=|u|^{r-1}\,u$ is a **signed $r$-power** ($r>0$) of the unknown function of (a positive variable) $u\in(0,\infty)$ which depends on the point $x\in\Omega$; $r=q(x)-1$, for instance.

We will briefly present basic methods for treating the semilinear problem (18) with a ***convex*** and ***concave*** nonlinear reaction $f(x,\,\cdot\,)\colon s\longmapsto |s|^{q(x)-2}s\colon \mathbb{R}_+$ $\subset\mathbb{R}\to\mathbb{R}$ which (for $s\geq 0$) is ***convex*** in a nonempty open subset $\Omega_+ \stackrel{\text{def}}{=} \{x\in\Omega\colon$ $q(x)>2\}$ and ***concave*** in another nonempty open subset $\Omega_- \stackrel{\text{def}}{=} \{x\in\Omega\colon q(x)$ $<2\}$ of a bounded domain $\Omega\subset\mathbb{R}^N$. Here, $\lambda\in\mathbb{R}_+$ is a nonnegative spectral parameter which decides about the existence and multiplicity of positive weak solutions (at least two) to problem (18) in case we take $f\equiv 0$.

The Dirichlet Laplace operator being linear, we observe that the elliptic equation in problem (18) is ***convex*** (***concave***, respectively) at a given point $x\in\Omega$, provided $q(x)\geq 2$ $(1<q(x)\leq 2)$. This is a typical problem with variable powers (exponents).

Finally, if time permits, we will discuss also the classical question of ***uniqueness*** for a related problem with the $p(x)$-Laplacian provided $\sup_{x\in\Omega} q(x)$ $<\inf_{x\in\Omega} p(x)$, holds, and the ***multiplicity*** of large solution branches bifurcating from infinity as $\lambda\searrow 0+$.

## References

[1] P. Takáč and J. Giacomoni, *A $p(x)$-Laplacian extension of the Díaz-Saa inequality and some applications*. Proc. Roy. Soc. Edinburgh, Sect. **A 150**(1) (2020), 205–232.

[2] J. I. Díaz and J. E. Saa, *Existence et unicité de solutions positives pour certaines équations elliptiques quasilinéaires.* Comptes Rendus Acad. Sc. Paris, Série I, **305** (1987), 521–524.

[3] Benedikt, J., Girg, P., Kotrla, L., and Takáč, P. *On the Existence of Two Positive Solutions for Spatially Nonhomogeneous Elliptic Problems.* Mathematica Slovaca, (2026), to appear.

## Geometric Determinants of Oxygen and Heat Exchange in Placental Mammals

Igor Chernyavsky

Department of Mathematics, University of Manchester, Manchester, UK
IGOR.CHERNYAVSKY@MANCHESTER.AC.UK

### Abstract

The mammalian placenta is one of the most complex and unique exchange organs. Despite its transient nature, this crucial life-support system not only nourishes growing fetuses but also determines their lifelong health. The human placenta features an intricate disordered micro-porous structure that packs a large surface area ($\sim 10$ m$^2$) into a relatively thin disc connected to the fetus via a helically coiled umbilical cord. The placenta also exhibits unusually high evolutionary diversity across placental mammals, while the impact of its architecture, and in particular of cord coiling, on human pregnancy pathologies, such as fetal growth restriction, remains poorly understood [1].

The talk summarises recent progress in hyper-multiscale 3D imaging (spanning $\mu$m to cm) [2] and associated image-based modelling of placental exchange. Our combined analytical and numerical approaches reduce complexity and offer direct insights into the dominant geometrical determinants of maternal–fetal solute and heat transfer in humans and other species. The models demonstrate a certain universality of upscaled approximations for a wide class of transported solutes [3] and explore the surprising role of the helical vascular configurations of the umbilical cord in modulating solute and heat exchange [4]. These results clarify the role of placental and cord morphology in the context of evolution and fetal resilience.

**Acknowledgements:** Joint work with Tianran Wan, Davis Laundon, Carl Whitfield, Shier Nee Saw, Nicholas Cheng, Hwee Kuan Lee, Edward D. Johnstone, Rohan M. Lewis, Alys R. Clark and Oliver E. Jensen. Supported by A*STAR, EPSRC, BBSRC and Wellcome Leap.

### References

[1] A. R. Clark, I. L. Chernyavsky, O. E. Jensen. The complexities of the human placenta. *Phys. Today* **76**(4): 26–32, 2023. doi: 10.1063/PT.3.5216.

[2] W. M. Tun, *et al.* A massively multi-scale approach to characterizing tissue architecture by synchrotron micro-CT applied to the human placenta. *J. R. Soc. Interface* **18**(179): 20210140, 2021. doi: 10.1098/rsif.2021.0140.

[3] A. Erlich, P. Pearce, R. P. Mayo, O. E. Jensen, I. L. Chernyavsky. Physical and geometric determinants of transport in fetoplacental microvascular networks. *Sci. Adv.* **5**(4): eaav6326, 2019. doi: 10.1126/sciadv.aav6326.

[4] T. Wan, E. D. Johnstone, S. N. Saw, O. E. Jensen & I. L. Chernyavsky. A functional shunt in the umbilical cord: the role of coiling in solute and heat transfer. *J. R. Soc. Interface* **22**: 20250148, 2025. doi: 10.1098/rsif.2025.0148.

## Homogenization of a Stokes-Transport System in an Anisotropic Porous Medium with Thin, Evolving Tubes

Kilian Hacker

Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany
KILIAN.HACKER@FAU.DE

### Abstract

In this contribution, we study the asymptotic behavior of a Stokes flow in an array of thin, evolving tubes, coupled with advective-diffusive transport within the tubes and in the surrounding medium. The domain evolution is induced by the transport process via chemical reactions at the microscopic lateral boundaries of the tubes, leading to a fully coupled problem. Our approach aims to model the thermal control of blood flow in human skin, including temperature-dependent production of biochemical substances and the subsequent dilation and constriction of blood vessels.

After a transformation of the microscopic problem onto a periodic microscopic reference domain, we rigorously derive a fully coupled effective model, by means of the methods of homogenization. It consists of an anisotropic Darcy model coupled to an effective reaction-diffusion-advection system of partial and ordinary differential equations. The effective coefficients (permeability, diffusivity) of the macroscopic model are computed from solutions to standard cell problems involving the time and space dependent macroscopic deformation gradient. The anisotropies induced by the special features of the geometry (array of thin tubes) are exhibited in the effective model. This is joint work with Markus Gahn and Maria Neuss-Radu.

## A Lumen–Tissue Magneto-Two-Phase Flow Model with Transendothelial Transport for Magnetic Drug Targeting

Jonas Knoch

Department Mathematik, FAU Erlangen-Nürnberg, Erlangen, Germany
JONAS.KNOCH@FAU.DE

### Abstract

Motivated by magnetic drug targeting, we develop a thermodynamically consistent magneto-two-phase flow model describing the dynamics of superparamagnetic iron oxide nanoparticles (SPIONs) in the blood vessel lumen as well as their transendothelial transport into the surrounding tissue. The strongly coupled system consists of a modified Navier–Stokes equation with an additional inertia term for the volume-averaged velocity, a transport equation for the SPION volume fraction, and a quasi-static nonlinear elliptic equation for the magnetic potential. Using Onsager's variational principle, we derive constitutive relations, including a transmission condition for the SPION phase at the lumen–tissue interface. Finally, we present a numerical framework and experiment-inspired simulations that pave the way toward optimized therapy strategies.

## On Some Multifluid Systems

Milan Pokorný

Charles University, Praha, Faculty of Mathematics and Physics
pokorny@karlin.mff.cuni.cz

### Abstract

We consider certain classes of models for multicomponent fluids of the Baer–Nunziato type and consider questions of existence of weak/dissipative measure-valued solutions, its weak/measure valued–strong uniqueness questions as well as low Mach number limits. It is a joint work with Ewelina Zatorska from University of Warwick.

## Global Solvability for a Stochastic Hyperbolic Keller - Segel System

Nikolai V. Chemetov

Department of Computing and Mathematics, University of São Paulo,
Ribeirão Preto - SP, Brazil
NVCHEMETOV@USP.BR

### Abstract

In this work we study the stochastic Stratonovich hyperbolic Keller-Segel model with an infinite dimensional multiplicative noise in a bounded domain $\mathcal{O}$ in $\mathbb{R}^2$. The evolution system on a time interval $(0,T)$ reads for a.s. in $\Omega$ as

$$\begin{cases} du = -\mathrm{div}(g(u)\ \nabla h)\,dt + \sigma(u) \circ d\mathcal{W}_t, \\ \\ -\Delta h + h = u, \end{cases} \qquad \text{for} \quad (t,\mathbf{x}) \in (0,T) \times \mathcal{O}$$

where $u = u(t,\mathbf{x})$ denotes the cell density and $g(u) = u - u^2$; $\mathcal{W}_t$ is a Wiener process given on the probability space $(\Omega, \mathcal{F}, P)$ and $\sigma = \sigma(u)$ is the diffusion coefficient.

This system is closed by the boundary condition

$$h = a \qquad \text{on} \quad (0,T) \times \partial\mathcal{O}$$

on the boundary $\partial\mathcal{O}$ of the domain $\mathcal{O}$, the influx boundary condition

$$u = b \quad \text{on} \quad (0,T) \times \partial\mathcal{O}^-$$

and the initial condition

$$u = u_0 \qquad \text{in} \quad \mathcal{O} \quad \text{at} \quad t = 0,$$

where

$$\partial\mathcal{O}^- = \{\mathbf{x} \in \partial\mathcal{O} : \quad g'(u)\,(\nabla h \cdot \mathbf{n})\,(t,\mathbf{x}) < 0\}$$

is the influx part of $\partial\mathcal{O}$ and $\mathbf{n} = \mathbf{n}(\mathbf{x})$ is the external normal at $\mathbf{x} \in \partial\mathcal{O}$.

We establish the global-in-time existence result for weak entropy martingale solutions to this stochastic Keller-Segel model with general boundary-initial data $(a, b, u_0)$.

The proof combines the stochastic a priori estimates with compactness techniques based on the kinetic theory [1] and on the Jakubowski-Skorokhod representation theorem [2, 3]. The dependence of the noise function makes more difficult for the analysis of a priori estimates, giving rise to nonlinear terms induced by the martingale part of the equation and the Stratonovich correction term.

### References

[1] Chemetov N. V., Nonlinear hyperbolic-elliptic systems in the bounded domain. *Communications on Pure and Applied Analysis*, **10**, 1079-1096, (2011).

[2] Arruda L. K., Chemetov N. V., Cipriano F., Solvability of the Stochastic Degasperis-Procesi Equation. *J. Dynamics and Differential Equations*, **35**, n. 1, 523-542 (2023).

[3] Chemetov N. V., Cipriano F., Weak solution for stochastic Degasperis-Procesi equation. *J. Differential Equations*, **382**, n. 15, 1-49 (2024).

## Structure-Preserving LDG Methods for Linear and Nonlinear Transport Equations with Gradient Noise

Thomas Christiansen

Department of Mathematics, University of Oslo, Norway
THOMCHR@MATH.UIO.NO

### Abstract

We develop a framework for constructing local discontinuous Galerkin (LDG) discretizations that preserve the underlying hyperbolic structure of stochastic partial differential equations of the form

$$du + \mathrm{div}_x f(x,u)\,dt + \sum_{\ell \in L} \mathrm{div}_x\big(\sigma_\ell(x) g_\ell(u)\big) \circ dW_t^\ell = 0, \qquad \text{in } (0,T] \times \mathbb{R}^d.$$

Here $f = (f_1, ..., f_d) : \mathbb{R}^d \times \mathbb{R} \to \mathbb{R}^d$ is a given deterministic heterogenous flux vector and the stochastic perturbations enter through Stratonovich gradient noise (or as stochastic heterogenous fluxes), where each $\sigma_\ell : \mathbb{R}^d \to \mathbb{R}^d$ is a prescribed spatial vector field and $g_\ell : \mathbb{R} \to \mathbb{R}$ is a scalar flux function. Moreover, $L$ is a finite index set and the driving processes $\{W^\ell\}_{\ell \in L}$ are mutually independent real-valued Wiener processes.

Starting from the Itô formulation we design semi-discretizations that reproduce the exact cancellation between the quadratic variation and the Itô–Stratonovich correction term (modulo fluxes) which is the key hyperbolic stability mechanism of gradient noise. By selecting suitable numerical fluxes, we obtain discrete energy conservation or energy dissipation, valid either pathwise or in expectation. Furthermore, by combining this energy stability with a Khasminskii-type argument we can prove well-posedness of the resulting high-order schemes without relying on commonly adopted global linear growth assumptions. We end this talk by presenting a few numerical results that showcase the stability of the LDG schemes and how various numerical fluxes influence the approximation accuracy.

The talk is based on joint work with Kenneth H. Karlsen (UiO).

### Moving Boundary Problems in Fluid-Poroelastic Structure Interaction

Jeffrey Kuan

University of Maryland, College Park
JEFFREYK@UMD.EDU

#### Abstract

This is joint work with Sunčica Čanić (University of California, Berkeley) and Boris Muha (University of Zagreb).

In this talk, we discuss recent progress in the well-posedness of moving boundary models of fluid-poroelastic structure interaction. Poroelastic materials are porous substances whose porous properties are coupled to their elastodynamics, and these include materials such as biological tissues and soil. There are many important applications of poroelasticity to fluid dynamics problems in biomedical engineering and geosciences, and hence, it is of particular interest to study the coupled dynamics of how poroelastic materials interact with surrounding fluid flows.

We introduce a fully coupled moving boundary model for an immersed poroelastic material (modeled by the Biot equations of poroelasticity) interacting with a viscous fluid (modeled by the incompressible Navier-Stokes equations), where deformations of the poroelastic material are rigorously accounted for, through a moving boundary model. We introduce a new regularized interface method, which uses a minimal regularization of the fluid-poroelastic interface to define a notion of solution, in order to establish the existence of a suitable notion of (finite energy) regularized interface weak solutions to this coupled and highly nonlinear moving boundary problem of mixed type. The proof of this existence result uses geometric bounds on Lagrangian maps to control moving domain behavior, in addition to a constructive splitting scheme and compactness arguments, for showing convergence of suitable approximate solutions. We conclude by discussing recent progress in showing how regularized interface weak solutions relate to classical solutions when such classical solutions exist (weak-classical consistency), in the sense of convergence to a classical solution as the regularization parameter tends to zero.

#### References

[1] J. Kuan, S. Čanić, and B. Muha. A regularized interface method for fluid-poroelastic structure interaction problems with nonlinear geometric coupling. *arXIv:2508.18065*, 2025.

[2] J. Kuan, S. Čanić, and B. Muha. Fluid-poroviscoelastic structure interaction problem with nonlinear geometric coupling. *J. Math. Pures Appl.*, 188: 345-445, 2024.

## A Smoluchowski–Kramers Approximation for the Stochastic Variational Wave Equation

Julien Vovelle

UMPA, CNRS, ENS de Lyon, Lyon, France
julien.vovelle@ens-lyon.fr

### Abstract

We investigate the Smoluchowski–Kramers approximation for the one-dimensional periodic variational wave equation with state-dependent damping and additive noise. We show that weak dissipative solutions converge to solutions of a stochastic quasilinear parabolic equation. Joint work with Billel Guelmame.

### References

[1] Billel Guelmame and Julien Vovelle. Global dissipative martingale solutions to the variational wave equation with stochastic forcing. *Stoch. Partial Differ. Equ. Anal. Comput.*, 13(1):465–530, 2025.

[2] Billel Guelmame and Julien Vovelle. A Smoluchowski-Kramers approximation for the stochastic variational wave equation. November 2025. preprint hal-0536952.

### Controllability Problem of a Differential Equation with Memory

Sumit Arora and Rodrigo Ponce

Universidad de Talca Instituto de Matemáticas, Casilla 747, Talca, Chile
arorasumit10623@gmail.com

### Abstract

In this talk we addresses control problems governed by a semilinear evolution equation with memory kernel $\kappa \in L^1_{loc}(\mathbb{R}^+)$ We examine the existence of a mild solution and the approximate controllability of both linear and semilinear control systems. To this end, we introduce the concept of a resolvent family associated with the linear evolution equation with memory and develop some of its essential properties. Subsequently, we consider a linear-quadratic regulator problem to determine the optimal control that yields approximate controllability for the linear control system. Furthermore, we derive sufficient conditions for the existence of a mild solution and the approximate controllability of a semilinear system in a reflexive Banach space having uniformly convex dual. Additionally, we present an approximate controllability result within the framework of a general Banach space. Finally, we apply our theoretical findings to investigate the approximate controllability of the heat equation with singular memory.

Recently, Pandolfi [1] studied the approximate controllability of a general class of control systems with singular memory by applying boundary control. The considered system particularly includes systems with fractional derivatives and integrals as well as the standard heat equation.

### References

[1] Pandolfi, L.: Controllability properties for equations with memory of fractional type, *Appl. Math. Optim.*, **84**(1) (2021) 325–353.

## On Parameter-Dependent Inhomogeneous Boundary-Value Problems in Sobolev Spaces

Olena Atlasiuk, Vladimir Mikhailets, Jari Taskinen

University of Helsinki, Helsinki, Finland; King's College London, London, UK;
Institute of Mathematics, NAS of Ukraine, Kyiv, Ukraine
olena.atlasiuk@helsinki.fi, mikhailets@imath.kiev.ua,
jari.taskinen@helsinki.fi

### Abstract

We study a wide class of linear inhomogeneous boundary-value problems for $r$th order ODE-systems depending on a parameter $\mu$ belonging to a general metric space. The solutions belong to the Sobolev spaces $(W_p^{n+r})^m$, $n \in \mathbb{N} \cup \{0\}$, $m, r \in \mathbb{N}$, $1 \leq p \leq \infty$. The boundary conditions are of a most general form $By = c$, where $B$ is an arbitrary continuous operator from $(W_p^{n+r})^m$ to $\mathbb{C}^{rm}$. Thus, they may contain derivatives of the unknown vector function of integer and/or fractional orders $\geq r$. We find necessary and sufficient conditions for the continuity of solutions with respect to the parameter $\mu$. We also prove that the solutions of the original problems can be approximated in the space $(W_p^{n+r})^m$ by solutions of ODE-systems with polynomial coefficients, right-hand sides of the equation, and multipoint boundary conditions, which are independent of the right-hand sides of the original problem.

### References

[1] O. Atlasiuk, V. Mikhailets, J. Taskinen, Parameter-dependent inhomogeneous boundary-value problems in Sobolev spaces, Preprint (2025) https://doi.org/10.48550/arXiv.2512.21361

## Morse Decomposition for Semi-Dynamical Systems with an Application to Systems of State-Dependent Delay Differential Equations

István Balázs

University of Szeged, Hungary
BALAZSI@MATH.U-SZEGED.HU

### Abstract

Understanding the structure of the global attractor is crucial in the field of dynamical systems, where Morse decompositions provide a powerful tool by partitioning the attractor into finitely many invariant Morse sets and gradient-like connecting orbits. Building on Mallet-Paret's pioneering use of discrete Lyapunov functions for constructing Morse decompositions in delay differential equations, similar approaches have been extended to various delay systems, also including state-dependent delays. In this talk, we develop a unified framework assuming the existence and some properties of a discrete Lyapunov function for a semi-dynamical system on an arbitrary metric space, and construct a Morse decomposition of the global attractor in this general setting. We demonstrate that our findings generalize previous results; moreover, we apply our theorem to a cyclic system of differential equations with threshold-type state-dependent delay.

### References

[1] Balázs, I., Garab, Á., Rauscher, T.: Morse Decomposition for Semi-Dynamical Systems with an Application to Systems of State-Dependent Delay Differential Equations. Journal of Dynamics and Differential Equations (2025)

### Lagrangian Flows Associated to Nonstandard Singular Rough Vector Fields

Henrique Borrin de Souza, João Fernando da Cunha Nariyoshi

Postdoc at FFCLRP-USP, Ribeirão Preto, Brazil
HENRIQUEBORRIN@USP.BR

**Abstract**

In this presentation, we provide quantitative estimates associated to vector fields which can be written as convolution of a nonstandard singular kernel with a Besov function. The aforementioned kernels do not satisfy Calderón-Zygmund estimates, nor have cancellations in order to ensure good properties on its Fourier estimates, so we present a new technique in order to deal with this case. Moreover, we consider rough kernels, in the sense that its angular part is not Lipschitz, as previously considered in [1], but only in Sobolev spaces.

**References**

[1] Henrique Borrin. Quantitative estimates for flows of Lipschitz and Dini type singular vector fields. Discrete and Continuous Dynamical Systems, accepted.

## Functional Equations Involving Reflection and Piecewise Constant Arguments

Paula Cambeses-Franco[1,2], Alberto Cabada[1,2]

[1] CITMAga, 15782, Santiago de Compostela, Galicia, Spain
[2] Department of Statistics, Mathematical Analysis and Optimization
Faculty of Mathematics, University of Santiago de Compostela, Spain.
paula.cambeses.franco@usc.es

### Abstract

Differential equations remain a fundamental tool for describing phenomena across mathematics, physics, engineering, and the life sciences. In recent decades, increasing attention has been given to models incorporating nonstandard functional dependence, leading to the development of differential equations with involution and equations with piecewise constant arguments. Equations with involution, characterized by a self-inverse transformation of the independent variable, naturally arise in problems involving symmetry and nonlocal effects [2]. Meanwhile, equations with piecewise constant arguments provide a suitable framework for systems where state variables are evaluated at discrete moments, as occurs in control processes, epidemiology, and economic dynamics [3]. These classes of equations offer a flexible setting for modeling hybrid and nonstandard dynamical behaviors.

In this work, we establish the theoretical framework for the study of a general two point $n$-th order differential equation incorporating both a reflection term $m\,v(-t)$ and a piecewise continuous dependence $M\,v([t])$ [1]. The analysis is carried out using Green's function theory, which allow us to obtain its explicit expression and study its qualitative properties. In particular, we obtain comparison results that allow us to determine regions where the related Green's function maintains a constant sign. We then focus on a periodic first-order equation with reflection and piecewise constant arguments. Once the corresponding Green's function and its sign structure are characterized, we address the existence of solutions for the related nonlinear problem by applying fixed point techniques, with special emphasis on the monotone method.

### References

[1] Cabada, A., Cambeses-Franco, P.: Reflection equation with piecewise constant arguments. *Commun. Nonlinear Sci. Numer. Simul.* 109488 (2025).

[2] Cabada, A., Tojo, F.A.F.: *Differential equations with involutions.* Atlantis Briefs in Differential Equations, vol. 2. Atlantis Press, Paris (2015).

[3] Dai, L.: *Nonlinear dynamics of piecewise constant systems and implementation of piecewise constant arguments.* World Scientific, Singapore (2008).

## The Least-Squares Method in the Theory of Boundary Value Problems with Delay

Peter Benner, Sergey Chuiko, Viktor Chuiko

Max Planck Institute, Magdeburg, Germany
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
benner@mpi-magdeburg.mpg.de, chujko-slav@ukr.net

**Abstract**

The present work addresses the issue of solution existence

$$z(t,\varepsilon):\ z(\cdot,\varepsilon)\in\mathbb{C}^1[0,T],\ z(t,\cdot)\in\mathbb{C}[0,\varepsilon_0]$$

for nonlinear periodic boundary value problem with delay [1]

$$dz(t,\varepsilon)/dt = A(t)z(t,\varepsilon) + B(t)z(t-\Delta,\varepsilon) + f(t) + \tag{19}$$

$$+\varepsilon\, Z(z(t,\varepsilon), z(t-\Delta,\varepsilon), t, \varepsilon)$$

in a small neighborhood of the solution $z_0(t)\in\mathbb{C}^1[0,T]$ of the generating problem

$$dz_0(t)/dt = A(t)z_0(t) + B(t)z_0(t-\Delta) + f(t), \quad \Delta\in\mathbb{R}^1. \tag{20}$$

Here, $A(t), B(t)$ are $(n\times n)$ dimensional matrices, $Z(z(t,\varepsilon), z(t-\Delta,\varepsilon), t, \varepsilon)$ is a nonlinear vector function, $T$-periodic to the independent variable $t$, analytic with respect to the unknown $z(t,\varepsilon)$ and $z(t-\Delta,\varepsilon)$ in a small neighborhood of the solution of the generating problem (20) and continuous with respect to the small parameter $\varepsilon$ on the interval $[0,\varepsilon_0]$. In addition, the function $f(t)$ is continuous with respect to the $t\in[0,T]$.

By applying the Adomian decomposition method and the least squares method scheme, we have derived the necessary and sufficient conditions for the existence of solutions of the weakly nonlinear periodic boundary value problem for a system of differential equations with concentrated delay in the critical case [3]. The efficiency of the iterative schemes we have developed is demonstrated using an example of solving the problem of approximating periodic solutions to an equation with concentrated delay, which models a non-isothermal chemical reaction [1, 2].

### References

[1] Benner P., Chuiko S., Chuiko V. *The lest square method in the theory of nonlinear periodic boundary-value problems with delay.* Functional differential equations, 32, 7-36 (2025).

[2] Benner P., Chuiko S., Zuyev A. *A periodic boundary value problem with switchings under nonlinear perturbations.* Boundary Value Problems, 50, 1-12 (2023).

[3] Chuiko S.M., Chuiko A.S. *On the approximate solution of periodic boundary value problems with delay by the least-squares method in the critical case.* Nonlinear Oscillations, 14, 445-460 (2012).

## Robustness of Sacker-Sell Trichotomy for Nonautonomous Dynamics

Davor Dragičević

Faculty of Mathematics, University of Rijeka, Rijeka, Croatia
DDRAGICEVIC@MATH.UNIRI.HR

### Abstract

We discuss the robustness property of the notion of a uniform exponential dichotomy (introduced by Sacker and Sell [2]) for nonautonomous dynamics. More precisely, we formulate conditions under which this notion persists under sufficiently small linear perturbations. It turns out that we need to restrict to small perturbations in $\ell^1$-norm. The talk will be based on the joint work with A. L. Sasu and B. Sasu [1].

### References

[1] D. Dragičević, A. L. Sasu and B. Sasu, *Robustness of uniform exponential trichotomy: An input-output approach*, J. Differential Equations **439** (2025), 113409, 36pp.

[2] R. J. Sacker and G. R. Sell, *Existence of dichotomies and invariant splittings for linear differential systems III*, J. Differential Equations **22** (1976), 497–522.

## Multiplicity of Solutions for Nonlinear Differential Systems Using Fixed Point Theory in Product Spaces

Laura Mª Fernández–Pardo

Universidade de Santiago de Compostela, Santiago de Compostela, Spain.
laura.fernandez.pardo@usc.es

### Abstract

In this talk we present sufficient conditions for the existence and multiplicity of solutions to nonlinear differential systems using fixed point theory in cones. To establish these conditions, the problem is reformulated as an equivalent fixed point problem for compact operators of the form $T = (T_1, T_2)$ acting on a product space $X \times Y$.

In recent works [1, 3], component-wise Krasnosel'skii-type fixed point conditions have been imposed on operators of this form. In this direction, we show that these conditions can be combined with classical multiplicity results, such as the Leggett–Williams theorem [2], yielding multiplicity results in product spaces. Many of the fixed points obtained by these methods have the important property that both components are positive. Therefore, when applied to nonlinear differential systems, these results yield multiple solutions with all components nontrivial.

### References

[1] L. M. Fernández-Pardo and J. Rodríguez-López, Component-wise Krasnosel'skii type fixed point theorem in product spaces and applications, *Nonlinear Anal. Real World Appl.*, **88** (2026), No. 104506, 1-12.

[2] R. W. Leggett and L. R. Williams, Multiple positive fixed points of nonlinear operators on ordered Banach spaces, *Indiana Univ. Math. J.*, **28** 4 (1979), 673-688.

[3] J. Rodríguez-López, A fixed point index approach to Krasnosel'skiĭ-Precup fixed point theorem in cones and applications, *Nonlinear Anal.*, **226** (2023), No. 113138, 1–19.

## Chaos in the Hyperion Tumbling Model

Anna Gierzkiewicz

Jagiellonian University in Krakow, Poland
ANNA.GIERZKIEWICZ@UJ.EDU.PL

### Abstract

In a joint work with Amadeu Delshams (UPC Barcelona) and Piotr Zgliczyński (JU) we study the three-axis inner rotation (*tumbling*) of an ellipsoidal satellite in a Keplerian orbit. In our previous paper [1] we considered a simplified uniaxial case, with the axis of rotation perpendicular to the orbital plane [2]. The system of three ordinary differential equations is periodic with respect to one of the variables (*true anomaly* $f$), which allowed us to study the Poincaré section $f = 0 = 2\pi$. The system exhibited chaotic behavior for some parameters (the shape of the ellipsoid and the orbital eccentricity $e$). We were particularly interested in the values corresponding to Hyperion, one of Saturn's moons, known for its non-spherical shapeand apparently chaotic inner rotation.

The uniaxial model does not provide a full physical description of Hyperion's tumbling, which is clearly triaxial. The full model, based on Euler's rigid-body equations, leads to a much more complicated system of seven ordinary differential equations (omitted here for reasons of size; see [3]), of which the simplified uniaxial model is an invariant subspace. For this reason, it too is chaotic, but we expect the structure of this chaos to be more complex in the 7-dimensional space: perhaps it is divided into chaotic regions connected by homo- and heteroclinic orbits.

Using the Interval Newton Method implemented in `C++` with the use of `CAPD` library [4] for computer-assisted proofs, we found a number of periodic orbits for the analogous Poincaré map in the full space. The current work is to study the behavior of their stable and unstable manifolds which seem to intersect transversely. This, in turn, we expect to prove rigorously with `CAPD`. The poster presents the up-to date numerical results and plans for future work.

### References

[1] Anna Gierzkiewicz and Piotr Zgliczyński. *A computer-assisted proof of symbolic dynamics in Hyperion's rotation.* Celestial Mechanics and Dynamical Astronomy, 131(7):33, 2019.

[2] J. Wisdom, S. J. Peale, and F. Mignard. *The chaotic rotation of Hyperion.* Icarus, 58(2):137 – 152, 1984.

[3] James Jay Klavetter. *Rotation of Hyperion. II – Dynamics.* Astronomical Journal, 98:1855–1874, 11 1989.

[4] T. Kapela, M. Mrozek, D. Wilczak and P. Zgliczyński. *CAPD::DynSys: a flexible C++ toolbox for rigorous numerical analysis of dynamical systems.* Communications in Nonlinear Science and Numerical Simulation, 101:105578, 2021.

## Periodic Solutions of Hamiltonian Systems with Symmetries

Anna Gołębiewska

Nicolaus Copernicus University in Toruń, Poland
ANIAR@MAT.UMK.PL

### Abstract

The aim of this talk is to show the results concerning global bifurcation of periodic solutions for autonomous Hamiltonian and Newtonian systems. We consider also the situation when the potential has additional symmetries of some compact Lie group. Such results allow to generalize the classical Lyapunov center theorem to the case of symmetric systems.

The main tool that we use is the degree theory for equivariant gradient maps. For the problem with symmetries of some compact Lie group $G$, this invariant takes values in the Euler ring of $G$. Due to the sophisticated structure of this ring, using our tool we are able to prove global bifurcation also in the case, when the classical methods cannot be applied.

### References

[1] A. Gołębiewska, S. Rybicki: Generalization of Lyapunov center theorem for Hamiltonian systems via normal forms theory, Journal of Differential Equations, 421 (2025), 241-–263.

[2] A. Gołębiewska, S. Rybicki, P. Stefaniak: Periodic solutions of autonomous $S^1$-symmetric Newtonian systems, arXiv:2510.16073

## Periodic Solutions of Cosmology Inspired Hamiltonian System

Agnieszka Gołembiewska

Nicolaus Copernicus University in Toruń, Poland
agagolembiewska@mat.umk.pl

### Abstract

This is a joint work with prof. Sławomir Rybicki.
In this talk, we investigate the existence and structure of sets of nonstationary periodic solutions of a Hamiltonian system motivated by cosmology. The system is given by

$$\dot{u}(t) = JH'(u(t)), \tag{21}$$

where $u = (x, y, z, X, Y, Z)$ and the Hamiltonian $H$ is defined as

$$H(x,y,z,X,Y,Z) = \tag{22}$$
$$= \frac{1}{2}\left(x^2 + X^2 - y^2 - Y^2 + z^2 + Z^2\right) +$$
$$+\epsilon^2\left(\frac{\alpha}{4}\left(x^4 + y^4 + z^4\right) + \frac{\beta}{2}\left(x^2y^2 + x^2z^2 + y^2z^2\right)\right).$$

We use two approaches to study sets of nonstationary periodic solutions of system (21) with Hamiltonian (22). The first is based on bifurcation theory, while the second relies on equivariant topology and the representation theory of compact Lie groups. First, we introduce the parametrized system $\dot{u}(t) = \lambda JH'(u(t))$ and search for $2\pi$-periodic solutions, which correspond to $2\pi\lambda$-periodic solutions of the original system (21). We prove the existence of global bifurcation points of nonstationary $2\pi$-periodic solutions of parametrized Hamiltonian system and describe how their number depends on the parameters $\alpha$ and $\beta$. Furthermore, we provide lower bounds on the number of unbounded continua of nonstationary $2\pi$-periodic solutions bifurcating from stationary ones. In addition, we study nonstationary periodic solutions of system (21) that bifurcate from nonisolated stationary solutions. Exploiting the $S^1$-invariance of the Hamiltonian (22), we establish the existence of connected families of nonstationary periodic solutions bifurcating from the manifold of stationary solutions.

## Dynamics of Delayed Rational Difference Equations in Higher Dimensions

Yacine Halim

Department of Mathematics, Abdelhafid Boussouf University, Mila, Algeria
y.halim@centre-univ-mila.dz

### Abstract

In this paper, we study the dynamical properties of a class of $p$-dimensional rational difference equation systems that generalize earlier two and three dimensional models. The considered system includes a power-type nonlinearity characterized by a parameter $q > 0$, which significantly affects the qualitative behavior of the system.

Our main focus is on proving the boundedness of solutions and analyzing the local asymptotic stability of the equilibrium points. We establish sufficient conditions ensuring stability and investigate the influence of the system parameters on the local behavior of solutions. Through extending several existing results to higher-dimensional cases, this work develops a more general theoretical framework and provides further understanding of the local dynamics, structural characteristics, and qualitative behavior of this family of nonlinear discrete systems.

### References

[1] S. Alaydi, *An introduction to difference equations*, Springer New York (2005).

[2] T. D. Alharbi, Y. Halim, *Multi-dimensional rational recurrence models: Local analysis with non-linear effects*, AIMS Mathematics **10(9)** (2025), 20050-20065.

[3] H. Bao, *Dynamical behavior of a system of second-order nonlinear difference equations*, International Journal of Differential Equations **ID 679017** (2015).

[4] G. Papaschinopoulos and C. J. Schinas, *On the system of two nonlinear difference equations* $x_{n+1} = A + \frac{x_{n-1}}{y_n}$, $y_{n+1} = A + \frac{y_{n-1}}{x_n}$, International Journal of Mathematics and Mathematical Sciences **23(12)** (2000), 839–848.

[5] Q. Zhang, S. Zhang, Z. Zhang, and F. Lin, *On three-dimensional system of rational difference equations with second-order*, Electronic Research Archive **33(4)** (2025), 2352–2365.

## On Impulsive Differential Equations with Adaptive State-Dependent Delays

Ferenc Hartung

University of Pannonia, Veszprém, Hungary
HARTUNG.FERENC@MIK.UNI-PANNON.HU

### Abstract

In this talk we consider a class of impulsive nonlinear differential equations with adaptive state-dependent delays. We discuss the existence and uniqueness of solutions of the initial value problem using a Picard–Lindelöf type argument where we define approximate solutions with the help of equations with piecewise-constant arguments (EPCAs). Moreover, we show that the solutions of the associated EPCAs approximate the solutions of the original impulsive DDE with adaptive state-dependent delay uniformly on compact time intervals. The key assumption underlying both results is that the delayed time function is monotone, or piecewise strictly monotone. We also discuss the problem of differentiability of the solutions with respect to parameters.

### References

[1] F. Hartung. On differentiability of solutions with respect to parameters in impulsive differential equations with adaptive state-dependent delay. J. Differential Equations, 413 (2024) 751–804.

[2] F. Hartung. On existence, uniqueness and numerical approximation of impulsive differential equations with adaptive state-dependent delays using equations with piecewise-constant arguments. arXiv:2601.01670 [math.DS], 2026.

## Threshold Points and Oscillation Criteria for Differential and Difference Equations

Petr Hasil, Michal Veselý

Department of Mathematics and Statistics, Faculty of Science,
Masaryk University, Brno, Czech Republic
hasil@math.muni.cz, michal.vesely@mail.muni.cz

### Abstract

Some of the presented results were obtained in collaboration with Kōdai Fujimoto and Jiřina Šišoláková (see the references below). The results describe the oscillatory properties of second-order equations. The aim is to use the adapted Prüfer angle (differential equations) and variants of the Riccati method (difference equations) to obtain oscillatory criteria based on measuring the size and comparing the coefficients of these equations. The ideal result is an unimproveable situation where a threshold value is identified, traditionally called the critical oscillation constant (although it is not a constant in the usual sense, i.e., it is not a number, but a combination of coefficients that change for the studied equations). The results are obtained for half-linear equations (in fact, we proved them for linear equations as well).

### References

[1] P. Hasil, M. Veselý: *Oscillation and non-oscillation of half-linear differential equations with coefficients determined by functions having mean values.* Open Mathematics **16** (2018), no. 1, pp. 507–521.

[2] K. Fujimoto, P. Hasil, M. Veselý: *Riccati transformation and nonoscillation criterion for linear difference equations.* Proceedings of the American Mathematical Society **148** (2020), no. 10, pp. 4319–4332.

[3] P. Hasil, M. Veselý: *Riccati technique and oscillation of linear second-order difference equations.* Archiv Der Mathematik **117** (2021), no. 6, pp. 657–669.

[4] K. Fujimoto, P. Hasil, M. Veselý: *Riccati transformation and non-oscillation criterion for half-linear difference equations.* Bulletin of the Malaysian Mathematical Sciences Society **47** (2024), no. 5, art. 146.

[5] P. Hasil, J. Šišoláková, M. Veselý: *Oscillation of half-linear second-order difference equations via the adapted Riccati transformation.* Bulletin des Sciences Mathematiques **204** (2025), art. 103711.

## Oscillation Theorems for Linear Differential Equations with a Proportional-Derivative Controller

Kazuki Ishibashi

Institute of Science and Engineering, Academic Assembly, Shimane University,
Matsue 690-8504, Japan
K.ISHIBASHI.MATH@YMAIL.NE.JP

### Abstract

This study is based on joint work with Professor M. Onitsuka of Okayama University of Science (including [5]). Recently, considerable attention has been devoted to oscillation theory for linear differential equations with proportional-derivative (PD) control (see, e.g., [1, 2, 3, 4]); nevertheless, this theory remains in its early stages of development. In this work, we investigate the oscillatory and nonoscillatory behavior of solutions to these equations, with the aim of extending classical oscillation theory to this framework.

In particular, we establish a Leighton–Wintner-type oscillation theorem and Hille–Kneser-type oscillation and nonoscillation criteria in the presence of a PD controller. The Leighton–Wintner-type result is obtained by means of the Riccati technique, suitably adapted to the PD structure of the equation. Furthermore, by clarifying the oscillation constant for Euler-type equations with PD control, we derive Hille–Kneser-type criteria that characterize both oscillatory and nonoscillatory behavior. This work was supported in part by JSPS KAKENHI Grant No. JP22K13933 and by the Sumitomo Foundation Grant for Basic Science Research Projects, Grant No. 2402156.

### References

[1] D. R. Anderson, *Second-order self-adjoint differential equations using a proportional-derivative controller*, Appl. Nonlinear Anal., 24 (2017), 17–48.

[2] F. A. Çetinkaya and T. Cuchta, *Sturm–Liouville and Riccati conformable dynamic equations*, Adv. Dyn. Syst. Appl., 15 (2020), 1–13.

[3] K. Ishibashi, *Nonoscillation of damped linear differential equations with a proportional derivative controller and its application to Whittaker–Hill-type and Mathieu-type equations*, Opuscula Math., 43 (2023), 67–79.

[4] K. Ishibashi, *Wintner-type nonoscillation theorems for conformable linear Sturm–Liouville differential equations*, Opuscula Math., 44 (2024), 727–748.

[5] K. Ishibashi and M. Onitsuka, *Oscillation criteria of the Leighton–Wintner-type and the Hille–Kneser-type for linear differential equations with a proportional derivative controller*, Math. Methods Appl. Sci., 48 (2025), 16690–16699.

## Sharp Oscillation Criteria for $n$th Order Linear Delay Differential Equations

Irena Jadlovská

Mathematical institute, Slovak Academy of Sciences, Košice, Slovakia
JADLOVSKA@SASKE.SK

**Abstract**

We establish a sharp oscillation criterion for $n$-th order linear delay differential equations with positive continuous coefficients and general delay argument. The decisive oscillation constant is the maximum of an explicitly constructed exponential–polynomial, determined by the order and the asymptotic time–delay ratio. Its optimality is shown by exact agreement with an Euler-type proportional-delay differential equation, extending recent sharp results for lower orders: $n = 2$ (in a more general half-linear setting) by Jadlovská and Džurina [1], $n = 3$ by Graef, Jadlovská, and Tunç [2], and $n = 4$ by Jadlovská et al. [3]. If this constant exceeds the critical value, Property A holds: every solution oscillates for $n$ even, while for odd $n$ every solution either oscillates or tends to zero together with all derivatives up to $n - 1$. Beyond identifying this constant, we develop an iterative process, built from suitably chosen $\liminf$ constants, that yields two-sided comparison inequalities between arbitrary derivative orders of any nonoscillatory solution, in a precise binomial-coefficient form. We illustrate the scope of our results on broad classes of equations with asymptotically proportional, sublinear, or piecewise-proportional delays, as well as slowly varying or oscillatory coefficients. *This is a joint work with Agacik Zafer and Tongxing Li [4].*

### References

[1] I. Jadlovská, J. Džurina, *Kneser-type oscillation criteria for second-order half-linear delay differential equations*, Appl. Math. Comput. **380** (2020), 1–15.

[2] J. R. Graef, I. Jadlovská, and E. Tunç, *Sharp asymptotic results for third-order linear delay differential equations*, J. Appl. Anal. Comput **11.5** (2021): 2459–2472.

[3] I. Jadlovská, J. Džurina, J. R. Graef, S. R Grace, *Sharp oscillation theorem for fourth-order linear delay differential equations*, Journal of Inequalities and Applications **2022.1** (2022): 122.

[4] I. Jadlovská, T. Li, A. Zafer, *Sharp oscillation of n-th order linear delay differential equations,* submitted.

## Robust Stability of a Class of Complex Balanced Chemical Reaction Networks with Time Delays and Time Varying Perturbations

Hirokazu Komatsu

Faculty of Engineering, Kindai University, Hiroshima, Japan
KOMATSU@HIRO.KINDAI.AC.JP

### Abstract

Chemical reaction network theory (CRNT) provides methods for analyzing the dynamics of solutions to ordinary differential equations (ODEs) describing the time evolution of species concentrations in chemical reaction networks (CRNs) based on their topological structures. In particular, the Deficiency Zero Theorem provides a sufficient condition for the ODEs to possess a unique asymptotically stable equilibrium point, known as a complex balanced equilibrium [1]. Recently, CRN systems have been extended to incorporate production delays and environmental fluctuations in reactions. Consequently, the dynamics of such systems are described by non-autonomous delay differential equations (DDEs). While the asymptotic stability of an equilibrium for autonomous DDEs of complex balanced CRNs [2] and the persistence of non-autonomous DDEs for delayed CRNs with time-varying kinetics [3] have been proven, robust stability for systems with both delays and time-varying perturbations has not yet been established. In this talk, we consider a class of complex balanced and conservative CRNs satisfying the conditions of the DZT and provide a sufficient condition for the robust stability of non-autonomous DDEs subjected to both arbitrary production delays and time-varying perturbations in reaction rates. In order to prove this, we define a Lyapunov-Krasovskii functional for the non-autonomous DDEs, extending the one used for autonomous systems [2]. Under the assumption that the perturbations are bounded within a specific range, we evaluate the time derivative along the solutions and prove that the solutions to the DDEs are globally ultimately bounded. Moreover, we also show that the solutions to the DDEs converge to an equilibrium point when the perturbations vanish as time goes to infinity.

### References

[1] M. Feinberg, Foundation of chemical reaction network theory, Springer, New York, 2019.

[2] H. Komatsu and H. Nakajima, The Deficiency Zero Theorem and global asymptotic stability for a class of chemical reaction networks with arbitrary time delays, Systems& Control Letters, Vol. 136, No. 104601 (2020).

[3] H. Komatsu and H. Nakajima, Persistence in delayed chemical reaction network systems with time-varying kinetics, Systems & Control Letters, Vol. 204, No. 106206 (2025).

[4] M. Chaves, Input-to-state stability of a class of chemical reaction networks, SIAM Journal on Control and Optimization, Vol. 43, No. 5, pp. 1700-1721 (2005).

## Bifurcations of Limit Cycles in DDEs: Theory & Software

Lentjes Bram

Department of Mathematics, Hasselt University, Diepenbeek Campus, Belgium
BRAM.LENTJES@UHASSELT.BE

### Abstract

Until now, no robust and efficient (numerical) method has been available to study bifurcations of limit cycles in delay differential equations (DDEs). In this talk, I will present explicit computational formulas for the critical normal form coefficients of all codimension 1 bifurcations of limit cycles (fold, period-doubling and Neimark-Sacker) in DDEs. These formulas are essential for detecting codimension 2 points and distinguishing between sub- and supercritical bifurcations.

The approach consists of four steps. First, we prove the existence of a smooth periodic finite-dimensional center manifold near a nonhyperbolic cycle [1]. Second, we show that the dynamics near such a cycle on the periodic center manifold can be described in terms of so-called periodic normal forms [2]. Third, we derive explicit formulas for the critical normal form coefficients in terms of the periodic (adjoint) (generalized) eigenfunctions of the characteristic operator, a generalization of the well-known characteristic matrix [3]. Finally, we illustrate the effectiveness of the method with numerical examples [4].

### References

[1] B. Lentjes, L. Spek, M. M. Bosschaert, and Yu. A. Kuznetsov, *Periodic Center Manifolds for DDEs in the Light of Suns and Stars*, Journal of Dynamics and Differential Equations, vol. 37, no. 1, pp. 815–858, 2023, doi:10.1007/s10884-023-10289-9.

[2] B. Lentjes, L. Spek, M. M. Bosschaert, and Yu. A. Kuznetsov, *Periodic normal forms for bifurcations of limit cycles in DDEs*, Journal of Differential Equations, vol. 423, pp. 631–694, 2025, doi:10.1016/j.jde.2025.01.064.

[3] M. M. Bosschaert, B. Lentjes, L. Spek, and Yu. A. Kuznetsov, *Numerical Periodic Normalization at Codim 1 Bifurcations of Limit Cycles in DDEs*, arXiv preprint, 2025, doi:10.48550/arXiv.2505.19786.

[4] M. M. Bosschaert, B. Lentjes, L. Spek, and Yu. A. Kuznetsov, *PeriodicNormalizationDDEs*, GitHub repository, 2024. Available at: https://github.com/mmbosschaert/PeriodicNormalizationDDEs

## On Observability of Limit Cycles in the Chemostat

Torsten Lindström

Linnæus University Växjö, Sweden
Torsten.Lindstrom@lnu.se

### Abstract

Ever since May (Science, 1974) pointed out that the dynamics of equations describing commonly appearing equations in ecology contains the seeds of chaos and nonlinear dynamics, it has been a debate whether nonlinear dynamics and chaos actually can be observed in ecological systems. The chemostat is a very basic experimental setting describing of an ecosystem with an explicitly modelled resource flow. Presence of autonomous limit cycles is the simplest nonlinear dynamical phenomenon that can be studied and the Rosenzweig and MacArthur (Am. Nat. 1963) criterion divides the parameter space of the deterministic two-species chemostat into a one regime describing global stability and one with autonomous limit cycles. Deterministic systems of differential equations can, however, only be viewed only as approximations of the Markov processes describing the ecological phenomena (Renshaw, Stochastic Population Processes, 2011). It is likely that deterministic stable equilibria describing sufficiently large populations are described by approximately normal distributions and this can indeed, be proved in the univariate case. The objective of this talk is to study in what sense limit cycles approximate the quasi-stationary distributions arising in the Markov process describing the two-species chemostat.

## Oscillation Criterion for Generalized Euler Difference Equations

Ludmila Linhartová

Masaryk University Brno, The Czech Republic
474383@mail.muni.cz

### Abstract

I would like to present new results in the field of generalized Euler difference equations and speak about the paper Oscillation criterion for generalized Euler difference equations. The major part of this paper is formed by the proof of the main theorem. During my studies I analyze the Euler type half-linear difference equation with bounded and asymptotically periodic coefficients. The main goal of my work was to prove an oscillation criterion that complements a large number of previously known oscillation criteria. It is already proved that the Euler-type equation with positive coefficients is conditionally oscillatory. There are also already identified the corresponding oscillation constants which are given by the coefficients of the equation. In our work we extended the results for the coefficients from more general classes and identify the corresponding oscillation constants.

By using the Sturmian theorem we obtain that any non-zero solution of the general half-liner difference equation has either infinitely many generalized zeros or none in a neighborhood of infinity and we can divide considered equations into two types: oscillatory and non-oscillatory equations. We use this fact and work with the generalized adapted Riccati transformation together with known auxiliary results. To illustrate the fact that the presented criterion is new even for linear equations with periodic coefficients, we finish this paper with examples of simple equations whose oscillatory properties do not follow from previously known criteria.

### References

[1] P. Hasil; L. Linhartová; M. Veselý: *Oscillation criterion for generalized Euler difference equations.* Acta Mathematica Hungarica. Springer. **174** (2024), 94–115.

[2] P. Řehák: *Generalized discrete Riccati equation and oscillation of half-linear difference equations.* Math. Comput. Mod. **34** (2001), no. 3-4, 257–269.

[3] J. Šišoláková: *Oscillation of linear and half-linear difference equations via modified Riccati transformation.* J. Math. Anal. Appl. **528** (2023), art. no. 127526, 1–19.

## Nonlinear Systems of ODEs Subject to Non-Local Boundary Conditions

Lucía López-Somoza

Universidade de Santiago de Compostela, Santiago de Compostela, Spain
LUCIA.LOPEZ.SOMOZA@USC.ES

### Abstract

In this talk we will present a method to derive an explicit formula for the Green's function corresponding to a particular class of systems of differential equations subject to linear non-local boundary conditions. The novelty of our work lies in the fact that the unknown functions are not decoupled, neither in the equations nor in the boundary conditions of the system. Furthermore, we show that the aforementioned decomposition makes it possible to enhance certain results concerning the existence of solutions for nonlinear systems with linear non-local boundary conditions.

### References

[1] A. Cabada, L. López-Somoza, M. Yousfi, *Existence of solutions of nonlinear systems subject to arbitrary linear non-local boundary conditions*, Journal of Fixed Point Theory and Applications, 25 (81), 1–24 (2023).

## Voter Model with Time Dependent Dynamics

Peter Malárik

Brno University of Technology, Brno, Czech Republic
PETER.MALARIK@VUTBR.CZ

### Abstract

The Voter model is one of the paradigmatic models of interacting particle systems and a standard framework for the study of opinion dynamics [1, 3]. We consider its time-dependent modification in which the update rules depend on the internal age of agents, that is, on how long they have kept their current opinion [1, 2]. The main question is how this age-dependent dynamics affects ordering, in particular whether the system approaches consensus, understood as a state in which all agents share the same opinion, or whether it exhibits coexistence of opinions or frozen disordered states [1]. Our analysis is based on dynamical equations derived for the model and, in particular, on a scalar differential equation obtained by linearization around the consensus, or absorbing, state. This reduction provides the main tool for studying stability and asymptotic behavior by standard qualitative methods of differential equations [4]. As an extension of the mean-field setting, we also study the model on a complete bipartite graph and examine how this interaction structure influences the long-time dynamics and the emergence of consensus.

### References

[1] PERALTA, Antonio F., KHALIL, Nagi a TORAL, Raúl. Ordering dynamics in the voter model with aging. *Physica A: Statistical Mechanics and its Applications*. 2020, vol. 552, art. 122475. DOI: 10.1016/j.physa.2019.122475.

[2] PERALTA, Antonio F., KHALIL, Nagi a TORAL, Raúl. Reduction from non-Markovian to Markovian dynamics: the case of aging in the noisy-voter model. *Journal of Statistical Mechanics: Theory and Experiment*. 2020, 024004. DOI: 10.1088/1742-5468/ab6847.

[3] THURNER, Stefan, HANEL, Rudolf a KLIMEK, Peter. *Introduction to the Theory of Complex Systems*. First edition. Oxford: Oxford University Press, 2018. ISBN 978-0-19-882193-9. DOI: 10.1093/oso/9780198821939.001.0001.

[4] HALE, Jack K. a VERDUYN LUNEL, Sjoerd M. *Introduction to Functional Differential Equations*. New York: Springer, 1993. ISBN 978-1-4612-8741-4. DOI: 10.1007/978-1-4612-4342-7.

## Floquet Theory and Exponential Stabilization of Impulsive Second-Order Delay Differential Equations

Sergey Malev

Ariel University, Ariel, Israel
sergeyma@ariel.ac.il

### Abstract

We present a new approach to the exponential stabilization of impulsive second-order delay differential equations based on a finite-dimensional Floquet theory. The proposed framework preserves equality between the order of the equation and the dimension of the associated monodromy matrix, making it possible to extend several classical results of Floquet theory for ordinary differential equations to impulsive delay equations. We establish new criteria for exponential stability and instability and demonstrate how suitable choices of periodic coefficients, delays, and impulsive actions can stabilize systems that are unstable within the standard nonimpulsive setting. The approach also provides explicit representations of solutions and constructive algorithms for stability analysis.

### References

[1] Martin Bohner, Alexander Domoshnitsky, Sergey Malev, Tsahi Shavit and Cemil Tunc *Exponential Stability of Impulsive Second-Order Delay Differential Equations through Floquet Theory* Preprint (2026).

### Stable and Unstable Periodic Motions in a Bouncing Ball Model

Stefano Marò

University of Oviedo, Oviedo, Spain
marostefano@uniovi.es

#### Abstract

We consider the vertical motion of a free falling ball bouncing elastically on a racket moving in the vertical direction according to a regular 1-periodic function $f$. For fixed coprime $p$, $q$ we study existence, Lyapunov stability and multiplicity of $(p,q)$-periodic motions. These are $p$-periodic motions making $q$ bounces in a period. We prove that if $f \in C^3$ and $p/q$ is large, there always exist at least two $(p,q)$-periodic motions.

We are interested in the stability, in the sense of Lyapunov, of these motions. First we prove that if $f$ is analytic, at least one is unstable. Then, under quantitative conditions on $f$ we prove that there exists an infinite sequence of stable motions with $q = 1$. As a consequence, we get an open and unbounded set of initial conditions giving rise to bounded motions.

#### References

[1] S. Marò. *Some remarks on the periodic motions of a bouncing ball.* Bollettino dell'Unione Matematica Italiana 16: 397-409 (2023)

[2] S. Marò, R. Ortega *Stability of some periodic configurations of discrete Lagrangian equations.* Nonlinear Analysis 264: 114002 (2026)

## Weakly Nonlinear Degenerate Differential-Algebraic Boundary Value Problems

Sergey Chuiko, Olga Nesmelova

Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany
Institute of Applied Mathematics and Mechanics of NAS of Ukraine, Cherkasy, Ukraine
chujko-slav@ukr.net, star-o@ukr.net

**Abstract**

We have studied the problem of constructing solutions

$$z(t,\varepsilon): \ z(\cdot,\varepsilon) \in \mathbb{C}^1[a,b], \ z(t,\cdot) \in \mathbb{C}[0,\varepsilon_0]$$

of the weakly nonlinear differential-algebraic boundary value problem [1]

$$A(t)z'(t,\varepsilon) = B(t)z(t,\varepsilon) + f(t) + \varepsilon\, Z(z,t,\varepsilon), \ \ell z(\cdot,\varepsilon) = \alpha + \varepsilon\, J(z(\cdot,\varepsilon),\varepsilon). \tag{1}$$

We have found the solutions of the boundary value problem (1) in a small neighborhood of the solution $z_0(t) \in \mathbb{C}^1[a,b]$ of the generating Noetherian $(n = q)$ boundary value problem [1]

$$A(t)z_0'(t) = B(t)z_0(t) + f(t), \quad \ell z_0(\cdot) = \alpha. \tag{2}$$

Here, $A(t),\ B(t) \in \mathbb{C}_{m\times n}[a,b]$ are continuous matrices, $f(t) \in \mathbb{C}[a,b]$ is a continuous vector; $Z(z,t,\varepsilon)$ is a nonlinear function which is continuously differentiable with respect to the unknown $z(t,\varepsilon)$ in a small neighborhood of the solution of the generating problem, continuous in $t \in [a,b]$, and continuous in a small parameter; $\ell z(\cdot,\varepsilon)$ and $J(z(\cdot,\varepsilon),\varepsilon)$ are, correspondingly, a linear and nonlinear vector functionals $\ell z(\cdot,\varepsilon),\ J(z(\cdot,h,\varepsilon),\varepsilon) : \mathbb{C}[a,b(\varepsilon)] \to \mathbb{R}^q$. Moreover, the second functional is continuously differentiable with respect to the unknown $z(t,\varepsilon)$ and continuous in the small parameter $\varepsilon$ in a small neighborhood of the solution of the generating problem (2) and on the interval $[0,\varepsilon_0]$. The weakly nonlinear differential-algebraic boundary value problem (1) is a generalization of numerous statements of nonlinear boundary value problems [1]. We have studied the case of degeneration [1, 2] of the generating boundary value problem (2), namely: $P_{A^*}(t) \neq 0$; here, $P_{A^*}(t)$ is the orthoprojector [1]: $P_{A^*}(t) : \mathbb{R}^m \to \mathbb{N}(A^*(t))$. Generally speaking, the degenerate system (2) is not solvable with respect to the derivative. The necessary and sufficient conditions of solvability of weakly nonlinear differential-algebraic boundary value problem (1) and the convergent iterative scheme of construction of approximations to their solutions are found.

### References

[1] Chuiko S.M., Nesmelova O.V. *Nonlinear boundary-value problems for degenerate differential-algebraic systems*, Journal of Mathematical Sciences **252** (2021), 463-471. https://doi.org/10.1007/s10958-020-05174-5

[2] Chuiko S.M. *On a reduction of the order in a differential-algebraic system*, Journal of Mathematical Sciences **235** (2018), 2-14. https://doi.org/10.1007/s10958-018-4054-z

## Distributional Adjoint Theory for Retarded Differential Operators: Uncovering the Structural Origin of Hale's Bilinear Form

Junya Nishiguchi

Setsunan University, Osaka, Japan / Tohoku University, Miyagi, Japan
junya.nishiguchi@setsunan.ac.jp / junya.nishiguchi.b1@tohoku.ac.jp

### Abstract

The bilinear form introduced by Hale plays a pivotal role in the adjoint theory and spectral analysis of retarded functional differential equations (RFDEs). However, in the case of systems of RFDEs, this bilinear form is often presented as a prescribed definition, and its structural origin as a boundary term—especially regarding the rigorous ordering of matrix-valued measures—has not been fully elucidated.

In this talk, we provide a self-contained reconstruction of the linear RFDE theory from a distributional perspective. We characterize the fundamental matrix as a distributional matrix solution by considering an RFDE with a Dirac delta function as an external force. This result provides a rigorous formulation of the impulse response matrix for general retarded systems. In addition, we demonstrate that Hale's bilinear form naturally emerges as a boundary term in a Green-type identity. Our approach ensures the rigorous ordering of matrix-valued integrals and clarifies the essential role of constant extensions of functions at the boundaries. These results provide a solid mathematical foundation for the adjoint structures in systems of RFDEs.

## A Leighton-Wintner Type Oscillation Theorem for $p(t)$-Laplacian Dynamic Equations on Time Scales

Masakazu Onitsuka

Department of Applied Mathematics, Okayama University of Science, Okayama 700-0005, Japan
ONITSUKA@OUS.AC.JP

**Abstract**

This study is based on joint work (including [3, 4]) with Professors K. Fujimoto (Shimane University) and K. Ishibashi (Hiroshima Institute of Technology). It is known that applying the Riccati method to discrete oscillation problems is challenging (see [2]). Recently, however, the authors established oscillation criteria for difference equations involving the $p(t)$-Laplacian by utilizing the Riccati technique (see [4]). Based on the ideas developed there, we establish a Leighton-Wintner type oscillation theorem via the Riccati method for dynamic equations on time scales, which are applicable to both continuous and discrete cases. This work was supported by JSPS KAKENHI Grant Number JP25K07092.

### References

[1] M. Bohner and A. Peterson, Dynamic equations on time scales, Birkhäuser Boston, Inc., Boston, MA, 2001.

[2] O. Došlý and P. Řehák, Half-linear differential equations, North-Holland Math. Stud., 202, Elsevier Science B.V., Amsterdam, 2005.

[3] K. Fujimoto and M. Onitsuka, *Leighton-Wintner type oscillation criteria for second-order differential equations with p(t)-Laplacian*, Electron. J. Qual. Theory Differ. Equ., 2024 (2024), Paper No. 19, 9pp.

[4] K. Fujimoto, K. Ishibashi, and M. Onitsuka, *Leighton-Wintner-type oscillation theorem for the discrete $p(k)$-Laplacian*, Appl. Math. Lett., 163 (2025), Paper No. 109465, 6 pp.

## Some Oscillation Criteria for the Second-Order Linear Advanced Differential Equations

Zdeněk Opluštil

Institute of Mathematics, FME, Brno University of Technology, Brno, Czech Republic
OPLUSTIL@FME.VUTBR.CZ

### Abstract

On the half-line $\mathbb{R}_+ = [0, +\infty[$, we consider the second-order linear differential equation with argument deviation

$$u''(t) + p(t)u(\sigma(t)) = 0,$$

where $p\colon \mathbb{R}_+ \to \mathbb{R}_+$ is a locally Lebesgue integrable function and $\sigma\colon \mathbb{R}_+ \to \mathbb{R}_+$ is a continuous function such that $\sigma(t) \geq t$, for $t \geq 0$.

New oscillation criteria for solutions to the considered equation are presented. These criteria extend and build upon those established in [3]. The proofs are based on the Riccati technique and suitable estimates of nonoscillatory solutions. Some of the results may be regarded as an analogy of the Hille-Nehari type oscillation criteria for ODEs introduced in [1] and [2].

### References

[1] Hille, E. *Non-oscillation theorems*, Trans. Amer. Math. Soc. **64** (1948), No. 2, 234–252.

[2] Nehari, Z *Oscillation criteria for second-order linear differential equations*, Trans. Amer. Math. Soc. **85** (1957), No. 2, 428–445.

[3] Opluštil, Z. *Oscillation criteria for the second-order linear advanced differential equation*, Applied Mathematics Letters **157** (2024), No. 157.

### Delay-Induced Instabilities in Neural Field Equations

Mónika Polner

Bolyai Institute, University of Szeged, Hungary
polner@math.u-szeged.hu

**Abstract**

We investigate neural field equations with transmission delays as infinite-dimensional dynamical systems, analyzing their spectral properties, bifurcation scenarios, and symmetry-induced mechanisms of pattern formation. On bounded two-dimensional domains with exponential connectivity kernels, the characteristic equation is reformulated as a linear partial differential equation, permitting a complete characterization of the point spectrum and resolvent for single-exponential kernels. This approach yields explicit conditions for Hopf bifurcations and an analytical expression for the first Lyapunov coefficient, providing a rigorous description of delay-induced oscillatory instabilities [1].

Building on this foundation, we further study two-population systems on the sphere incorporating both transmission delays and diffusive coupling, following the framework of [2]. By employing spherical harmonic decompositions in combination with equivariant bifurcation theory, we derive analytical conditions for the onset of bifurcations and determine the associated normal form coefficients up to third order. The interplay between delay and diffusion mechanisms is shown to drive symmetry-breaking Hopf bifurcations and the selective stabilization of particular spherical harmonic modes.

A space–time discontinuous Galerkin method supports the theoretical analysis, [3].

#### References

[1] L. Spek, K. Dijkstra, S. A. van Gils, and M. Polner, J. Differ. Equ., 317 (2022), 439–473.

[2] L. Spek, S. A. van Gils, Y. A. Kuznetsov, and M. Polner, SIAM J. Appl. Dyn. Syst., 2024.

[3] M. Polner, J. J. W. van der Vegt, and S. A. van Gils, SIAM J. Sci. Comput., 39 (2017), B797–B818.

### Localization of Solutions in Annular Regions Times Conical Shells for Systems with Sign-Changing Nonlinearities

Jorge Rodríguez–López

Universidade de Santiago de Compostela, Santiago, Spain
JORGERODRIGUEZ.LOPEZ@USC.ES

**Abstract**

In this talk, we provide an existence result for abstract nonlinear operator systems of the form

$$\begin{cases} u_1 = T_1(u_1, u_2), \\ u_2 = T_2(u_1, u_2), \end{cases}$$

where $T = (T_1, T_2) : \mathcal{A} \times \mathcal{K} \subset X \times Y \to X \times Y$ is a compact map and $\mathcal{A}$ and $\mathcal{K}$ are an annular and a conical region in the normed spaces $X$ and $Y$, respectively. To localize the solutions of the system, compression-expansion type conditions are imposed in each component of the operator $T$. Our approach [2] relies on the topological methods and, more concretely, it is based on the multiplicativity property of the Leray-Schauder fixed point index.

As an application, a system of ODEs with two point boundary conditions is studied. We consider the case in which one of the nonlinearities in the system is nonnegative whereas the other is sign-changing. In this setting, the solution is located within the product of annular regions and conical shells and thus both of its components are nontrivial.

#### References

[1] G. Infante, G. Mascali and J. Rodríguez–López, An existence result in annular regions times conical shells and its applications to nonlinear Poisson systems, *Ann. Mat. Pura Appl.*, to appear (2026).

## Inverse Problem for Abstract Differential Inclusions

Santosh Ruhil

Department of Mathematics, Ariel University, 40700 Ariel, Israel
ruhilsantosh11@gmail.com

### Abstract

Inverse problems arise naturally in many areas of mathematics and applied sciences, where the goal is to recover unknown components of a system from indirect or incomplete observations. These problems become more challenging when the governing dynamics are described by differential inclusions, since such systems involve multivalued nonlinearities and uncertainty in their evolution. In this talk, we study an inverse problem for abstract semilinear differential inclusions, where the aim is to identify unknown elements of the system using additional observational data. We establish the existence of mild solutions for the considered inverse problem and further investigate the continuous dependence of solutions on the initial data. Our analysis is mainly based on semigroup theory, set-valued analysis, Banach fixed point theorem, and Grönwall's inequality. In addition, an example will be presented to demonstrate the applicability and effectiveness of the obtained results.

### References

[1] Ruhil, S., & Malik, M. (2025). Results on inverse problem for abstract semilinear differential inclusions. Journal of Inverse and Ill-posed Problems, 33(6), 735-748.

[2] Aubin, J. P., & Frankowska, H. (2008). Differential inclusions. In Set-Valued Analysis (pp. 1-27). Boston: Birkhäuser Boston.

[3] Kirsch, A. (2011). An introduction to the mathematical theory of inverse problems (Vol. 120). New York: Springer.

## Sharp Delay-Independent Global Stability for a Lotka-Volterra System with Multiple Delays

Yasuhisa Saito

Department of Mathematics, Shimane University, Matsue 690-8504, Japan
ysaito@riko.shimane-u.ac.jp

### Abstract

Delays are known to destabilize dynamical systems and to generate complex behaviors such as oscillations and chaos, especially in nonlinear delay differential equations. Even for the classical Lotka-Volterra predator-prey model, whose global dynamics are well understood in the absence of delay, the introduction of time lags can drastically complicate the qualitative behavior, and sharp characterizations of global stability remain a challenging problem in functional differential equations.

This talk considers a symmetric Lotka-Volterra predator-prey system with four discrete delays appearing in the intraspecific interaction terms, where both delayed and undelayed feedbacks are incorporated and a unique positive equilibrium exists under natural assumptions. Here, the parameter $a$ represents the strength of instantaneous self-limitation, while $\alpha$ and $\beta$ denote the strengths of delayed interaction effects. The main result establishes a necessary and sufficient condition for delay-independent global asymptotic stability: the positive equilibrium is globally asymptotically stable for all nonnegative delays if and only if $|\alpha| + |\beta| \leq a$ ([1]). The sufficiency is proved by constructing an appropriate Lyapunov functional and applying LaSalle's invariance principle, while the necessity follows from a local analysis near the equilibrium.

A comparison with the corresponding two-delay system, for which the sharp stability condition is given by $\sqrt{\alpha^2 + \beta^2} \leq a$ (see [2]), reveals a clear discrepancy between the two settings. This highlights that the global dynamics of delayed Lotka-Volterra systems depend sensitively not only on the magnitude of interaction coefficients but also on how delays are distributed within the nonlinear structure.

### References

[1] Y. Saito, Sharp global stability in a delayed Lotka-Volterra predator-prey system: a delay-placement discrepancy, submitted.

[2] Y. Saito, T. Hara, and W. Ma, Necessary and sufficient condition for permanence and global stability of Lotka-Volterra system with two delays *J. Math. Anal. Appl.*. **236** (1999) 534-556.

## Applications of Coincidence Degree Theory to Fractional Boundary Value Problems and Non-Autonomous Ecological ODE Models

Satyam Narayan Srivastava

Department of Mathematics and Statistics, Faculty of Science, Masaryk University, Brno, Czech Republic
srivastava@math.muni.cz, satyamsrivastava983@gmail.com

**Abstract**

This talk highlights coincidence degree theory as a powerful tool in nonlinear analysis, proving existence results in both fractional boundary value problems and non-autonomous ecological dynamical systems. We first consider higher-order Riemann–Liouville fractional differential equations subject to Riemann–Stieltjes integral boundary conditions at resonance. The main idea is to transform the nonlinear problem into a semilinear operator equation of the form $Lx = Fx$, where $L$ is a linear Fredholm operator of index zero and $F$ is a nonlinear operator. By constructing appropriate operators and verifying the conditions of Mawhin's coincidence degree theory, we establish existence results for various classes of fractional boundary value problems (problems involving Caputo, Hadamard, and Katugampola derivatives). Related Publications [1, 2, 4].

Using the same analytical methodology, we also study non-autonomous ecological ODE models. In particular, we present recent results on allelopathic phytoplankton systems and bioeconomic fishery models with periodic coefficients. By carefully defining suitable function spaces, norms, and operator settings, we derive sufficient conditions for the existence of positive periodic solutions. Related Publications [3].

### References

[1] Martin Bohner, Alexander Domoshnitsky, Seshadev Padhi, and Satyam Narayan Srivastava. Existence of solutions by coincidence degree theory for Hadamard fractionaldifferential equations at resonance. *Turkish Journal of Mathematics*, 48(2):296–306, 2024.

[2] Alexander Domoshnitsky, Satyam Narayan Srivastava, and Seshadev Padhi. Existence of solutions for a higher order Riemann–Liouville fractional differential equation by Mawhin's coincidence degree theory. *Math. Methods Appl. Sci.*, 46(11):12018–12034, 2023.

[3] Satyam Narayan Srivastava, Alexander Domoshnitsky, Seshadev Padhi, Rana D. Parshad, and John R. Graef. Existence of periodic solution of a non-autonomous allelopathic phytoplankton model with fear effect. *Electronic Research Archive*, 34(2):723–737, 2026.

[4] Satyam Narayan Srivastava, Smita Pati, John R Graef, Alexander Domoshnitsky, and Seshadev Padhi. Existence of solution for a Katugampola fractional differential equation using coincidence degree theory. *Mediterranean Journal of Mathematics*, 21(4):1–16, 2024.

## Levinson-Pliss Theorem for Periodic Lattice Dynamical Systems

David Cheban and Andrei Sultan

Moldova State University, Chisinau, Republic of Moldova
david.ceban@usm.md, andrew15sultan@gmail.com

**Abstract**

In this talk we study the compact global attractors of periodic dynamical systems. We establish the relation between periodic compact dissipative dynamical systems and the compact dissipative discrete dynamical (Poincare map) generated by given periodic system. Using this relation we establish some important facts for compact dissipative periodic systems. We apply the obtained general results for the periodic lattice dynamical systems [1] of the form

$$u_i' = \nu(u_{i-1} - 2u_i + u_{i+1}) - \lambda u_i + F(u_i) + f_i(t) \ (i \in \mathbb{Z}). \tag{23}$$

In particular we show that under some conditions the periodic lattice dynamical system (35) admits a compact global attractor which contains at least one periodic solution of the system (35).

We have established the following result.

**Theorem.** Assume that the following conditions hold:

1. the functions $f_i \in C(\mathbb{R}, \mathbb{R})$ (for all $i \in \mathbb{Z}$) are $\tau$-periodic;
2. the function $F \in C(\mathbb{R}, \mathbb{R})$ is Lipschitz continuous on bounded sets and $F(0) = 0$;
3. there exists a positive number $\alpha$ such that $sF(s) \leq -\alpha s^2$ for any $s \in \mathbb{R}$.

Then the lattice dynamical system (35) is compact dissipative [2], i.e., there exists a nonempty compact subset $I \subset \ell_2$ possessing the following properties:

1. $P(I) = I$, i.e., the set $I$ is invariant w.r.t. the Poincare transformation $P$;
2. $I$ is orbitally stable;
3. the set $I$ is globally attracting;
4. the Poincare mapping $P$ is asymptotically compact;
5. the map $P$ has at least one fixed point $x_0 \in I$, i.e., $P(x_0) = x_0$.

### References

[1] Petr W. Bates, Kening Lu and Bixiang Wang, Attractors for Lattice Dynamical Systems. International Journal of Bifurcation and Chaos, Vol. 11, No. 1 (2001), pp.143-153.

[2] Cheban D. N. *Global Attractors of Nonautonomous Dynamical and Control Systems. 2nd Edition.* Interdisciplinary Mathematical Sciences, vol.18, River Edge, NJ: World Scientific, 2015, xxv+589 pp.

### Multiple Solutions of Dirichlet Problems in Billiard Spaces with Impulses at Fixed Times

David Švec

Faculty of Science, Palacký University Olomouc, Czechia
david.svec02@upol.cz

**Abstract**

This is a joint work with Jan Tomeček (Faculty of Science, Palacký University Olomouc, Czechia). We investigate a Dirichlet problem for an ODE of the second order with state-dependent impulses and with impulses at fixed times in the form

$$\begin{aligned}
&\ddot{x}(t) = f(t, x(t)), \quad \text{for a.e. } t \in [0, T],\ x(t) \in \operatorname{int} K,\\
&\dot{x}(s+) = -\dot{x}(s-), \quad \text{if } s \in (0, T),\ x(s) \in \partial K,\\
&\dot{x}(t_i+) = \dot{x}(t_i) + I_i(x(t_i)), \quad \text{if } x(t_i) \in \operatorname{int} K,\ i = 1, \dots, N,\\
&x(0) = A, \quad x(T) = B,
\end{aligned}$$

where $K$ is a compact interval in $\mathbb{R}$, $f : [0, T] \times K \to \mathbb{R}$ satisfies Carathéodory conditions, $A, B \in \operatorname{int} K$, $0 < t_1 < t_2 < \cdots < t_N < T$ and $I_i : K \to \mathbb{R}$, for $i = 1, \dots, N$. We can understand this impulsive differential equation as an one-dimensional billiard space with added impulses at specific times. We give existence and multiplicity result for solutions with prescribed number of impacts.

### Symbolic Dynamics in Pseudospectral Projection of Delay Differential Equations

Robert Szczelina

Jagiellonian University, Krakow, Poland
ROBERT.SZCZELINA@UJ.EDU.PL

**Abstract**

We study the following class of Delay Differential Equations (DDEs):

$$\begin{aligned} x'(t) &= f(x(t), x(t-\tau)) && t \geq 0 \\ x(t) &= \psi(t) && t \in [-\tau, 0], \end{aligned}$$

with $\tau > 0$ fixed, $x \in \mathbb{R}^d$, $\psi \in \mathcal{C}^0([-\tau, 0], \mathbb{R}^d) =: \mathcal{C}$. While significant work has been done on understanding the dynamics of such systems–most notably regarding chaos in the Mackey-Glass equation[1]–the infinite-dimensional nature of DDEs presents some challenges for carrying out the rigorous computations and obtaining estimates good enough for computer-assisted proofs remains a challenge, see [2] and references therein.

This talk will present a recent attempt to reduce the number of unknown factors affecting the quality of rigorous numerics by using a pseudospectral approximation [3], which reduces the DDE to a relatively small system of ODEs while preserving numerically observed dynamical features. Due to the low-dimensionality of the resulting approximation and its great theoretical accuracy, the computations are less demanding and can be done using known tools, such as CAPD [4], to efficiently verify some dynamical phenomena that closely mirror those of the full system. We present a proof of existence of symbolic dynamics in the ODE approximating DDE and we discuss lessons learned that might be used to guide a similar proof for the full DDE system. **Acknowledgments:** the author acknowledge the support of Polish National Science Center (NCN) grant no. 2023/49/B/ST6/02801.

#### References

[1] M. C. Mackey and L. Glass. Oscillation and chaos in physiological control systems. *Science*, 197(4300), pp. 287–289, 1977.

[2] A. Gierzkiewicz, R. Szczelina. Sharkovskii theorem for infinite dimensional dynamical systems. *Commun. Nonlinear Sci. Numer. Simul.*, Vol. 146, 108770, 2025.

[3] D. Breda, O. Diekmann, M. Gyllenberg, F. Scarabel, and R. Vermiglio. Pseudospectral Discretization of Nonlinear Delay Equations: New Prospects for Numerical Bifurcation Analysis. *SIAM J. Appl. Dyn. Sys.*, Vol. 15(1), pp. 1–23, 2016.

[4] T. Kapela, M. Mrozek, D. Wilczak, and P. Zgliczyński. CAPD::DynSys: A flexible C++ toolbox for rigorous numerical analysis of dynamical systems. *Commun. Nonlinear Sci. Numer. Simul.*, 101:105578, 2021.

## Oscillation Results for First Order Neutral Delay Differential Equations with Several Positive and Negative Coefficients

Tóth Gergő

Bolyai Institute, University of Szeged, Szeged, Hungary
tothgergo411@gmail.com

### Abstract

We provide sufficient criteria for the oscillation of all solutions of neutral delay differential equations of the form

$$\left[x(t) - \sum_{i=1}^{N_r} R_i(t)x(t - r_i(t))\right]' + \sum_{i=1}^{N_p} P_i(t)x(t - \tau_i(t)) - \sum_{i=1}^{N_q} Q_i(t)x(t - \delta_i(t)) = 0,$$

with both positive and negative terms and time-variable delays. Our results improve and generalize several existing criteria available in the literature that address restricted cases, such as constant delays or the absence of negative coefficients. Under additional assumptions on slowly varying parameters, we derive sharper oscillation conditions. We demonstrate the applicability of our findings through illustrative examples.

This is a joint work with Ábel Garab (University of Szeged)

### Finitude of Limit Cycles of Linear Piecewise ODEs in the Cylinder

Roberto Trinidad Forte

Departament de Ciencies Matematiques i Informatica, IAC3 Institute of Applied Computing & Community Code, Universitat de les Illes Balears, Palma 07122, Spain

ROBERTO.TRINIDAD@UIB.CAT

**Abstract**

Let $x' = S(t, x)$ be a differential equation in the cylinder, linear piecewise in $x$ and with trigonometric coefficients in $t$. In this talk, we provide an upper bound on the number of limit cycles in terms of the number of zones of the piecewise equation and the degree of the coefficients, that is, an analog of Hilbert's 16th problem in this context.

## About Properties of Ultrametric Meromorphic Solutions of Some Types of Difference Equations

Zerzaihi Tahar

Laboratory of Pure and Applied Mathematics (LMPA), Mohamed Seddik Ben Yahia University, Jijel, Algeria.
zerzaihi@yahoo.com

### Abstract

We consider an algebraically closed field $\mathbb{K}$, complete for an ultrametric absolute value. We denote by $\mathcal{A}(\mathbb{K})$ the $\mathbb{K}$-algebra of entire functions and by $\mathcal{M}(\mathbb{K})$ the field of meromorphic functions on $\mathbb{K}$.

In recent years, many works have focused on the behavior of meromorphic solutions to difference equations, both in the complex and ultrametric cases. Significant results have been obtained regarding the growth of such solutions.

In this talk, We extend some results from the complex case to ultrametric difference equations of Malmquist and Painlevé types. In particular, we study equations of the form

$$\sum_{j=1}^{n} A_j\, y(x+c_j) = \frac{P(x, y(x))}{Q(x, y(x))},$$

where $P$ and $Q$ are relatively prime polynomials in $y$ with rational coefficients.

### References

[1] W. Bergweiler, K. Ishizaki, N. Yanagihara, Meromorphic solutions of some functional equations, *Methods and Applications of Analysis*, **5**(3), 248–258 (1998).

[2] S. Bourourou, A. Boutabaa, and T. Zerzaihi, "On the growth of solutions of difference equations in ultrametric fields, "Indagationes Mathematicae. New Series **27**(1), 112–123 (2016).

[3] H. Boughaba, S. Bouternikh, and T. Zerzaihi, "Results on the Growth of Meromorphic Solutions of some Functional Equations of Painlevé and Schróder Type in Ultrametric Fields, "P-Adic Num Ultrametr Anal Appl **16**, 14–22 (2024)

**Perturbations of the Kepler Problem under Dirichlet Boundary Conditions**

Tingyu Zhong

School of Mathematical Science, Shanghai Jiao Tong University, China
TUNGYU-CHONG@SJTU.EDU.CN

**Abstract**

We study the forced Kepler problem in dimension two and three:

$$\ddot{u} = -\frac{u}{|u|^3} + \epsilon \nabla_u U(t,u), \qquad u \in \mathbb{R}^d, \quad t \in [0,1], \quad d = 2,3, \tag{$K_\epsilon$}$$

combined with the Dirichlet boundary conditions

$$u(0) := \lim_{t \to 0^+} u(t) = 0 = \lim_{t \to 1^-} u(t) =: u(1), \tag{$C$}$$

where $\epsilon > 0$ is the perturbation parameter and $U : [0,1] \times \mathbb{R}^d \to \mathbb{R}$ is a given $C^1$ function. Our solutions are allowed to collide with the singularity.

We combine the use of Levi-Civita regularization together with a perturbation result of critical manifolds to study multiplicity of solutions for problem $(K_\epsilon)$–$(C)$.

This is a joint work with professor Antonio J. Ureña at the Department of Applied Mathematics, University of Granada, Spain.

## Phase-Field Modeling of Fluid-Driven Fracture in Porous Media with Application to Desiccation Cracking

Sayahdin Alfat[1,2], Michal Beneš[2], Masato Kimura[3]

[1]Physics Education Department, Halu Oleo University, Kendari, Southeast Sulawesi, Indonesia
[2] Department of Mathematics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague
[3]Division of Mathematical and Physical Sciences, Kanazawa University, Ishikawa, Japan
SAYAHDIN.ALFAT@YAHOO.COM

### Abstract

Desiccation cracking due to fluid pressure reduction is a key issue in porous media. We study this phenomenon using a phase-field model (PFM) coupled with Biot's poroelastic theory to formulate a constitutive framework for desiccation-driven fracture and its associated energy dissipation. Fluid transport is described by a modified consolidation model with damage-dependent intrinsic permeability following an Arrhenius-type law, where the activation energy decreases with the damage variable $z$. The resulting system comprises the momentum balance, crack evolution, and fluid mass balance equations, written as follows.

$$\begin{cases} -\operatorname{div}(\sigma_z^\dagger[u,\varrho]) = 0 & \text{in } \Omega, \\ \alpha \dfrac{\partial z}{\partial t} = \Big(\epsilon \operatorname{div}(\gamma_* \nabla z) - \dfrac{\gamma_*}{\epsilon} z + (1-z)\lambda^* (\operatorname{div} u)_+^2 + 2\mu |e_D[u]|^2\Big)_+ & \text{in } \Omega, \\ \dfrac{1}{Q}\dfrac{\partial \varrho}{\partial t} = \operatorname{div}\big(\kappa^\dagger(z,\epsilon)\nabla \varrho\big) - \alpha_p (1-z)^2 \operatorname{div}\dot{u} & \text{in } \Omega. \end{cases}$$

For the numerical implementation, we employ FEM with P1 elements and adaptive mesh refinement to resolve crack evolution in two-dimensional settings. A semi-implicit time discretization is used to solve the coupled nonlinear system. Numerical results show that the model captures the initiation and propagation of desiccation cracks, as well as the strong coupling between fluid pressure and damage evolution in porous media.

Keywords: Poroelasticity, Desiccation Cracking, Phase Field Model, Anisotropic Adaptive Mesh, Arrhenius law, Random Heterogeneous Materials.

### References

[1] Biot, M. A. (1941). General theory of three-dimensional consolidation. Journal of Applied Physics, 12(2), 155-164.

[2] Kimura, M., Takaishi, T., Alfat, S., Nakano, T., & Tanaka, Y. (2021). Irreversible phase field models for crack growth in industrial applications: thermal stress, viscoelasticity, hydrogen embrittlement. SN Applied Sciences, 3(9), 781.

[3] S. Alfat, M. Beneš, J. F. T. Rabago, and M. Kimura Energy-Consistent Phase Field Model for Fluid-Driven Fracture in Porous Medium: Part I. Hydraulic Fracturing. On Preparation.

## A Posteriori Algebraic Error Estimates and Nonoverlapping Domain Decomposition in Mixed Formulations: Energy Coarse Grid Balancing, Local Mass Conservation on Each Step, and Line Search

Akram Beni-Hamad

1. Inria, 48 rue Barrault, 75647 Paris, France
2. CERMICS, Ecole nationale des ponts et chaussées, IP Paris, 77455 Marne la Vallée, France.
AKRAM.BENI_HAMAD@MINESPARIS.PSL.EU

### Abstract

We focus on iterative algebraic solvers for saddle-point mixed finite element discretizations of the model Darcy flow problem. We propose a posteriori error estimators of the algebraic error as well as a nonoverlapping domain decomposition algorithm. The estimators control the algebraic error from above and from below in a guaranteed and fully computable way. The distinctive feature of the domain decomposition algorithm is that it produces a locally mass conservative approximation on each iteration. Both the estimate and the algorithm rely on a coarse grid solver, a subdomain Neumann solver, and a subdomain Dirichlet solver. The algorithm also employs a line search to determine the optimal step size, leading to a Pythagoras formula for the algebraic error decrease in each iteration. We suppose that the fine mesh is a refinement of a coarse mesh, where both meshes need to be formed by simplices or rectangular parallelepipeds. Numerical experiments illustrate the theoretical developments and confirm the efficiency of the algebraic error estimates and of the domain decomposition algorithm. This work was carried out in collaboration with M. Bastidas Olivares, M. Vohralík, and I. Yotov. [1].

### References

[1] M. B ASTIDAS OLIVARES , A. BENI HAMAD, M. VOHRALÍK , AND I. YOTOV, A posteriori algebraic error estimates and nonoverlapping domain decomposition in mixed formulations: energy coarse grid balancing, local mass conservation on each step, and line search, Computer Methods in Applied Mechanics and Engineering. 444 (2025), 118090.

### Stability of Equilibria in an Infinite Dimensional Network of Theta Neurons with Time Delay

Lavinia Florina Rodica Birdac

#### Abstract

We consider a network of identical all-to-all coupled theta neurons interacting through a synaptic current with distributed delay. Using the Watanabe–Strogatz transformation, we derive a reduced description of the system in terms of global variables.
In the thermodynamic limit of infinitely many neurons and under appropriate assumptions on the constants of motion, the dynamics reduces to a two-dimensional system with distributed delay, or equivalently to a complex delay differential equation describing the macroscopic behavior of the neuronal population.
Our results show that the introduction of delay can lead to significant changes in the dynamical behavior of the system, including delay-induced oscillations and changes in basin geometry.
The analytical results are complemented by numerical simulations illustrating how the delay reshapes the long-time dynamics of the neuronal population.

## Graph-Based Semi-Supervised Learning via PDEs and Total Variation

Farid Bozorgnia

New Uzbekistan University, Tashkent city, 100007, Uzbekistan
F.BOZORGNIA@NEWUU.UZ

### Abstract

Graph-based semi-supervised learning seeks to propagate class labels from a small labeled set to a large unlabeled graph by exploiting its intrinsic geometry. This talk surveys a line of work developing increasingly refined PDE-theoretic frameworks for this problem [1, 4].

We begin with segregation-based classification [1], where a quadratic Dirichlet energy is minimized subject to pairwise disjointness constraints motivated by spatially competitive reaction–diffusion systems. The resulting model admits a unique minimizer whose fixed-point characterization yields a simple convergent algorithm. We then address class imbalance and convergence issues arising in Laplace and Poisson learning through modified random-walk schemes [2], and extend the segregation paradigm to the infinity Laplacian setting connecting label propagation to Lipschitz extensions on graphs.

The central contribution is Total Variation Segregated Learning (TVSL) [4], which replaces the quadratic energy with graph total variation. This yields piecewise-constant label functions with sharp decision boundaries, an exact min-cut equivalence in the binary case, and a tractable proximal-gradient algorithm with PDHG inner solvers enforcing competition at each iteration. Convergence guarantees and numerical experiments on benchmark datasets are presented.

### References

[1] F. Bozorgnia, M. Fotouhi, A. Arakelyan, and A. Elmoataz, *Graph based semi-supervised learning using spatial segregation theory*, Journal of Computational Science **74** (2023), 102153. `https://doi.org/10.1016/j.jocs.2023.102153`

[2] F. Bozorgnia, *Improved graph-based semi-supervised learning schemes*, Computational and Applied Mathematics **45** (2026), 83. `https://doi.org/10.1007/s40314-025-03482-7`

[3] F. Bozorgnia, Y. Belkheiri, and A. Elmoataz, *Graph-based semi-supervised segregated Lipschitz learning*, https://arxiv.org/abs/2411.03273 (2025).

[4] F. Bozorgnia and H. Ghaolipour, *Graph-based semi-supervised learning using total variation and spatial segregation theory*, (2026).

### Time Filtered Finite Difference Schemes for Hyperbolic Problems

K. Boatman†, K. Courtney‡, F. Courtney-Pahlevani†, L. Davis⋆, C. Drapaca‡, T. Rajan†

†Penn State University Abington, Abington, PA, USA
‡Penn State University, University Park, PA, USA
⋆Montana State University, Bozeman, MT, USA
LISA.DAVIS@MONTANA.EDU

**Abstract**

The focus of this presentation is to investigate the effects of combining a time filtering scheme with various finite difference methods for partial differential equations in conservation law form. Time filters have been used for more than fifty years, with the first version introduced by Robert in 1966 [6], and subsequently analyzed and expanded by many scholars [1, 2, 4, 5, 7, 9]. The work presented here is motivated by a time filter proposed by Guzel and Layton in [3]. We show that the filter can be combined with three classical algorithms when applied to the prototype linear hyperbolic equation to result in filtered methods that are modular and require minimal modification (adding only one line of code) to achieve increased accuracy without increased computational expense. When assessing the stability and convergence properties of the filtered versions of the schemes, the examples demonstrate different possible outcomes when one attempts to apply filtering. In some cases, the filter technique can be successfully combined with an explicit scheme to yield one that is more accurate than its unfiltered counterpart. Von Neumann analysis [8] proves useful for determining the stability properties of the filtered versions of the upwind and leapfrog methods as it allows one to deal with the filter parameter systematically. With a careful choice of this parameter, the filtered upwind scheme is shown to be more accurate than its upwind counterpart, revealing that a new CFL condition is required for stability.

Our analysis demonstrates that the type of filter considered here cannot remedy certain kinds of stability properties. The filtered leapfrog scheme is shown to inherit the same type of spurious mode that its original unfiltered counterpart is known to possess. We show that combining the filter with the Crank-Nicolson method leads to an implicit method where no value of the filter parameter provides for a consistent filtered version of the method. While the analysis of the aforementioned methods is carried out for the linear hyperbolic equation, we also demonstrate that the filtering scheme can be combined with both fully explicit and semi-implicit treatments of an approximation scheme for the nonlinear Lighthill-Whitham-Richards model. The results for this case show that the improvements gained through filtering are less pronounced. Simulation results are given for both a shock wave example and a rarefaction wave case. The presentation includes a survey of computational results that support the theoretical results for the improved accuracy of the filtered upwind scheme along with examples of the time filter's limited effectiveness for the nonlinear model case.

**References**

[1] Amezcua, J. and Williams, P. D. (2015) The composite-tendency Robert–Asselin–Williams (RAW) filter in semi-implicit integrations. *Quarterly Journal of the Royal Meteorological Society*, Vol. 141, No. 688, pp. 764–773.

[2] Asselin, R. (1972) Frequency Filter for Time Integrations. *Monthly weather review*, Vol. 100, No. 6, pp. 487–490.

[3] Guzel, A. and Layton, W. (2018) Time filters increase accuracy of the fully implicit method. *BIT Numerical Mathematics*, Vol. 58, No. 2, pp. 301–315.

[4] Guzel, A. and Trenchea, C. (2018) The Williams step increases the stability and accuracy of the hoRA time filter. *Appl. Numer. Math.*, Vol. 131, pp. 158–173.

[5] Li, Y. and Trenchea, C. (2014) A higher-order Robert-Asselin type time filter. *J. Comput. Phys.*, Vol. 259, pp. 23–32.

[6] Robert,A. J. (1966) "The Integration of a Low Order Spectral Form of the Primitive Meteorological Equations," *Journal of the Meteorological Society of Japan. Ser. II*, Vol. 44, No. 5, pp. 237-245.

[7] Schlesinger, R. E., Uccellini, L. W. and Johnson, D. R. (1983) The Effects of the Asselin Time Filter on Numerical Solutions to the Linearized Shallow-Water Wave Equations. *Monthly weather review*, Vol. 111, No. 3, pp. 455–467.

[8] Strikwerda, J. C. (2004) *Finite difference schemes and partial differential equations* (2nd ed.). Philadelphia: Society for Industrial and Applied Mathematics.

[9] Williams, P. D. (2013) Achieving Seventh-Order Amplitude Accuracy in Leapfrog Integrations. *Monthly Weather Review*, Vol. 141, No. 9, pp. 3037–3051.

## A Parabolic-Elliptic-Hyperbolic System of Keller-Segel Type: Numerical Analysis and Simulation

Augusto Fernandes, José Augusto Ferreira, Gonçalo Pena

University of Coimbra, CMUC, Department of Mathematics, FCTUC
Largo D. Dinis, 3000–143 Coimbra, Portugal
augusto98@outlook.com, ferreira@mat.uc.pt, goncalo.pena@uc.pt

**Abstract**

This talk aims to present a numerical study of a system of partial differential equations consisting of a parabolic, an elliptic, and a hyperbolic equation, which can be used to describe cell migration in elastic tissues by integrating chemotaxis, interstitial fluid pressure, and tissue displacement.

The system couples a Keller–Segel-type model for cell density and chemical concentration with an elliptic equation for fluid pressure and a wave-type equation for tissue displacement. The convective cell velocity depends nonlinearly on the chemoattractant gradient, Darcy's velocity induced by fluid pressure, and the velocity of tissue displacement, whereas the convective velocity of the chemoattractant depends only on Darcy's velocity.

We propose a semidiscrete nonuniform finite difference method to approximate the continuous system, which can be interpreted as a fully discrete-in-space, piecewise linear finite element method.

Furthermore, we derive second-order spatial error estimates for all system variables using a suitable discrete norm. Finally, numerical simulations illustrate the theoretical results and demonstrate the interplay between chemotaxis and tissue mechanics.

We remark that the present work improves upon the results presented in [2], where we studied, from a numerical perspective, a Keller–Segel–flow model describing cell migration driven by the spatial distribution of a chemoattractant enhanced by interstitial fluid pressure. For a review of several extensions of the classical Keller–Segel model for cell migration, we refer the reader to [1].

### References

[1] Gurusamy Arumugam, Jagmohan Tyagi. Keller-Segel chemotaxis models: a review. *Acta Applicandae Mathematicae*, 171 (1):6, 2021.

[2] Augusto Fernandes, José Augusto Ferreira, Luís Pinto. Numerical Analysis of a Keller-Segel-Flow model for tumor cell migration. *Mathematics and Computers in Simulation*, 238: 65–81, 2025.

## A Priori and a Posteriori Estimates for Vectorial Problems via Convex Duality

P. A. Gazca-Orozco

Charles University, Prague, Czechia
GAZCA@KARLIN.MFF.CUNI.CZ

**Abstract**

By exploiting remarkable properties of the Crouzeix-Raviart and Raviart-Thomas finite elements, numerous works in recent years have been able to employ convex duality theory to derive error estimates for a diverse set of problems, including total variation minimisation, the p-Laplacian, the obstacle problem, elastoplastic torsion, among others. However, virtually all of the available results have been developed for scalar problems with homogeneous Dirichlet boundary conditions. This work extends the existing results in three directions, taking the incompressible Stokes and linear elasticity systems as prototypical examples: it considers vectorial as opposed to just scalar problems, it includes non-homogeneous mixed boundary conditions, as well as loads in the dual of the energy space. This is joint work with Alex Kaltenbach (TU Berlin).

### References

[1] P. A. Gazca-Orozco, A. Kaltenbach, *A priori and a posteriori error identities for vectorial problems via convex duality.* Submitted, 2026. ArXiv:2602.04368.

### Order Reduction of Exponential Runge–Kutta Methods: Non-Commuting Operators

Trung Hau Hoang

Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic
TRUNGHAUGG@GMAIL.COM

**Abstract**

Nonlinear parabolic equations are central to numerous applications in science and engineering, posing significant challenges for analytical solutions and necessitating efficient numerical methods. Exponential integrators have recently gained attention for handling stiff differential equations. This paper explores exponential Runge–Kutta methods for solving such equations, focusing on the simplified form $u'(t) + Au(t) = Bu(t)$, where $A$ generates an analytic semigroup and $B$ is relatively bounded with respect to $A$. By treating $A$ exactly and $B$ explicitly, we derive error bounds for exponential Runge–Kutta methods up to third order. Our analysis shows that the first- and second-order methods maintain their order under mild regularity conditions on the initial data $u_0$, while also addressing the phenomenon of order reduction in third-order methods. Through careful convergence analysis and numerical investigations, this study provides a comprehensive understanding of the applicability and limitations of exponential Runge–Kutta methods in solving linear parabolic equations involving two unbounded and non-commuting operators.

#### References

[1] M. Hochbruck and A. Ostermann. Exponential integrators. *Acta Numerica*, 19 (2010), 209–286.

[2] M. Hochbruck and A. Ostermann. Explicit exponential Runge–Kutta methods for semilinear parabolic problems. *SIAM Journal on Numerical Analysis* 43 (2005), 1069–1090.

[3] T. H. Hoang. Order Reduction of Exponential Runge–Kutta Methods: Non-Commuting Operators. *Advances in Applied Mathematics and Mechanics* , to appear.

## Mixed-Dimensional Modeling for Flow in Fractured Porous and Poroelastic Media

Maximilian Hörl*, Kundan Kumar†, Christian Rohde*

*University of Stuttgart, Germany
†University of Bergen, Norway
maximilian.hoerl@mathematik.uni-stuttgart.de

### Abstract

We consider fluid flow in porous and poroelastic media containing thin heterogeneities such as debris-filled fractures. When the fracture aperture is sufficiently small, it is natural to model fractures as lower-dimensional manifolds embedded in the bulk medium, leading to discrete fracture models at the Darcy scale. The fracture's material parameters, such as hydraulic conductivity and elasticity, are assumed to scale with powers of the width-to-length ratio $\varepsilon$ of the fracture. Based on a priori estimates, we rigorously derive limit models as $\varepsilon \to 0$ and identify different limit regimes [1, 2]. We illustrate our results with numerical experiments based on discontinuous Galerkin discretizations.

### References

[1] M. Hörl, K. Kumar, and C. Rohde. Fractured poroelastic media in the limit of vanishing aperture. *Submitted for publication*, 2025. doi: 10.48550/arXiv.2512.04978.

[2] M. Hörl and C. Rohde. Rigorous derivation of discrete fracture models for Darcy flow in the limit of vanishing aperture. *Netw. Heterog. Media* 19(1):114–156, 2024. doi: 10.3934/nhm.2024006.

## Unconditionally Stable Numerical Solution for the High-Order Allen–Cahn Equation

Seungyoon Kang

Department of Industrial Mathematics, National Institute for Mathematical Sciences,
Daejeon 34047, Republic of Korea
SYKANG@NIMS.RE.KR

### Abstract

We present an unconditionally stable algorithm for the Allen–Cahn (AC) equation incorporating a high-order free energy formulation. This high-order AC model enhances the preservation of interfacial dynamics and effectively suppresses noise. The proposed scheme guarantees unconditional stability, which is crucial for accurate phase transition modeling and maintaining fine structural details. To solve the governing equation efficiently, it is split into two subproblems that are solved independently. The nonlinear term is treated using a frozen coefficient approach, followed by an analytical solution. The linear term is solved using the discrete cosine transform. To demonstrate the effectiveness of the method, we perform several numerical simulations in both two- and three-dimensional domains. Thanks to its unconditional stability, the algorithm allows large time steps without compromising stability. Furthermore, we explore the distinct features of the high-order AC equation, particularly its improved capability to model phase separation under strong noise and intricate interface structures.

### References

[1] S. Kang, Y. Hwang, J. Kim, Unconditionally stable method for the high-order Allen–Cahn equation. Journal of Computational Science, (2025) 102636.

[2] J. Kim, Maximum principle preserving the unconditionally stable method for the Allen–Cahn equation with a high-order potential. Electronic Research Archive, (2025) 33(1).

## A Numerical Algorithm for Solving Singularly Perturbed Boundary Value Problem Arising in Neuronal Variability Model

Ashifa Khan

Jamia Millia Islamia (A Central University) New Delhi-110025, INDIA
ashifakhan101@gmail.com

**Abstract**

In this research, we present a robust computational approach that effectively employs an exponentially fitted mesh cubic spline for the numerical treatment of a mathematical model associated with neuronal variability. This model decisively determines the expected time required for generating an action potential in nerve cells in response to random synaptic inputs into dendrites, framed as a general boundary value problem for a singularly perturbed differential-difference equation with small shifts. We construct a highly effective numerical scheme and establish a parameter-uniform error estimate that clearly demonstrates its reliability. Through a series of test examples, we thoroughly investigate the performance of this scheme, specifically analyzing how the size of the delay argument and the coefficient of the delay term influence the solution's layer behavior. The results obtained unequivocally align with the theoretical predictions, validating the efficacy of our approach.

### References

[1] Aziz, T., Khan, A.: A spline method for second-order singularly perturbed boundary-value problems. Journal of Computational and Applied Mathematics 147(2), 445–452 (2002)

[2] Kadalbajoo, M., Sharma, K.: Numerical treatment of a mathematical model arising from a model of neuronal variability. Journal of mathematical analysis and applications 307(2), 606–627 (2005)

[3] Lange, C.G., Miura, R.M.: Singular perturbation analysis of boundary value problems for differential-difference equations. v. small shifts with layer behavior. SIAM Journal on Applied Mathematics 54(1), 249–272 (1994)

[4] Stein, R.B.: Some models of neuronal variability. Biophysical journal 7(1), 37–68 (1967)

[5] Tuckwell, H.C.: Synaptic transmission in a model for stochastic neural activity. Journal of theoretical biology 77(1), 65–81 (1979)

# Adaptive Thresholded Physics-Informed CNN for Mixed Noise Removal Using Anisotropic Diffusion

Sanjeev Kumar

Department of Mathematics Indian Institute of Technology Roorkee, Roorkee 247667, India
SANJEEV.KUMAR@MA.IITR.AC.IN

## Abstract

Many image restoration tasks are modeled using partial differential equations (PDEs) of different classes. Images are often degraded by various types of noise, and in practice, mixed noise occurs more frequently than a single type [1, 2]. This work proposes an adaptive, pixel-dependent thresholding strategy for the anisotropic diffusion equation to remove mixed speckle–Gaussian noise. In this approach, the image gradient is used to design the thresholding function, making the process adaptive to local image structures [3]. Furthermore, the potential of CNN-based solvers to handle mixed noise is explored. In this context, two CNN-based solvers—namely, the Soft-Constrained Physics-Informed Convolutional Neural Network (SPICN) and the Hard-Constrained Physics-Informed Convolutional Neural Network (HPICN)—are employed to implement the Perona–Malik model as a representative case. Additionally, the stability analysis of neural network-based approximations for such diffusion equations is investigated. The experimental results demonstrate that the proposed strategy enhances the denoising capability of the diffusion equation and that the CNN-based solvers have the potential to outperform the traditional finite difference method in image denoising tasks.

## References

[1] H. Kumar and S. Kumar, A data-driven algorithm for solving image despeckling PDE model using physics-informed convnet, Computers & Mathematics with Applications 200 (2025) 202–227.

[2] N. Namaki, M. Eslahchi, R. Salehi, The use of physics-informed neural network approach to image restoration via nonlinear pde tools, Computers & Mathematics with Applications 152 (2023) 355–363.

[3] P. Shi, Z. Zeng, T. Liang, Physics-informed convnet: Learning physical eld from a shallow neural network, Communications in Nonlinear Science and Numerical Simulation 132 (2024) 107911.

### A Compact Higher-Order Formulation for Unsteady Two-Dimensional Convection–Diffusion Systems with RK-Type Time Integration

Sumit Kumar[1,*], Pratham Singh[1], Jiten C. Kalita[1]

[1]Department of Mathematics, Indian Institute of Technology Guwahati, Assam 781039, India
*k.sumit@iitg.ac.in

**Abstract**

The work is concerned with developing a high-order compact (HOC) finite-difference method for the two-dimensional unsteady convection–diffusion equation in conjunction with the classical fourth-order Runge–Kutta (RK4) strategy for time advancement on a nine-point stencil. The fully explicit time integration enables straightforward implementation of the HOC-RK4 formulation, ensuring computational efficiency without the need to solve large linear systems. A detailed von Neumann analysis demonstrates the method's robustness under highly convective conditions. Its accuracy and versatility are further evaluated using benchmark cases that include Dirichlet, Neumann, and periodic boundary conditions. In a diffusion-dominated test of a Gaussian pulse, the method effectively captures both diffusion and anti-diffusion transport without introducing artificial oscillations. Numerical results demonstrate fourth-order spatial accuracy and stability while employing RK4 time integration for Reynolds number ($Re$) up to 7500. At high $Res$, lid-driven cavity simulations accurately resolve secondary and tertiary vortices as well as complex recirculation patterns, demonstrating their suitability for large-scale computations. We examine the flow around a diamond-shaped cylinder, where our method successfully reproduces the well-known $\alpha$ and $\beta$ phenomena in the wake at higher $Re$. In the case of the impulsively started circular cylinder, a recognized unsteady laminar benchmark, the method accurately captures the temporal development of primary, secondary, and tertiary vortices, revealing intricate wake dynamics. The technique effectively identifies sub-$\alpha$ and sub-$\beta$ phenomena, including a previously unreported simultaneous occurrence with the $\alpha$-phenomenon at $Re = 3000$, showcasing its robustness in capturing complex flow behaviors. The computed flow characteristics align closely with experimental results, and in both above scenarios, the predicted vortex patterns and symmetry breaking correspond well with reference studies.

## Thermodynamical Allen–Cahn Equation with General Polynomial Free Energy

Gyeonggyu Lee

National Institute for Mathematical Sciences, Daejeon 34047, Republic of Korea
ssoss787@naver.com

### Abstract

The Allen–Cahn equation was originally introduced to model of antiphase domain coarsening in a binary mixture in materials science. It is applied to various scientific fields such as data classification, volume reconstruction, image segmentation, multiphase flow, and biological transport networks, and dendritic crystal growth. To describe more natural and complex dynamics, we consider the thermodynamical conductivity into the Allen–Cahn equation with the general polynomial free energy. The conductivities are chosen based on the Fick's law, Werede's law, and Chapman's law, which are based on the thermodynamics. Some properties of the governing equation are analyzed based on the modified Ginzburg–Landau energy. To numerically solve the proposed governing equation, we construct a numerical scheme based on the linear stabilized splitting scheme which treats the nonlinear term explicitly and the linear term as implicitly. It is an efficient, unconditionally energy-stable, and property-preserving scheme. We numerically analyze and simulate some properties of the proposed numerical scheme in one-, two-, and three-dimensional spaces.

### References

[1] Allen, Samuel M and Cahn, John W, "A microscopic theory for antiphase boundary motion and its application to antiphase domain coarsening", *Acta metallurgica*, Vol. 27, No. 6, 1979, pp. 1085–1095

[2] Kim, Yong-Jung and Seo, Hyowon, "Model for heterogeneous diffusion", *SIAM Journal on Applied Mathematics*, Vol 81, No. 2, 2021, pp. 335–354

### Invariant Region Preserving Schemes for Mixed Hyperbolic-Parabolic SPDEs

Kristian Nekstad Maroni

Department of Mathematics University of Oslo, Norway
KRISNM@MATH.UIO.NO

#### Abstract

Mixed hyperbolic-parabolic SPDEs encapsulate a large spectrum of equations that have broad applications in various fields. When the diffusion matrix is positive semi-definite, the resulting SPDE can experience strong degeneracy, inducing non-uniqueness and shock waves. To ensure uniqueness of weak solutions, appropriate entropy conditions that allow for discontinuous solutions are required. In addition, one has to account for stochastic perturbations. It is then a nontrivial task to design numerical schemes that preserve pointwise invariant regions, where the entropy solution remains inside the same domain as the initial condition. In this talk we showcase two invariant region preserving numerical schemes that relies on an operator-splitting of the deterministic and stochastic part of the equation

$$du + \left( \sum_{i=1}^{d} \partial_{x_i} f_i(u) - \sum_{i,j=1}^{d} \partial_{x_i} \big( a_{ij}(u) \partial_{x_j} u \big) \right) dt = \sigma(u)\, dB,$$

where $a$ is a symmetric and positive semi-definite matrix, $f$ is a non-linear flux, and $\sigma(u)dB$ is a cylindrical Wiener noise. The first scheme discretizes the deterministic part with a local discontinuous Galerkin method, while the stochastic part is discretized using a Lamperti transformation combined with an explicit Runge–Kutta method. Meanwhile, for the second scheme, the deterministic part is discretized by a finite difference scheme with cross derivative correctors, while the stochastic part is discretized with an artificial barrier method combined with the Euler–Maruyama scheme. The splitting approach is justified by theoretical results that shows that semi-discrete splitting solutions converge to the unique entropy solution of more general stochastic convection-diffusion equations. Moreover, numerical experiments confirm that the numerical schemes are stable and preserve both positivity and invariant regions.

## An Alternative Model of Multicomponent Diffusion Based on a Combination of the Maxwell-Stefan Theory and Continuum Mechanics

Jiří Mikyška

Czech Technical University in Prague, Czech Republic
JIRI.MIKYSKA@FJFI.CVUT.CZ

**Abstract**

We present a theory of multicomponent mixtures which does not employ any splitting of component fluxes into convective and diffusive parts. Instead, momentum balance is formulated individually for each component in which both 1) viscous friction within a component, and 2) momentum exchange among different components, are taken into account. While the viscous friction is described using the Newtonian stress tensor, the Maxwell-Stefan theory is used to describe the momentum exchange among different components. When the viscosity is neglected, the model of ideal mixture of ideal gases leads to a hyperbolic system of conservation laws. For the non-ideal mixtures, we obtain a first-order system in a non-conservative form. A simplified version of the model is discretized using a combination of the finite volume method and the mixed-hybrid finite element method. Numerical examples are provided to show typical behavior of the solution of the model equations. This is a joint work with Jan Francu̇.

### References

[1] J. Francu̇, J. Mikyška: An alternative model of multicomponent diffusion based on a combination of the Maxwell-Stefan theory and continuum mechanics, Journal of Computational Physics, 400, Article Number, 108962, 2020, DOI: 10.1016/j.jcp.2019.108962.

## Trefftz Discontinuous Galerkin Approximation of the T-Matrix for Scattering by Periodic Layered Structures

Armando Maria Monforte

University of Pavia, Pavia, Italy
ARMANDOMARIA.MONFORTE01@UNIVERSITADIPAVIA.IT

### Abstract

We study the scattering of time-harmonic electromagnetic waves by periodic layered gratings [1], modelled by the two-dimensional Helmholtz equation with piecewise-constant relative permittivity $\varepsilon$. The periodic obstacle may include penetrable and impenetrable regions, and consists of a finite number of stacked layers. The problem is formulated on a single periodic cell using quasi-periodic boundary conditions. Radiation condition in the vertical directions is imposed through Dirichlet-to-Neumann (DtN) operators.

To efficiently treat multilayer configurations, we adopt a formulation based on the T-matrix method [3]. The global scattering problem is decomposed into boundary value problems posed on individual layers, where the field is expressed in terms of quasi-periodic modal expansions, and the T-matrix describes the mapping between incoming and outgoing wave modes on the layer boundaries. The T-matrix is approximated numerically using a Trefftz Discontinuous Galerkin (TDG) method [2], providing an efficient discretization for the computation of the layer scattering response. The resulting framework combines a mathematically consistent formulation of quasi-periodic scattering with a flexible high-frequency numerical method for wave propagation in periodic layered media.

### References

[1] A.-S. Bonnet-Bendhia and F. Starling, Guided waves by electromagnetic gratings and nonuniqueness examples for the diffraction problem. *Math. Methods Appl. Sci.*. **17**, 305-338 (1994)

[2] A. Moiola and A. M. Monforte, Trefftz Discontinuous Galerkin methods for scattering by periodic structures. *ArXiv: 2505.23216*. (2025)

[3] P. Waterman, Matrix formulation of electromagnetic scattering. *Proceedings Of The IEEE*. **53**, 805-812 (1965)

## Transport of Scalar Quantities Along Evolving Planar Curves and its Numerical Solution

Maneesh Narayanan, Michal Beneš

Department of Mathematics, Czech Technical University in Prague, Czechia
narayman@cvut.cz

### Abstract

We investigate the dynamics of scalar quantities distributed along planar curves evolved by the constrained curvature flow. Such coupled geometric and transport processes arise naturally in a wide range of physical, chemical and biological systems, including interfacial dynamics, phase transitions, and mechanobiological modeling (see [1, 2]). In this work, we formulate a geometric framework that captures the interplay between curve evolution and scalar transport at moving interfaces. The governing system of partial differential equations is derived and analyzed, highlighting structural properties such as conservation mechanisms and curvature-driven effects (see [3, 4]). For a numerical solution, we use the flowing finite volume method in space and a higher-order time solver. We further present numerical simulations that demonstrate how the constrained curvature flow influences the redistribution and long-term behavior of scalar quantities along evolving curves. The results provide insight into the mutual interaction between geometry and transport phenomena and offer a foundation for accurate and stable computational modeling of such coupled systems.

### References

[1] C. M. Elliott, H. Garcke, and B. Kovács, "Numerical analysis for the interaction of mean curvature flow and diffusion on closed surfaces," *Numerische Mathematik*, vol. 151, pp. 873–925, 2022, https://doi.org/10.1007/s00211-022-01301-3

[2] J. Escher and G. Simonett, "Qualitative properties for a system coupling scaled mean curvature flow and diffusion," *Journal of Differential Equations*, vol. 349, pp. 236–268, 2023, https://doi.org/10.1016/j.jde.2022.12.019

[3] M. Kolář and D. Ševčovič, "Mean curvature flow of closed curves evolving in two dimensional manifolds," *Japan Journal of Industrial and Applied Mathematics*, vol. 43, article 16, 2026, https://doi.org/10.1007/s13160-026-00772-3

[4] M. Beneš, M. Kolář, and D. Ševčovič, "On diffusion and transport acting on parameterized moving closed curves in space," *Mathematical Methods in the Applied Sciences*, 2025, pp. 1–15, https://doi.org/10.1002/mma.70102

# Impact of Turbulence Modeling on Hemodynamic Risk Indicators for Aortic Dissection: a Quantitative Comparison of Predicted Risk Sites with Laminar Flow Model

Anne von Nida[1], Tristan Probst[1], Franz Masseli[2], Anna Hundertmark[1]

[1]RPTU University Kaiserslautern-Landau, Germany
a.vonnida@rptu.de
[2]University Medical Center, Mainz, Germany

## Abstract

Aortic dissection is one of the most dangerous cardiovascular conditions, it demands early diagnosis and fast treatment to improve patient outcomes. The pathology is characterized by a tear in the inner layer of the aorta, allowing blood to enter and create a false lumen between the inner and middle layers of the aortic wall. Early diagnosis and patient specific risk assessment of aortic dissections pose challenges for clinical treatment and appropriate interventions. Numerical simulation based on patient specific geometries promise to support the clinical analysis and improve systematic risk assessment of aortic dissections. Since Reynolds numbers in the aorta, especially in the aortic arch reach values of 3.500 and more in physiological settings, numerical hemodynamic modeling necessitates the consideration of turbulence models. In this study, the impact of turbulence modeling based on Smagorinsky Large-Eddy-Simulation model [1] on clinically significant hemodynamic risk indicators is investigated. The main focus for this study lies in the comparison of frequently studied surface risk parameters for prediction of mechanical wall damage caused by complex flow patterns such as Oscillatory Shear Index [2]. The predictive capacity of the turbulent approach is compared against laminar flow modeling across a series of patient-specific geometries. We analyze these patterns across the ascending aorta wall surface, the aortic arch, and the proximal descending aorta wall to account for localization-specific disease patterns. To evaluate the clinical relevance of turbulence modeling, simulations are performed on geometries in pre-dissection state with known positions of the dissection localization based on patient data. This enables an assessment of the predictive accuracy of laminar and turbulence models by comparing simulated risk sites with patient-specific dissection sites, thereby demonstrating whether complex turbulence models offer added value in a clinical context.

## References

[1] S. B. Pope (2000) *Turbulent Flows*, Cambridge University Press.

[2] K. Richter, A. Hundertmark, T. Probst, P. Eulzer, and K. Lawonn (2023) *Longitudinal wall shear stress evaluation using centerline projection approach in the numerical simulations of the patient-based carotid artery*, *Comput. Methods Biomech. Biomed. Eng.*, vol. 0, no. 0, pp. 1–18. https://doi.org/10.1080/10255842.2023.2185478 DOI:10.1080/10255842.2023.2185478

## Optimal Time-Adaptivity for Parabolic Problems with applications to Model Order Reduction

Michael Feischl, Fernando Henríquez, David Niederkofler

Institute of Analysis and Scientific Computing, TU Wien, Austria
DAVID.NIEDERKOFLER@ASC.TUWIEN.AC.AT

### Abstract

Since the first optimality proofs for adaptive mesh refinement algorithms in the early 2000s, the theory of optimal mesh refinement for PDEs was inherently limited to stationary problems. The reason for this is that time-dependent problems usually do not exhibit the necessary coercive structure that is used in optimality proofs to show a certain quasi-orthogonality, which is crucial for the theory. Recently, by using a new equivalence between quasi-orthogonality and inf-sup stability of the underlying problem, it was shown that an adaptive Crank-Nicolson scheme for the heat equation is optimal under a severe step size restriction. In this work, we use this new approach towards quasi-orthogonality together with a Radau IIA method that combines the advantages of the Crank-Nicolson and implicit Euler schemes. We obtain the first adaptive time stepping method for non-stationary PDEs that is provably rate optimal with respect to number of time steps vs. approximation error. Together with a reduced basis method that leverages the Laplace transform for building tailored subspaces of reduced dimension, we obtain a very efficient method.

# A Fully Discrete Weak Galerkin Mixed Approximation for Parabolic Interface Problems on Polygonal Meshes

Amit Kumar Pal

Department of Mathematics, BITS Pilani, Hyderabad, India
p20210064@hyderabad.bits-pilani.ac.in

## Abstract

The main aim of this talk is to present a priori error analysis for the fully discrete Crank-Nicolson weak Galerkin (WG) mixed approximation for parabolic interface problems on polygonal meshes. The central idea of the WG method is to approximate the underlying differential operator, precisely the divergence operator in the weak sense. Using appropriate mixed elliptic projection operators and approximation properties of the $L^2$-projection operator, we obtain optimal-order a priori error bounds for both the solution and the flux in the $L^\infty(L^2)$-norm. A series of numerical results are presented to support the theoretical conclusions.

### The Linearized Westervelt–Pennes Bioheat System: Numerical Analysis and Simulation

J. A. Ferreira, F. Oliveira and G. Pena

University of Coimbra, CMUC, Department of Mathematics, FCTUC
Largo D. Dinis, 3000–143 Coimbra, Portugal
ferreira@mat.uc.pt, filipaholiveira1@gmail.com, goncalo.pena@uc.pt

**Abstract**

Ultrasounds have been a staple in clinical diagnostic for decades. More recently, its mechanical and thermal effects have led to its use in several medical applications, through high intensity focused ultrasounds (HIFU), see [1, 2]. We present a robust and accurate numerical methodology for a coupled system of partial differential equations arising in the modelling of HIFU. The system combines the linearized Westervelt equation for acoustic propagation,

$$p_{tt} + \alpha_1 p_t = \alpha_2 \Delta p + \alpha_3 \Delta p_t + f, \tag{24}$$

with the Pennes bioheat equation governing temperature evolution induced by acoustic absorption,

$$T_t + \gamma_1 T = \gamma_2 \Delta T + Q(p^2, p_t^2, \langle p^2 \rangle, \langle p_t^2 \rangle) + g. \tag{25}$$

The heat source in the bioheat equation is allowed to depend on the acoustic field through $p^2$ and $p_t^2$, or suitable time-averages of these functions, thus capturing relevant physical regimes.

The proposed scheme admits two equivalent interpretations: a finite difference discretization and a fully discrete, piecewise linear finite element method in space. The scheme is based on nonuniform finite differences in space and employs a uniform discretization in time. We establish second-order convergence in space for the discrete time derivative and gradient of the pressure and the discrete temperature, and first-order convergence in time, even in the presence of nonlinear heat sources driven by the acoustic pressure. The convergence analysis is based on suitable discrete counterparts of the $L^p$, $p \geq 1$, and $H^1$ norms and relies on Bramble-Hilbert type arguments, allowing us to avoid standard high-regularity assumptions on the exact solution.

We will present numerical experiments that confirm the theoretical convergence rates as well as a converge rate drop when the use less regular solutions. We also illustrate the qualitative behavior of the acoustic and thermal fields in a HIFU application.

#### References

[1] Samaneh Haddadi and Mohammad Taghi Ahmadian. Analysis of nonlinear acoustic wave propagation in HIFU treatment using Westervelt equation. *Scientia Iranica*, pages 2087–2097, September 2017.

[2] Gail ter Haar and Constantin Coussios. High intensity focused ultrasound: Physical principles and devices. *International Journal of Hyperthermia*, 23(2):89–104, 2007.

**Data-Driven Stabilization Parameters for CDR Equations on General Meshes**

Manoj Prakash, Petr Knobloch

Charles University, Prague, Czech Republic.
MANOJ@KARLIN.MFF.CUNI.CZ

**Abstract**

Numerical simulation of fluid flows frequently requires solving convection–diffusion–reaction (CDR) equations, which model the transport of quantities such as temperature, concentration, or vorticity in a moving fluid. In convection-dominated regimes—where advective transport overwhelms diffusion—standard finite element methods often suffer from spurious oscillations and unstable solutions. The Streamline Upwind/Petrov–Galerkin (SUPG) method addresses this by augmenting the standard Galerkin variational formulation with a residual-based stabilization term controlled by an element-wise parameter $\delta_K$. Consider the CDR problem

$$-\varepsilon\,\Delta u + \mathbf{b}\cdot\nabla u + cu = f \quad \text{in } \Omega, \qquad u = u_b \quad \text{on } \partial\Omega, \tag{26}$$

where $u$ is the transported variable, $\mathbf{b}$ the velocity field, $\varepsilon$ the diffusion coefficient, $c$ a reaction rate, and $f$ a source term. The SUPG method seeks $u_h \in V_h$ such that

$$a(u_h, v_h) + \sum_{K\in\mathcal{T}_h} \delta_K\big(-\varepsilon\,\Delta u_h + \mathbf{b}\cdot\nabla u_h + c\,u_h - f,\ \mathbf{b}\cdot\nabla v_h\big)_K = 0 \quad \forall\, v_h \in V_h, \tag{27}$$

where $a(\cdot,\cdot)$ is the standard Galerkin bilinear form, $v_h$ the test function, and $\delta_K > 0$ the stabilization parameter that controls the amount of artificial diffusion added along the streamlines. Selecting an appropriate value of $\delta_K$ is critical: a too small value leads to oscillatory solutions, while a too large value introduces excessive numerical diffusion. Classical choices perform poorly on most of the mesh geometries.

In this work, we extend our machine learning–based framework for computing optimal SUPG stabilization parameters to unstructured and anisotropic meshes, which are far more prevalent in practical CFD applications than the structured setting. A neural network model is trained within the SUPG framework to learn local flow and mesh features and to output element-wise optimal parameters $\delta_K$. The network is trained on a diverse range of mesh configurations, enabling it to generalize robustly to structured, unstructured, and strongly anisotropic meshes alike. Comparative numerical experiments demonstrate that the proposed ML-enhanced SUPG method yields more accurate solutions than the standard SUPG method on all tested mesh types, successfully suppressing spurious oscillations to a degree consistent with the quality obtained on structured grids. These results underscore the potential of data-driven stabilization strategies for delivering reliable, oscillation-free finite element solutions to convection-dominated problems on general computational meshes.

### Fractional SIR Model with Distributed Contacts

Bárbara Silva Rodrigues, Magda Rebelo, Paula Patrício

Center for Mathematics and Applications (NOVA Math) and Department of Mathematics,
NOVA FCT, Caparica, Portugal
BFS.RODRIGUES@CAMPUS.FCT.UNL.PT

**Abstract**

We propose a spatial epidemiological model extending the classical SIR framework by incorporating spatial dependence through a distributed contacts (DC) approach [1]. Spatially non-local interactions arising from long-range contacts are incorporated via a superdiffusive term, leading to a fractional DC–SIR model governed by the fractional Laplacian in the Riesz integral sense [2].

Motivated by empirical evidence that human mobility is more accurately characterised by superdiffusive processes than by classical diffusion, owing to long-distance movements (Lévy flights) [3], we compare the numerical results obtained from the fractional and classical diffusive DC–SIR models and analyse spatial dependence through variations in the fractional order $\alpha$, interpreted as a contact distribution parameter.

Numerical simulations are performed using a method-of-lines approach, combining finite-difference spatial discretisation with a third-order Runge–Kutta time integration scheme. The basic reproduction number $R_0$ is defined and numerically approximated, acting as a threshold for disease extinction or persistence. The convergence order of the numerical methods used to approximate the solution of the DC-SIR model and $R_0$ is illustrated through numerical examples.

**References**

[1] C. Cosner. *Models for the effects of host movement in vector-borne disease systems*, Math. Biosci. 270 (2015), 192–197.

[2] D. Brockmann. *Human mobility and spatial disease dynamics*, Rev. Nonlinear Dyn. Complex. 2 (2009), 1–24.

[3] D. Brockmann, L. Hufnagel, T. Geisel. *The scaling laws of human travel*, Nature 439.7075 (2006), 462–465.

### Hybrid-Dimensional Models for Fluid-Filled Fractured Porous Media

Iryna Rybak, Linheng Ruan, Paula Strohbeck

University of Stuttgart, Germany
IRYNA.RYBAK@IANS.UNI-STUTTGART.DE

#### Abstract

Fractured porous media appear in a wide range of environmental settings and industrial applications such as groundwater aquifers in bedrocks, contaminant transport in subsurface, nuclear waste disposal, flow and reaction in membranes. In these applications, fractures are thin inclusions in the surrounding porous media that often dominate flow and transport processes through the overall coupled system. To efficiently simulate such processes, hybrid-dimensional formulations, where the fractures are modelled as inclusions of co-dimension one, are developed. In most mathematical models, fluid flow in fractures and the surrounding porous matrix is described using Darcy-based formulations in both domains.

In this presentation, we deal with fluid-filled fractures embedded in a porous medium and advance the hybrid-dimensional Stokes–Darcy model developed in our previous work [1]. Here, flow through the porous material is described by Darcy's law, while viscous flow within the fractures is modelled by the Stokes equations. The model takes surface roughness of the fracture/matrix interfaces into account and allows for arbitrary flow directions. We prove the well-posedness of the resulting hybrid-dimensional problem, develop efficient numerical methods for its solution, and compare the proposed model with the ones existing in the literature.

#### References

[1] I. Rybak and S. Metzger, "A dimensionally reduced Stokes–Darcy model for fluid flow in fractured porous media", *Appl. Math. Comp.*, **384**, 125260, 2020. doi:10.1016/j.amc.2020.125260

## A Posteriori Error Estimation of the Immersed Weak Galerkin Method for Parabolic Problems with Non-Smooth Coefficients

Jhuma Sen Gupta

Assistant Professor, Department of Mathematics, BITS Pilani Hyderabad, India
jhuma@hyderabad.bits-pilani.ac.in

### Abstract

This talk aims to present a posteriori error estimation of the non-conforming immersed weak Galerkin (WG) method for parabolic problems with non-smooth coefficients, called the interface problems. These type of problems arise in a wide range of applications which includes fluid mechanics, materials science, biological sciences and many others. The key idea of the WG method is to approximate the underlying differential operator in the weak sense. We study a fully discrete scheme obtained by coupling the implicit backward Euler in time and the immersed WG method in space. Using the time-dependent Helmholtz decomposition of the error, we derive a reliable a posteriori error bound in the $L^2(H^1)$-norm without directly relying on the energy argument. Numerical results are reported to validate the theoretical findings.

## Data-Driven PDE Models of Tumor–Immune Dynamics in Tumor Micro-Environment

Leili Shahriyari

Department of Mathematics and Statistics, University of Massachusetts Amherst,
Amherst, MA, USA
LSHAHRIYARI@UMASS.EDU

### Abstract

We develop and analyze a family of data-driven partial differential equation (PDE) models that resolve how spatial heterogeneity and tumor–immune interactions shape disease progression across three cancer types: breast cancer [1], colon cancer [2], and osteosarcoma [3]. Building on calibrated kinetic (ODE/QSP) descriptions of key cellular and cytokine networks, we extend the reaction terms to reaction–diffusion–advection PDE systems to capture localization, transport, and proximity-driven regulation inside the tumor microenvironment. In breast cancer, we use spatio-temporal data from the MMTV-PyMT mouse model to study how immune infiltration patterns and boundary influx influence evolving tumor architecture and immune depletion. In colon cancer, we investigate spatial immune organization through a PDE system coupled with tissue mechanics (domain deformation via elasticity), enabling in silico experiments on how microenvironmental structure alters competitive/cooperative signaling. For osteosarcoma, we incorporate biomechanical effects (e.g., porous-media style growth and stress) and immune-context stratification to quantify progression and treatment response across distinct immune phenotypes.

Across these case studies, we emphasize parameter identification from experimental/omics-derived cell abundance estimates, sensitivity/uncertainty analyses, and predictive simulations that support patient- or cluster-specific hypotheses for therapy design and optimization.

### References

[1] N. M. Mirzaei, Z. Tatarova, et al., and L. Shahriyari, "A PDE Model of Breast Tumor Progression in MMTV-PyMT Mice," *Journal of Personalized Medicine*, 12:807, 2022. doi: 10.3390/jpm12050807.

[2] N. M. Mirzaei, W. Hao, and L. Shahriyari, "Investigating the spatial interaction of immune cells in colon cancer," *iScience*, 26(5):106596, 2023. doi: 10.1016/j.isci.2023.106596.

[3] Y. Hu, N. M. Mirzaei, and L. Shahriyari, "Bio-Mechanical Model of Osteosarcoma Tumor Microenvironment: A Porous Media Approach," *Cancers*, 14(24):6143, 2022. doi: 10.3390/cancers14246143.

### A Robin–Robin Domain Decomposition Method for Stokes–Darcy Problems with Generalised Coupling Conditions

Paula Strohbeck[1], Marco Discacciati[2], Iryna Rybak[1]

[1]University of Stuttgart, Germany
[2]Loughborough University, United Kingdom
PAULA.STROHBECK@IANS.UNI-STUTTGART.DE

**Abstract**

Coupled free-flow and porous-medium models appear in numerous environmental, industrial, and medical applications. Such flow problems are usually described by Stokes equations in the free-flow domain and Darcy's law in the porous medium. The choice of interface conditions plays a crucial role in accurately capturing the interaction between free flow and porous-medium flow. In this talk, we consider a generalisation of the classical Beavers–Joseph condition. Unlike the classical formulation, which is restricted to flows parallel to the fluid-porous interface, the generalised conditions are applicable for arbitrary flow directions.
To solve the resulting Stokes–Darcy models efficiently, we develop and analyse a Robin–Robin domain decomposition method for non-overlapping subdomains [1]. Fourier analysis is used to determine optimal weights in the Robin interface conditions. We investigate the efficiency and robustness of the proposed domain decomposition method and present numerical experiments that confirm the theoretical convergence results.

#### References

[1] P. Strohbeck, M. Discacciati, and I. Rybak, "Optimized Schwarz method for the Stokes–Darcy problem with generalized interface conditions", *J. Comput. Appl. Math.*, **477**, 117141, 2026. doi:10.1016/j.cam.2025.117141

## Boundary Control and Data Assimilation for Navier–Stokes Flows with Applications to Hemodynamic

Jorge Tiago

Instituto Superior Técnico, University of Lisbon, Portugal
jorge.tiago@tecnico.ulisboa.pt

### Abstract

In this talk we will present recent results on optimization problems associated with fluid flows governed by the Navier–Stokes equations, with applications to blood flow modeling.

First, we consider an optimal boundary control problem for the steady Navier–Stokes equations with mixed boundary conditions. We addressed the issue of correctly adjusting boundary conditions at artificial boundaries in truncated domains. In particular we study the minimization of a velocity tracking functional with controls acting on part of the boundary, combined with non-standard outflow conditions of the type directional do-nothing.

We then discuss a data-assimilation framework for time-dependent blood flow simulations, where boundary data are optimized using time-series observations. The proposed approach reduces the discrepancy between simulated and observed data and improves the reconstruction of relevant flow quantities such as velocity, pressure, and wall shear stress, even in the presence of noise.

### References

[1] Anwar, M. Adnan; Tiago, Jorge (2025) Time-dependent strategy for improving aortic blood flow simulations with boundary control and data assimilation. Applied Mathematics and Computation: Volume 493, 15, 129266. https://doi.org/10.1016/j.amc.2024.129266

[2] Nogueira, Pedro M.; Silvestre, Ana Leonor; Tiago, Jorge (2025) Steady Navier–Stokes Equations with Regularized Directional Do-Nothing Boundary Condition: Optimal Boundary Control for a Velocity Tracking Problem. Applied Mathematics and Optimization. Volume 91, 24. https://doi.org/10.1007/s00245-024-10216-4

## Criteria for Asymptotic Stability of Three-Term Linear Difference Equation with Two Parameters

Petr Tomášek

Faculty of Mechanical Engineering, Brno University of Technology, Brno, Czech Republic
Petr.Tomasek@vut.cz

### Abstract

Asymptotic stability analysis of linear difference equations with constant coefficients is closely connected to the study of asymptotic stability of numerical methods for initial value problems in ordinary differential equations. We investigate the asymptotic stability of the three-term difference equation

$$y(n+k)+\alpha y(n+k-m)+\beta y(n)=0, \quad n=0,1,2,\dots,$$

where $\alpha,\beta\in\mathbb{C}$, $k,m\in\mathbb{N}$, $k>m\geq 1$. Its asymptotic stability is characterized by all roots of the associated characteristic polynomial lying inside the unit circle. The talk surveys known results, presents graphical stability regions for selected real parameters, and comments on approaches to asymptotic stability analysis.

### References

[1] Čermák, J., Fedorková, L., Jánský, J., On moduli and arguments of roots of complex trinomials. Pacific Journal of Mathematics, 2024, vol. 332, no. 1, 39-67.

[2] Čermák, J., Tomášek, P., On delay-dependent stability conditions for a three-term linear difference equation. Funkcialaj Ekvacioj-Serio Internacia, 2014, vol. 57, no. 1, 91-106.

[3] Tomášek, P., Jánský, J., On complex trinomial roots distribution. Mathematics for Applications, 2023, vol. 12, no. 1, 49-58.

# Use of Repeated Integration in Efficient Algorithms for Numerical Analysis of Selected Problems of Computational Mechanics

Jiří Vala [1], Katarína Tvrdá [2]

[1] Brno University of Technology, Faculty of Civil Engineering,
Institute of Mathematics and Descriptive Geometry, Brno, Czech Republic
Jiri.Vala@vut.cz

[2] Slovak University of Technology, Faculty of Civil Engineering,
Institute of Structural Mechanics, Bratislava, Slovak Republic
katarina.tvrda@stuba.sk

## Abstract

Standard numerical methods for numerical analysis of such problems as structural vibrations or elastic wave propagation in computational mechanics are frequently inaccurate or rather complicated; cf. [1, 2]. Thus, it is reasonable to seek alternative approaches for handling such problems. A promising computational technique, namely for all Cauchy initial value problems, comes from the Cauchy formula for repeated integration, introduced by [3, 4]. For computation of repeated integrals and derivatives, appropriate modifications of Newton-Cotes formulae can be developed, as well as those for Lobato, Radau, etc., quadrature ones; see [5]. In general, the explicit methods of Runge-Kutta type, referring to the Butcher series by [6], as an algebraic tool for analysing solutions to ordinary differential equations or their systems, including approximate solutions, can be combined with repeated integration effectively. For partial differential equations of evolution, this approach is applicable, too. It can work, e.g., with the finite element technique from [2] in the 3-dimensional Euclidean space, which converts the original problem to the analysis of a sparse system of ordinary differential equations. In addition to existence and convergence considerations, a practical comparison of various numerical approaches will be presented.

## References

[1] Vala, J. On some composite schemes of time integration in structural dynamics. *Proc. Programs and Algorithms of Numerical Mathematics (PANM)* 19 in Hejnice (2018), Institute of Mathematics CAS, Prague, 2019, 159–168.

[2] Kolman, R., Okrouhlík, M., Kruisová, A. A review of temporal and spatial dispersions of linear and quadratic finite elements in linear elastic wave propagation problems. *Proc. Est. Acad. Sci.* 73 (2024), 279–316.

[3] Tvrdá, K., Minárová, M. Computation of definite integral over repeated integral. *Tatra Mt. Math. Publ.* 75 (2018), 1–14.

[4] El Haddad, R. Repeated integration and explicit formula for the $n$-th integral of $x^m \ln x^{m'}$. *Open J. Mat. Sci.* 6 (2022), 51–75.

[5] Tvrdá, K., Novotný, P. Modification of Newton-Cotes formulas for computation of repeated integrals and derivatives. *Czech. Math. J.* 73 (2023), 1175–1188.

[6] Butcher, J. C. *B-Series, Algebraic Analysis of Numerical Methods.* Springer, Cham, 2021.

## Stable Long-Horizon Prediction of Parametric PDE Solution with Latent-Tensor Vision Transformers

Domagoj Vlah

University of Zagreb Faculty of Electrical Engineering and Computing, Zagreb, Croatia
DOMAGOJ.VLAH@FER.HR

### Abstract

This is a joint work with Iva Mikuš and Boris Muha (University of Zagreb). We study deep-learning surrogates for time-dependent parametric PDEs in an autoregressive rollout setting, where long-horizon accuracy is limited by error accumulation. We propose a parameter-aware encoder–transformer–decoder architecture: a fully convolutional encoder compresses each snapshot to a latent tensor, a vision transformer advances it in time, and a decoder reconstructs the fields. PDE parameters are injected at multiple stages (via FiLM-type modulations and a parameter token), and spatial information is provided through coordinate channels. Training uses short-window scheduled sampling to mitigate exposure bias. On a parametric advection–diffusion–reaction benchmark and on 2D Navier–Stokes flow past a cylinder (joint velocity–pressure prediction), we obtain improved rollout accuracy over latent-vector ROM baselines and plain full-field transformer models, with fewer trainable parameters.

### References

[1] Alexey Dosovitskiy, Lucas Beyer, Alexander Kolesnikov, Dirk Weissenborn, Xiaohua Zhai, Thomas Unterthiner, Mostafa Dehghani, Matthias Minderer, Georg Heigold, Sylvain Gelly, Jakob Uszkoreit, Neil Houlsby, *An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale*, arXiv preprint, 2020.

[2] Samy Bengio and Oriol Vinyals and Navdeep Jaitly and Noam Shazeer, *Scheduled Sampling for Sequence Prediction with Recurrent Neural Networks*, arXiv preprint, 2015.

[3] Alberto Solera-Rico, Carlos Sanmiguel Vila, Miguel Gómez-López, Yuning Wang, Abdulrahman Almashjary, Scott T. M. Dawson, Ricardo, Vinuesa, *$\beta$-Variational autoencoders and transformers for reduced-order modelling of fluid flows*, Nature Communications 15, Article number: 1361 (2024)

### Evolutionary Equations with State-Dependent Delay

Bernhard Aigner

TU Bergakademie Freiberg, Institute for Applied Analysis, Germany
BERNHARD.AIGNER@DOKTORAND.TU-FREIBERG.DE

#### Abstract

In [3] well-posedness in $\mathrm{H}^1(0,T;H)$ of state-dependent ODEs of the form

$$\dot{x}(t) = F(t, x_{(t)}) \quad t \in [0,T]$$
$$x_{(0)} = \Phi$$

was established for Lipschitz-continuous prehistories $\Phi \in \mathrm{H}^1(-h,0;H)$ (where $H$ is some Hilbert space) and almost uniformly Lipschitz-continuous forcing terms $F\colon [0,T] \times \mathrm{L}^2(-h,0;H) \to H$. The proof utilizes a Hilbert space projection argument and exponentially weighted Bochner–Lebesgue-spaces to force contractivity of a fixed point map. This approach can be generalized to evolutionary partial differential equations in the sense of R. Picard, that is, to equations of the form

$$\big(\partial_t M(\partial_t) + A\big)u(t) = f(t) \quad t \in \mathbb{R},$$

where $A\colon H \supseteq \mathrm{dom}(A) \to H$ is an m-accretive (unbounded) linear operator and $M\colon \mathbb{C}_{\mathrm{Re}>\nu} \to \mathcal{L}_{\mathrm{b}}(H)$ is a material law. This regime is general enough to capture a vide variety of examples, including classical PDEs (heat, wave and Maxwell's equations), examples from semigroup theory, port-Hamiltonian systems, as well as equations featuring fractional derivatives and convolutions (in time) with bounded operators.
The corresponding generalized initial value problem — that stems from a distributional formulation — is

$$\big(\partial_t M(\partial_t) + A\big)u(t) = f(t) + F\big(t, u_{(t)}\big) \quad t \in \mathbb{R}.$$

Local well-posedness (in the sense of weak solutions) of this equation similarly requires prehistories in $\mathrm{H}^1$ with bounded derivative and a Lipschitz-property, but technical reasons additionally mandate a regularity increasing right-hand side and a consistency condition.

#### References

[1] *B. Aigner* and *M. Waurick*, "Evolutionary equations with state-dependent delay", Preprint, arXiv:2511.14883 (2025) — accepted for publication in Nonlinear Anal.

[2] *B. Aigner* and *M. Waurick*, "A quick guide to ordinary state-dependent delay differential equations", Preprint, arXiv:2410.20613 (2024) — accepted for publication in Oper. Theory Adv. Appl.

[3] *J. Frohberg* and *M. Waurick*, "State-dependent Delay Differential Equations on $\mathrm{H}^1$", J. Differ. Equations 410, 737–771 (2024)

### The Dirichlet Problem for the Laplacian in Lipschitz Domains

Amrouche Chérif

Université de Pau et des Pays de l'Adour, Pau, France
CHERIF.AMROUCHE@UNIV-PAU.FR

#### Abstract

We are interested here in the homogeneous and non-homogeneous Dirichlet problem for the Laplacian in bounded Lipschitz domains. Although it has been extensively studied by many authors, we would like to return to a number of fundamental questions and known results, such as the traces and the **maximal regularity** of solutions. First, to treat non-homogeneous boundary conditions, we rigorously define the notion of traces for non regular functions. This approach replaces the non-tangential trace notion that has dominated the literature since the 1980s. We identify a functional space

$$E(\nabla;\,\Omega) \;=\; \Big\{ v \in H^{1/2}(\Omega);\ \nabla v \in [\boldsymbol{H}^{1/2}(\Omega)]' \Big\}, \tag{28}$$

which satisfies the embeddings $H^{1/2}_{00}(\Omega) \hookrightarrow E(\nabla;\,\Omega) \hookrightarrow H^{1/2}(\Omega)$. The trace operator is well-defined and continuous from $E(\nabla;\,\Omega)$ into $L^2(\Gamma)$, leading to a new characterization of $H^{1/2}_{00}(\Omega)$ as the kernel of this operator. Second, we address the regularity of solutions to the Laplace equation with homogeneous Dirichlet conditions. Using specific equivalent norms in fractional Sobolev spaces and Grisvard's results for polygons and polyhedral domains, we prove that maximal regularity $H^{3/2}$ holds in any bounded Lipschitz domain $\Omega$, for all right-hand sides in the dual of $H^{1/2}_{00}(\Omega)$. This conclusion contradicts the prevailing claims in the literature since the 1990s. Third, we describe some criteria which establish new uniqueness results for harmonic functions in Lipschitz domains. In particular, we show that if $u \in H^{1/2}(\Omega)$ or $u \in W^{1,2N/(N+1)}(\Omega)$, with $N \geq 2$, is harmonic in $\Omega$ and vanishes on $\Gamma$, then $u \equiv 0$. These criteria play a central role in deriving regularity properties. Finally, we revisit the classical Area Integral Estimate of Dahlberg, and of Kenig, Pipher, and Verchota. For a harmonic function $u$ in $\Omega$ vanishing at some interior point, the estimate asserts

$$\int_\Gamma |u|^2 d\sigma \leq C \int_\Gamma |S(u)|^2 d\sigma \simeq C \int_\Omega \varrho |\nabla u|^2 dx, \tag{29}$$

where $S(u)$ is the area integral of $u$ and $\varrho$ is the distance to the boundary. Using Grisvard's work in polygons and an explicit function given by Nečas, we show that this inequality cannot hold in its stated form.

#### References

[1] C. Amrouche and M. Moussaoui. Maximal regularity of Dirichlet problem for the Laplacian in Lipschitz domains, *ArXiv* 2025, https://arxiv.org/abs/2509.08543

[2] C. Amrouche and M. Moussaoui. On the traces of harmonic functions $H^{1/2}$ and $H^{3/2}$ in Lipschitz domains, https://arxiv.org/abs/2601.05610

[3] B.E.J. Dahlberg, C.E. Kenig, J. Pipher and G.C. Verchota. Area integral estimates for higher order elliptic equations and systems. *Ann. Inst. Fourier*, **47-5**, 1425–1461, (1997).

[4] P. Grisvard.: Elliptic Problems in Nonsmooth Domains, Pitman, Boston, (1985).

[5] D. Jerison and C.E. Kenig. The Inhomogeneous Dirichlet Problem in Lipschitz Domains, *J. Funct. Anal.* **130**, 161–219, (1995).

[6] J.L. Lions and E. Magenes. *Problèmes aux limites non-homogènes et applications*, Vol. 1, Dunod, Paris, (1969).

[7] J. Nečas. *Direct methods in the theory of elliptic equations.* Springer Monographs in Mathematics. Springer, Heidelberg, (2012).

### Energy Consistent Hyperbolic Approximations for Some Higher Order PDEs

Rahul Barthwal

Institute for Applied Analysis and Numerical Simulation
University of Stuttgart, Stuttgart, Germany
RAHUL.BARTHWAL@MATHEMATIK.UNI-STUTTGART.DE

**Abstract**

We introduce novel energy-consistent hyperbolic relaxation models for approximating the solutions of third-order and fourth-order nonlinear PDEs. In particular, we first propose a hyperbolic and hyperbolic-parabolic system for a class of diffusive-dispersive PDEs using a novel relaxation approach. We derive an explicit energy structure of the proposed system, which converges to the energy (Lyapunov) functional of the original equation when relaxation parameters vanish. This, in particular, demonstrates that the relaxation system preserves the key structural properties of the underlying model. Relying on the relative entropy method, we prove the convergence of solutions of these approximate systems to the solutions of the diffusive-dispersive equations [2]. We then show the scope of this approach to a highly complex fourth-order thin-film type equation and propose a novel first-order system approximating the solutions of the fourth-order thin-film/ Cahn-Hilliard type equation in arbitrary space dimensions. Again, by utilizing the relative entropy framework, we succeed in proving the convergence of the weak entropy solutions of the first-order system to the sufficiently smooth solution of the original equation [1]. Some test cases for a variety of physical PDEs are provided in the end to validate the analysis and the scope of our approach.

This is a joint work with **Christian Rohde** (University of Stuttgart, Stuttgart, Germany) and **Firas Dhaouadi** (INRIA Bordeaux, France).

**References**

[1] R. Barthwal, F. Dhaouadi, and C. Rohde. Energy consistent hyperbolic approximation for a class of fourth-order degenerate partial differential equations. *In Preparation.*

[2] R. Barthwal, F. Dhaouadi, and C. Rohde. On hyperbolic approximations for a class of dispersive and diffusive-dispersive equations. *arXiv preprint arXiv:2512.04882*, 2025.

### Fourth-Order Diffusion Equations and Rényi Entropies

Mario Bukal

University of Zagreb, Croatia
MARIO.BUKAL@FER.HR

### Abstract

In this talk we discuss the asymptotic behavior of strong solutions to a family of nonlinear fourth-order diffusion equations [1, 5]

$$\partial_t u = -\frac{1}{p-1}\left(u^p\left(u^{p-1}-1\right)_{xx}\right)_{xx}, \quad x \in \mathbb{R},\ t > 0,$$

with particular focus on the thin-film equation

$$\partial_t u = -(u u_{xxx})_x,$$

which is obtained for $p = 3/2$. The method builds on the framework introduced by Carrillo and Toscani [4] for second-order diffusion equations — by introducing a time-dependent rescaling that preserves the second moment, we establish sharp convergence rates toward the steady state in terms of the relative Rényi entropy. Compared to rates derived from the dissipation of the classical relative entropy [3, 5], this approach yields improved estimates at early and intermediate times, and consequently a sharper convergence in the $L^1$-norm [2]. The method formally applies to the family of fourth-order equations, including the well-known Derrida-Lebowitz-Speer-Spohn (DLSS) equation ($p = 1$), and can be rigorously justified for strong solutions of the thin-film equation.

### References

[1] M. Bukal. A family of fourth-order $q$-logarithmic equations. *J. Math. Anal. Appl.* 438 (2016), 142-161.

[2] M. Bukal. Improved equilibration rates to self-similarity for strong solutions of a thin-film and related evolution equations. *Preprint, arXiv:* 2511.06553 (2025).

[3] J. A. Carrillo, G. Toscani. Long-time asymptotics for strong solutions of the thin film equation. *Comm. Math. Phys.* 225 (2002), 551-571.

[4] J. A. Carrillo, G. Toscani. Renyi entropy and improved equilibration rates to self-similarity for nonlinear diffusion equations. *Nonlinearity* 27 (2014), 3159-3177.

[5] D. Matthes, R. McCann, G. Savaré. A family of nonlinear fourth-order equations of gradient flow type. *Comm. Partial Differential Equations* 34 (2009), 1352-1397.

**Analysis of Anisotropic and Heterogeneous Models with Applications to Complex Materials**

Kateryna Buryachenko

Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany
BURYACHENKO@WIAS-BERLIN.DE

**Abstract**

We focus on developing new mathematical methods for the analysis of nonlinear processes in multiphase structures, modeled by elliptic and parabolic partial differential equations with anisotropic and heterogeneous behavior. These models describe complex phenomena in materials such as multiphase solid and liquid materials, porous media, semiconductor devices like Organic Light-Emitting Diodes (OLEDs), etc. Main attention will be given to the following models: evolution anisotropic porous medium equation:

$$u_t = \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \left(|u|^{m_i - 1} u_{x_i}\right) + f,$$

$$1 - \frac{2}{n} < m_1 \leq m_2 \leq \dots \leq m_n < \bar{m} + \frac{2}{n},\ \bar{m} = \frac{1}{n} \sum_{i=1}^{n} m_i,\ f \in L^1(\Omega_T),\ \Omega_T = \Omega \times (0, T),$$

and anisotropic $p-$ Laplace equation:

$$\sum_{i=1}^{n} \frac{\partial}{\partial x_i} \left(\left(|\nabla u|^{p_i - 2}\right) u_{x_i}\right) = f \in L^1(\Omega),$$

$$\frac{2n}{n+1} < p_1 \leq p_2 \leq \dots \leq p_n < 2 + \frac{k}{n},\ k = n(\bar{p} - 2) + \bar{p},\ \bar{p} = \frac{1}{n} \sum_{i=1}^{n} p_i.$$

We develop an approach ([1]) that eliminates the need to handle degenerate and singular cases separately. This approach provides potential estimates for solutions of anisotropic equations and, as a result, proves the regularity result (local continuity) of weak solutions for all cases.

## References

[1] K. Buryachenko, I. Skrypnik I, Local Continuity and Harnack's Inequality for Double-Phase Parabolic Equations.*Potential Anal* **56** (2022), no (1), 137–164.

## Criteria for Controllability of Semilinear Measure-Driven Evolution Systems with Impulses and Nonlocal Inclusions

Akhilesh Verma and Jaydev Dabas

Department of Applied Mathematics and Scientific Computing,
Indian Institute of Technology Roorkee, Roorkee, 247667, India.
JAYDEV@AS.IITR.AC.IN

### Abstract

We establish definitive criteria for the complete controllability of semilinear, measure-driven evolution systems in Banach spaces that incorporate impulsive effects and nonlocal inclusions. Our method boldly combines the Hausdorff measure of noncompactness with a Mönch-type fixed point argument, effectively circumventing any compactness assumptions. The conditions we provide are explicit, easily verifiable, and directly applicable to partial differential equations (PDEs). An illustrative example of a PDE unequivocally demonstrates how our abstract theory translates into concrete inequalities. This work decisively presents the controllability result for measure-driven systems that include both impulsive effects and nonlocal inclusions, significantly extending and unifying the existing controllability theory within a groundbreaking framework.

### References

[1] M. Cichoń, B.R. Satco, Measure differential inclusions between continuous and discrete, Adv. Differ. Equ. 2014 (2014) 1-18.

[2] Y. Cao, J. Sun, On existence of nonlinear measure driven equations involving non-absolutely convergent integrals, Nonlinear Anal. Hybrid Syst. 20 (2016) 72–81.

[3] A. Verma, J. Dabas, Controllability of impulsive differential inclusions with nonlocal condition driven by hemivariational inequalities, J. Differ. Equ. 447 (2025) 113622.

### Oseen Resolvent System with Traction Boundary Conditions

Paul Deuring

Laboratoire de Mathématiques Pures et Appliquées Joseph Liouville (LMPA),
Université du Littoral Côte d'Opale, Calais, France
PAUL.DEURING@UNIV-LITTORAL.FR

**Abstract**

Let $\Omega$ denote an exterior domain in $\mathbb{R}^3$ with connected $C^2$-boundary denoted by $\partial\Omega$. Let $p \in (1,\infty)$, $\tau \in \mathbb{R}\backslash\{0\}$ and $\lambda \in \mathbb{C}\backslash(-\infty, 0]$. Consider the Oseen resolvent system

$$-\Delta u + \tau\, \partial_1 u + \lambda u + \nabla\pi = f, \;\; \operatorname{div} u = 0 \quad \text{in } \Omega, \tag{30}$$

under the Neumann-type boundary conditions on $\partial\Omega$:

$$\sum_{k=1}^{3} (\partial_j u_k + \partial_k u_j - \delta_{jk}\, \pi - \tau\, \delta_{1k}\, u_j/2)\, n_k^{(\Omega)} = b_j \tag{31}$$

for $1 \le j \le 3$, $f \in L^p(\Omega)^3$ and $b \in W^{1-1/p,\,p}(\partial\Omega)^3$. Here $n^{(\Omega)}$ denotes the outward unit normal to $\Omega$. We look for solutions $(u, \pi)$ to (30), (31) with $u \in W^{2,p}(\Omega)^3$, $\pi \in D^{1,p}(\Omega)$ and $u(x) \to 0$ for $|x| \to \infty$, where $D^{1,p}(\Omega)$ denotes the set of all functions $\sigma \in W^{1,1}_{loc}(\Omega)$ such that $\nabla\sigma \in L^p(\Omega)^3$. (Functions from this set belong to $W^{1,p}(B_R \cap \Omega)$ for any $R > 0$ with $\mathbb{R}^3\backslash\Omega \subset B_R$.) We further require that at least one of the two side conditions $\int_{\partial\Omega} u \cdot n^{(\Omega)}\, do_x = 0$ or $\pi \in L^s(\Omega)$ for some $s \in (1,\infty)$ are fulfilled. Moreover we are interested in the estimate

$$|\lambda|\, \|u\|_p + \|u\|_{2,p} + \|\nabla\pi\|_{1,p} \le \mathfrak{C}\, (\|f\|_p + |\lambda|^{1/2-1/(2p)}\, \|b\|_{1-1/p,\,p}) \tag{32}$$

for $f$ and $b$ as above, provided that either $(\Im\lambda)^2/\tau^2 + \Re\lambda \ge \Lambda_0,\; |\lambda| < \Lambda_1$ or $|\lambda| \ge \Lambda_1,\; |\arg(\lambda)| \le \vartheta$, where $\vartheta \in [0,\pi),\;\; \Lambda_0, \Lambda_1 \in (0,\infty)$ with $\Lambda_0 < \Lambda_1$, The constant $\mathfrak{C}$ in (32) should depend only on $\Omega$, $\tau$, $\vartheta$, $\Lambda_0$ and $\Lambda_1$.

It turns out that solutions to this boundary value problem exist and in the case $\int_{\partial\Omega} u \cdot n^{(\Omega)}\, do_x = 0$ are unique and verify (32). If it is instead required that $\nabla\pi \in L^s(\Omega)^3$ for some $s > 1$, such a result holds only under certain restrictions. In particular the condition $p > 3/2$ must be imposed.

## Navier–Stokes–Cahn–Hilliard System in a 3D Perforated Domain with Free Slip and Source Term: Existence and Homogenization

Amartya Chakrabortty[1], Haradhan Dutta[2], Hari Shankar Mahato[2]

[1]Fraunhofer ITWM, Kaiserslautern, Germany
[2]Department of Mathematics, Indian Institute of Technology Kharagpur, India
HDUTTA1412@KGPIAN.IITKGP.AC.IN

**Keywords:** Navier–Stokes–Cahn–Hilliard system; periodic homogenization; perforated domain.

**Abstract**

We study a diffuse–interface model for a binary incompressible mixture in a periodically perforated porous medium, described by a time–dependent Navier–Stokes–Cahn–Hilliard (NSCH) system posed on the pore domain $\Omega_p^\varepsilon \subset \mathbb{R}^3$. The microscopic model involves a variable viscosity tensor, a non–conservative source term in the Cahn–Hilliard equation, and mixed boundary conditions consisting of no–slip on the exterior boundary and Navier slip with zero tangential stress on the surfaces of the solid inclusions. The capillarity strength $\lambda^\varepsilon > 0$ depends on the microscopic scale $\varepsilon > 0$.

For each fixed $\varepsilon > 0$, we establish the existence of weak solutions on a finite time interval $(0, T)$ together with a priori estimates that are uniform with respect to $\varepsilon$ (and $\lambda^\varepsilon$). A major analytical difficulty stems from the presence of the source term, which breaks mass conservation and prevents a direct application of the classical Poincaré–Wirtinger inequality. This issue is resolved through refined estimates for the spatial mean combined with suitable extension and restriction operators adapted to the perforated geometry.

We then investigate the asymptotic behavior of the system as $\varepsilon \to 0$ and perform a rigorous periodic homogenization using the periodic unfolding method. The uniform estimates provide the compactness required to identify the macroscopic limit fields and to pass to the limit in the nonlinear coupling terms, yielding effective tensors that encode the influence of the microscopic geometry. Depending on the limit value $\lambda = \lim_{\varepsilon\to 0} \lambda^\varepsilon \in [0, +\infty)$, two qualitatively different homogenized models emerge. In the vanishing capillarity regime $\lambda = 0$, the limit system is of Stokes–Cahn–Hilliard type and exhibits no macroscopic convection or phase advection, corresponding to creeping–flow behavior. In contrast, when $\lambda \in (0, +\infty)$, the homogenized equations retain nonlinear convection and advective transport, leading to a genuine Navier–Stokes–Cahn–Hilliard structure at the macroscopic scale.

Finally, we prove the convergence of the microscopic free energy towards a homogenized energy functional satisfying a corresponding dissipation law.

### References

[1] A. Chakrabortty, H. Dutta, and H. S. Mahato, Navier–Stokes–Cahn–Hilliard system in a 3D perforated domain with free slip and source term: Existence and homogenization, arXiv preprint, 2025.

## Something Other than the Usual Kind of Instability and Spatial Patterns

Jan Eisner

University of South Bohemia in České Budějovice, České Budějovice, Czechia
JEISNER@PRF.JCU.CZ

### Abstract

We introduce non-standard notions of instability and spatial patterns and discuss their robustness. In fact, these notions correspond to what is really usually done in numerical computations or in a laboratory. We describe situations when our patterns evolve due to the newly introduced instability of the basic homogeneous steady state even if it is stable and even if heterogeneous stationary solutions do not exist.

This is a joint work with Prof. M. Kučera (Institute of Mathematics, Czech Academy of Sciences, Prague, Czechia), V. Klika (Czech Technical University, Prague, Czechia) and M. Fencl (University of West Bohemia, Pilsen, Czechia).

### References

[1] M. Kučera, V. Klika, M. Fencl and J. Eisner, A new concept of instability and spatial patterns. *Nonlinear Analysis: Real World Applications*, 87:104445, 2026.

### The Vortex Filament Equation in the Energy-Variational Framework

Thomas Eiter

Freie Universität Berlin, Germany
Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany
THOMAS.EITER@WIAS-BERLIN.DE

**Abstract**

Strongly localized vortical structures are frequently observed in fluid dynamics. A classical model for the motion of vortex filaments is the equation for binormal curvature flow. However, this differential equation for the parametrization of a closed, oriented curve fails to capture the formation of corners and kinks, as well as the finite-time blow-up of smooth solutions. This limitation motivates a relaxation of the solution concept. We begin by considering a weak formulation, which naturally extends to divergence-free measures instead of curves. In this framework, the length of the curve, which is an invariant preserved by smooth solutions and serving as an energy, is replaced with the total variation of the measure. As global existence of weak solutions cannot be expected, we introduce energy-variational solutions, a concept that augments the combination of the energy inequality and the weak formulation with an additional defect term. A general result for abstract evolution equations enables to derive the global-in-time existence of energy-variational solutions in this framework. Additionally, we discuss a weak-strong uniqueness principle that guarantees that energy-variational solutions coincide with classical solutions as long as the latter exist.

#### References

[1] T. Eiter, R. Lasarzik, and M. Śliwiński. Existence and selection of solutions in the energy-variational framework with applications in fluid dynamics, arXiv:2601.20455, 2026.

### Singular Vortical Flows

Volker W. Elling

Academia Sinica, Taipei, Taiwan
VELLING@AS.EDU.TW

**Abstract**

Vortex spirals and cusps are special cases of self-similar singular solutions of the incompressible Euler equations. Existence of algebraic spirals has been shown in some cases, but additional cases remain open. New techniques for progress on these problems will be discussed. Some old and new numerical results show various obstacles to progress, including nonexistence and nonuniqueness.

### String Stability for Hysteretic Flows

Haitao Fan

Georgetown University, Washington, DC 20057, USA
fanh@georgetown.edu

#### Abstract

We investigate string stability in continuum flow models where information propagates to the left. String stability concerns whether oscillations decay or grow as they travel, with rates measured in the spatial variable. In traffic flow, this corresponds to how speed oscillations of successive vehicles evolve as disturbances move upstream.

Positive hysteresis loops may amplify flux oscillations and can generate shocks. For hysteretic LWR model, a solution containing a large enough hysteresis loop in the positive hysteresis region will develop stop-and-go waves, else the flux oscillations will decay as they propagate upstream. A necessary and sufficient condition for this "large enough" is found. In the region of negative hysteresis, hysteresis loops damp flux oscillations. For this case, we formally derive spatial decay rates for time-periodic string-stability solutions for several hysteretic models such as hysteretic LWR model, hysteretic ARZ model, transport with absorption and desorption model. Similar decay rates is also obtained for spatially periodic solutions of Amadori-Bressan-Shen hysteretic flow model.

### Large Time-Periodic Solutions to Nonlinear Wave Equations

Filip Ficek

Faculty of Mathematics, University of Vienna, Vienna, Austria
FILIP.FICEK@UNIVIE.AC.AT

**Abstract**

We study time-periodic solutions for nonlinear wave equations on an interval with Dirichlet boundary conditions and find intricate bifurcation structures forming fractal-like patterns [1, 2]. In addition to the small solutions with frequencies belonging to nowhere dense sets known from the classic works, see for example [3, 4], they include a new class of large-energy solutions with complex mode compositions. We perform the first proof of existence of solutions belonging to this class [5]. Similar phenomena can be observed for other systems [6, 7]. This is a joint work with Maciej Maliborski.

#### References

[1] F. Ficek and M. Maliborski, Periodic solutions for the 1d cubic wave equation with Dirichlet boundary conditions, *Nonlinearity* **38** (2025), 065016.

[2] F. Ficek and M. Maliborski, Trees, trunks, and branches – bifurcation structure of time-periodic solutions to $u_{tt} - u_{xx} \pm u^3 = 0$, to appear in *Advances in Theoretical and Mathematical Physics.*

[3] D. Bambusi and S. Paleari, Families of Periodic Solutions of Resonant PDEs, *Journal of Nonlinear Science* **11** (2001), pp. 69–87.

[4] M. Berti and P. Bolle, Cantor families of periodic solutions for completely resonant nonlinear wave equations, *Duke Mathematical Journal* **134** (2006), pp. 359–419.

[5] F. Ficek and M. Maliborski, New class of time-periodic solutions to the 1D cubic wave equation, arXiv:2506.10839.

[6] F. Ficek and M. Maliborski, Complex structure of time-periodic solutions decoded in Poincaré-Lindstedt series: the cubic conformal wave equation on $S^3$, *Physica D* **481** (2025), 134864.

[7] F. Ficek and M. Maliborski, Nonlinear oscillations of strings and beams, to appear in *Progress of Theoretical and Experimental Physics.*

## Regularity Lost: the Fundamental Limitations and Constraint Qualifications in the Problems of Elastoplasticity

Ivan Gudoshnikov

Institute of Mathematics of the Czech Academy of Sciences, Prague, Czech Republic
GUDOSHNIKOV@MATH.CAS.CZ

### Abstract

The connection between plasticity phenomenon in mechanics and the sweeping process was noticed since the very discovery of the sweeping process as a mathematical problem. Using the ideas of J.-J. Moreau, we describe abstract frameworks for elastoplasticity in terms of Helmholtz- or Beltrami-type decomposition, and convert them to equivalent formulations in terms of differential inclusions with normal cones.

However, with the seemingly simplest constitutive law of perfect plasticity it is, in general, impossible to find the strain rate as a $L_2$-function. Although some examples with such phenomenon are already known, we provide an example where the lack of a function-valued strain is caused by a displacement loading. This loss of regularity can be explained by the lack of additivity of the normal cones and the failure of Slater's constraint qualification [1].

Various constraint qualifications can help to establish the additivity of the normal cones and show well-posedness of the similar models, such as spatially discrete models and plasticty with asymptotically linear hardening.

### References

[1] Gudoshnikov I., Regularity lost: the fundamental limitations and constraint qualifications in the problems of elastoplasticity, Discrete. Contin. Dyn. Syst., early access, doi: 10.3934/dcds.2026034

### Self-Similar Solutions with Finite Time Blow-up for Reaction-Diffusion Equations

Razvan Gabriel Iagar

Universidad Rey Juan Carlos, Madrid, Spain
RAZVAN.IAGAR@URJC.ES

### Abstract

This talk focuses on the existence of self-similar solutions presenting finite time blow-up, in the form

$$u(x,t) = (T-t)^{-\alpha}u(|x|(T-t)^{-\beta}), \quad (x,t) \in \mathbb{R}^N \times (0,T), \tag{33}$$

for the classical quasilinear reaction-diffusion equation

$$\partial_t u = \Delta u^m + u^p, \quad p > m > 1. \tag{34}$$

For $m = 1$, Lepin [3] proposed the following critical exponent

$$p_L = \begin{cases} 1 + \frac{6}{N-10}, & \text{if } N \geq 11 \\ \infty, & \text{if } 1 \leq N \leq 10, \end{cases}$$

as the maximal value of $p$ for which the reaction-diffusion equation admits solutions in the self-similar form (33). This fact has been proved by Mizoguchi in [4]. One natural question was whether there is a similar exponent with $m > 1$, and we give a negative answer to this question: we prove that, for any $m > 1$ and $p > m$ large, Eq. (35) admits self-similar solutions in the form (33), in strong contrast to the semilinear case. We export the same techniques to the Hardy-Hénon equation in $\mathbb{R}^N \times (0, \infty)$

$$\partial_t u = \Delta u + |x|^\sigma u^p, \quad p > 1, \ \sigma > \max\{-2, -N\},$$

and show that no Lepin-type exponent exists for $\sigma \geq 2$, while we identify the expression of the critical Lepin-type exponent for $\sigma \in (-2, 2)$.

**Joint work with Ana Isabel Muñoz and Ariel Sánchez (URJC)**

### References

[1] R. G. Iagar and A. Sánchez, *Existence of blow-up self-similar solutions for the supercritical quasilinear reaction-diffusion equation*, Discrete Contin. Dyn. Syst., **45** (2025), no. 5, 1399-1433.

[2] R. G. Iagar, A. I. Muñoz and A. Sánchez, *Self-similar blow-up solutions for the supercritical parabolic Hardy-Hénon equation*, Preprint ArXiv no. 2511.02511.

[3] L. A. Lepin, *Self-similar solutions of a semilinear heat equation* (Russian), Mat. Model., **2** (1990), 63-74.

[4] N. Mizoguchi, *Nonexistence of backward self-similar blowup solutions to a supercritical semilinear heat equation*, J. Funct. Anal., **257** (2009), 2911-2937.

### On the Characteristic Form of $\mathfrak{g}$-Valued Zero-Curvature Representations

Jiřina Jahnová

Mathematical Institute, Silesian University in Opava,
Na Rybníčku 1, 746 01 Opava, Czech Republic
Jirina.Jahnova@math.slu.cz

**Abstract**

Zero-curvature representations (ZCRs) offer a geometric way of encoding nonlinear partial differential equations by means of auxiliary linear problems. In favorable cases, the equation can be reformulated as the flatness condition of a Lie algebra–valued connection. This viewpoint has proved particularly fruitful in the theory of integrable systems, where it leads to systematic constructions of symmetries, conservation laws, and related structures. In this talk, based on our recent preprint [1], we study $\mathfrak{g}$-valued ZCRs of partial differential equations in two independent variables from the viewpoint of their extension to the infinite jet space, focusing on their characteristic elements.

The notion of a characteristic element for $\mathfrak{g}$-valued ZCRs was introduced in a cohomological context by Marvan [2]. In the abelian case $\mathfrak{g} = \mathbb{R}$, this element coincides with the generating function of a conservation law. In the theory of conservation laws, a crucial step is the possibility of extending the generating function to the full jet space so that the conservation law assumes its characteristic form. This mechanism underlies effective computational approaches. We show that an analogous phenomenon holds for general matrix Lie algebras $\mathfrak{g} \subset \mathfrak{gl}(n)$. More precisely, every $\mathfrak{g}$-valued ZCR extended to the infinite jet space admits a characteristic representative in which the Maurer–Cartan equation takes a characteristic form generalizing the classical one known for conservation laws. This form is preserved under gauge transformations and may therefore be regarded as a natural normal form of a ZCR. As a consequence, we obtain a new necessary condition, independent of the Maurer–Cartan equation, that must be satisfied by any characteristic representative, as well as a sufficient condition for a function to serve as the characteristic element of a given ZCR. In contrast to the abelian case, the additional condition is genuinely restrictive for nonabelian Lie algebras. These results suggest a refinement of computational approaches to ZCRs, in analogy with the interplay of necessary and sufficient conditions in the algorithmic theory of conservation laws.

#### References

[1] J. Jahnová, *On the characteristic form of $\mathfrak{g}$-valued zero-curvature representations,* arXiv:2508.01224 (2025).

[2] M. Marvan, *On zero-curvature-representations of partial differential equations*, in: Differential Geometry and Its Applications, Proc. Conf. Opava, Czechoslovakia, Aug. 24?28, 1992, Math. Publ. 1 (Silesian University, Opava, 1993) 103–122.

## On Stabilization of Hyperbolic FitzHugh-Nagumo Model

Jamila Kalantarova

Koc University, Istanbul, Turkey
JKALANTAROVA@KU.EDU.TR

### Abstract

We study the following closed-loop feedback control problem for hyperbolic FitzHugh Nagumo equations

$$\begin{cases} \tau\partial_t^2 u + \partial_t u - \partial_x^2 u + f(u) + v = w, \; x \in (0,L), t > 0, \\ \partial_t v + bv - du = 0, \; x \in (0,L), t > 0, \end{cases}$$

under the homogeneous Dirichlet's boundary conditions, where $b > 0$, $d > 0$ and $\tau > 0$ are given numbers, $w$ is the control function, $f : \mathbb{R} \to \mathbb{R}$ is a continuously differentiable function satisfying the conditions

$$f(s)s - \mathcal{F}(s) \geq -r_1 s^2, \; \forall s \in \mathbb{R},$$

$$a_1|s|^{p+2} - r_2 s^2 \leq \mathcal{F}(s) \leq a_2|s|^{p+2} + r_3, \; \forall s \in \mathbb{R}.$$

We apply feedback controllers involving the first $N$ Fourier modes and finite volume elements of the function $u(x,t)$, to show global stabilization of the system to the zero stationary state.

## On the Continuity in Time of Solutions to a Generalized Navier-Stokes-Fourier System

Petr Kaplický

Charles University, Prague, Czech Republic
KAPLICKY@KARLIN.MFF.CUNI.CZ

### Abstract

We consider the flow of a generalized non-Newtonian incompressible heat-conducting fluid in a bounded domain $\Omega \subset \mathbb{R}^d$, $d = 2, 3$, subject to Dirichlet boundary conditions for velocity and temperature. The fluid obeys a power-law constitutive relation for the Cauchy stress with exponent $p$. For $p \geq (3d+2)/(d+2)$, we establish the existence of a global-in-time weak solution. We provide the proof of time continuity of the temperature in $L^1(\Omega)$. Furthermore, we investigate criteria for time continuity of the temperature component of a general weak solution. We prove time continuity for weak solutions of the entropy equation with a convective term and an $L^1$ right-hand side under minimal assumptions on the velocity regularity. We show that this continuity is equivalently described by vanishing dissipation on high level sets, a truncated variational inequality for admissible test functions, or the associated equality.

This is joint work with M. Bulíček and L. Wintrová, Charles University, Prague.

## On the Mean Curvature Flow of Closed Curves Evolving on Surfaces and Two Dimensional Manifolds

Miroslav Kolář

Czech Technical University in Prague, Czech Republic
Miroslav.Kolar@fjfi.cvut.cz

**Abstract**

We present an overview of our investigation of the flow of a family of closed curves evolving in the normal direction according to a prescribed geometric evolution equation either on a given two dimensional surface or on a two dimensional manifold which is embedded or immersed in the three-dimensional Euclidean space.

In the case of evolution on a surface given as a graph of two-dimensional function, we study two non-local geometric flows depending on the geodesic curvature and preserving either the total length of the surface curve, or its enclosed surface area. We treat these evolution equations by the method of vertical projection to the underlying plane. For such a projected flow, the particular planar normal velocity and external force are constructed. For a motion on two-dimensional manifolds, we derive a system of nonlinear parabolic equations that describe their motion and a possible attraction to the manifold.

Both considered geometric flows are treated by the parametric method, and we discuss some fundamental properties of such flows. The resulting evolution equations for parametrizations of those surface curves are solved numerically by means of the flowing finite volume method, which is modified by the suitable choice of the tangential velocity ensuring appropriate redistribution of the discretization points, which helps to stabilize the numerical algorithm. Qualitative and quantitative computational results demonstrating the spatio-temporal behavior of the solution, and the performance of the numerical algorithm, are included.

Our presented results are based on our papers [1, 2, 3, 4].

### References

[1] M. Kolář and D. Ševčovič, *Mean Curvature Flow of Closed Curves Evolving in Two Dimensional Manifolds*, Japan Journal of Industrial and Applied Mathematics, 43(16), (2026).

[2] M. Kolář, M. Beneš and D. Ševčovič, *Qualitative and numerical aspects of a motion of a family of interacting curves*, SIAM Journal on Applied Mathematics, 82(2), (2022), 549-575.

[3] M. Kolář, M. Beneš and D. Ševčovič, *On Surface Area and Length Preserving Flows of Closed Curves on a Given Surface*. In: Numerical Mathematics and Advanced Applications ENUMATH 2017, (2019).

[4] M. Kolář, M. Beneš and D. Ševčovič, *Area preserving geodesic curvature driven flow of closed curves on a surface*, Discrete and Continuous Dynamical Systems - B, 22 (2017), 3671–3689.

### Linearization for Fluid Structure Interaction

Christopher Körber

University of Würzburg, Germany and Charles University, Prague, Czech Republic
CHRISTOPHER.KOERBER@UNI-WUERZBURG.DE

**Abstract**

We consider an elastic body immersed into an incompressible viscous fluid, contained in a bounded lipschitz domain of dimension bigger or equal to two. The solid is partly attached to the boundary of the fluid domain. The fluid is modeled by the incompressible Navier-Stokes equations in Eulerian coordinates, while the solid is described by its deformation in Lagrangian coordinates. The existence of weak solutions for this problem was established in [1]. Now we assume smallness of external forces, meaning the force is given by $f_\delta = \delta f$ for some positive scaling parameter $\delta > 0$. We linearize the deformation describing the solid around the identity and the fluid velocity around zero. We prove that, assuming a suitable scaling of the initial conditions, the resulting first order perturbations converge weakly towards the unique weak solution of a corresponding linearized fluid-solid interaction problem with external force $f$ as $\delta \to 0$. Our result rigorously shows that, assuming smallness of external forces and initial conditions, fluid-structure interaction systems involving nonlinear elasticity can be approximated by corresponding much simpler, linearized systems.

#### References

[1] B. Benešová, M. Kampschulte and S. Schwarzacher, *A variational approach to hyperbolic evolutions and fluid-structure interactions*, Journal of the European Mathematical Society (JEMS), 2024.

### Second Positive Solution for Spatially Nonhomogeneous Elliptic Problem

Lukáš Kotrla

University of West Bohemia in Pilsen, Czech Republic
kotrla@kma.zcu.cz

#### Abstract

We consider the semilinear Dirichlet problem

$$\begin{cases} -\Delta u(x) = \lambda\, |u(x)|^{q(x)-2}\, u(x)\,, & x \in \Omega\,, \\ u(x) = 0\,, & x \in \partial\Omega\,. \end{cases}$$

Here, $\Omega \subset \mathbb{R}^N$ is a bounded domain with a sufficiently smooth boundary $\partial\Omega$ and $\lambda \geq 0$ is a spectral parameter. The variable exponent $q(x)$ is continuous and satisfies $1 < q^- \leq q(x) \leq q^+$. Specifically, we assume $q^- \leq q(x) < 2$ and $2 < q(x) \leq q^+$ on nonempty open subsets $\Omega_-$ and $\Omega_+$ of $\Omega$, respectively. Our attention is paid to the existence and multiplicity of positive weak solutions depending on the value of $\lambda$. The interplay between the convex and concave parts of the reaction term leads to the existence of thresholds $\lambda_0 > 0$ and $\lambda^* > 0$ such that a $C^1$-solution exists for any $\lambda \in (\lambda_0, \lambda^*)$. In addition, a second $C^1$-solution exists for $\lambda \in [a, b]$, where $\lambda_0 < a < b < \lambda^*$.

The talk extends the presentations of my co-authors Jiří Benedikt, Petr Girg, and Peter Takáč in minisymposium P11. This talk is focused particularly on the existence of a second solution, proven via Leray-Schauder degree theory.

#### References

[1] Benedikt, J., Girg, P., Kotrla, L., and Takáč, P. *On the Existence of Two Positive Solutions for Spatially Nonhomogeneous Elliptic Problems.* Mathematica Slovaca, to appear.

### Existence of Solutions to the Navier-Stokes Equations in Lorentz-Besov Spaces

Bholanath Kumbhakar, Jagmohan Tyagi

Indian Institute of Technology Gandhinagar, Gandhinagar, Gujrat, India
BKUMBHAKAR@IITGN.AC.IN, JTYAGI@IITGN.AC.IN

#### Abstract

In this paper, we investigate the existence of local mild solutions to the three-dimensional incompressible Navier-Stokes equations in the framework of Lorentz-Besov spaces. The external force is assumed to belong to the Lorentz space $L^{1,q}(0,T),\ T > 0$. We first establish maximal parabolic regularity in the Lorentz space $L^{1,q}(0,T;\dot{B}^0_{p,1})$, where $1 < p < \infty$ and $1 < q \leq \infty$. Using this regularity result, we prove the existence of a local mild solution. The obtained solution lies in the Lorentz space $L^{\alpha,q}(0,T), 1 < \alpha < \infty$ with respect to time and in the homogeneous Besov space $\dot{B}^0_{p,1}$ with respect to space. The principal difficulty arises from a time singularity appearing in the estimates within the Lorentz space $L^{1,q}(0,T)$.

## Optimal Control Problem for a Nonlinear, Nonlocal Evolution System Describing an Interacting Ternary Mixture

Arghya Kundu

Department of Mathematics and Computer Science, Karlstad University, Sweden
ARGHYA.KUNDU@KAU.SE

**Abstract**

This talk addresses an optimal control formulation for a continuum model describing morphology formation in interacting ternary mixtures, with one evaporating component. The state system consists of a coupled nonlinear, degenerate, and nonlocal parabolic evolution system originally derived in [1], where a phase-field indicator and a solvent concentration are linked through nonlinear, degenerating drift terms involving convolution operators. The evaporation is a new modeling feature of main interest here. This choice of problem is motivated by the need creating in silico optimal morphology designs for organic solar cells. The control problem is formulated with an $L^2$- type cost functional and is subject to PDE constraints given by a specific phase-field-concentration system introduced in [2, 3]. The control variable represents an external mechanism affecting the concentration of the solvent through suitable distributed sources, with the aim of steering the system towards desired configurations. We first discuss the well-posedness of the state system and establish the existence of optimal controls by using standard compactness and lower semi-continuity arguments. Furthermore, we analyze the control to state mapping and derive first order necessary optimality conditions via the corresponding adjoint system. The obtained results provide a theoretical framework for the systematic design of morphologies.

This is a joint work together with Nicklas Jävergård and Adrian Muntean (Karlstad University, Sweden).

### References

[1] R. Marra, and M. Mourragui. "Phase segregation dynamics for the Blume–Capel model with Kac interaction." Stochastic processes and their applications 88, no. 1 (2000): 79-124.

[2] R. Lyons, E.N.M. Cirillo, and A. Muntean. "Phase separation and morphology formation in interacting ternary mixtures under evaporation: Well-posedness and numerical simulation of a non-local evolution system." Nonlinear Analysis: Real World Applications 77 (2024): 104039.

[3] N. Jävergård, R. Lyons, and A. Muntean. "Semi-discrete finite volume approximations of coupled evolution equations for ternary mixtures: Convergence and 3D morphological studies." Discrete and Continuous Dynamical Systems-S (2025): 0-0.

### Initial Layer for the Boltzmann Equation

Hung-Wen Kuo

National Cheng Kung University, Tainan city, Taiwan
HWKUO@MAIL.NCKU.EDU.TW

**Abstract**

In studying the relation between the Boltzmann equation in kinetic theory and fluid dynamics, it is crucial to understand the singular layers that appear in the Boltzmann solutions. These include the boundary, shock, and initial layers, which become singular when the mean free path is small. The present work focuses on the initial layer, whose main feature is the propagation of discontinuities in the solutions of the Boltzmann equation. To analyze the global structure of Boltzmann solutions, it is essential to describe these discontinuities precisely in both the space-time variables $(x, t)$ and the microscopic velocity $\boldsymbol{\xi}$.

We construct the explicit form of the Green's function and use it to represent the Boltzmann solution. The quantitative structure of this Green's function enables a detailed analysis of the commutator between the nonlinear collision operator and the transport operator, providing a precise understanding of the propagation of discontinuities. Our results reveal both the explicit form and kinetic nature of these discontinuities, as well as the emergence of fluid-like waves in the initial layer. This explicit construction bridges the Boltzmann equation and hydrodynamic behavior. This work is in collaboration with Tai-Ping Liu and Shih-Hsien Yu.

# Modular Topology Approach to the Non–Homogeneous Dirichlet Problem for the $p(x)$-Laplacian

Jan Lang

The Ohio State University, Columbus, Ohio, USA
lang@math.osu.edu

## Abstract

Let $\Omega \subset \mathbb{R}^n$ be a bounded $C^2$ domain, and let $p : \Omega \to (1, \infty)$ be measurable with

$$p^- := \operatorname*{ess\,inf}_{x \in \Omega} p(x) > n, \qquad p^+ := \operatorname*{ess\,sup}_{x \in \Omega} p(x) = \infty.$$

For boundary data $\varphi \in W^{1,p(\cdot)}(\Omega)$, we consider the non-homogeneous Dirichlet problem for the $p(x)$-Laplacian,

$$\operatorname{div}\big(|\nabla u|^{p(x)-2} \nabla u\big) = 0 \quad \text{in } \Omega, \qquad u = \varphi \quad \text{on } \partial\Omega.$$

In the regime $p^+ = \infty$, norm-based methods and the classical space $W_0^{1,p(\cdot)}(\Omega)$ are not well suited to this problem. To overcome this difficulty, we work with the modular topology and introduce the space $V_0^{1,p(\cdot)}(\Omega)$, defined as the modular closure of $C_0^\infty(\Omega)$ in $W^{1,p(\cdot)}(\Omega)$. Within this framework, we establish existence, and under natural assumptions also uniqueness, of a weak solution $u$ such that

$$u - \varphi \in V_0^{1,p(\cdot)}(\Omega).$$

The solution is obtained as a minimizer of the Dirichlet energy in the modular setting.

The talk explains why the modular approach is more effective than the norm-based one when the $\Delta_2$-condition fails and $p(\cdot)$ is unbounded. In particular, we show that although the norm topology need not be uniformly convex, an appropriately defined modular topology may still possess uniform convexity.

### References

[1] M. A. Khamsi, J. Lang, O. Méndez, A. Nekvinda, *The non-homogeneous Dirichlet problem for the $p(x)$-Laplacian with unbounded $p(x)$ on a smooth domain*, J. Differential Equations 434 (2025), 113316.

[2] M. A. Khamsi, J. Lang, O. Méndez, *Modular topologies on vector spaces and their applications* (manuscript/preprint).

### Low Regularity Approach for Kinetic Equations

Donghyun Lee

POSTECH, South Korea
DONGLEE@POSTECH.AC.KR

**Abstract**

We discuss low regularity approach for the Boltlzmann equation and BGK model. In particular, Cauchy problem near Maxwellian (under physical boundary conditions), Hölder regularity and some ill-posedness results will be introduced. If time permits, the conditional large data problem will also be discussed.

## Asymptotic Behavior for the Maxwell-Dirac System in Lorenz and Coulomb Gauges

Kiyeon Lee

KAIST, Daejeon, South Korea and University of Michigan, Ann Arbor, USA
Kiyeonlee@kaist.ac.kr

### Abstract

In this talk, I will present the nonlinear scattering results for the Maxwell-Dirac system in the Coulomb gauge. In the Lorenz gauge, we obtained the nonlinear scattering results in Cho-Lee ([1]). Using this approach, we also prove nonlinear scattering in the Coulomb gauge. Although the Coulomb gauge lacks Lorentz invariance, it allows the elliptic nature of the scalar potential to be exploited. We introduce a spinorial null structure to overcome the lack of Lorentz invariance and to obtain the desired asymptotic behavior. Moreover, the vector field method combined with various resonance analyses adapted to the Coulomb gauge will be discussed.

### References

[1] Y. Cho and K. Lee, *Global dynamics for the Maxwell-Dirac system*, Preprint, https://arxiv.org/abs/2406.18887

## Optimal Korn-Maxwell-Sobolev Inequalities

Peter Lewintan

University of Duisburg-Essen, Germany
PETER.LEWINTAN@UNI-DUE.DE

**Abstract**

We present a complete picture of coercive Korn-type inequalities for generalised incompatible fields, and optimally extend and unify several previously known inequalities that are crucial to the existence theory for a variety of models in continuum mechanics. This talk is based on the recent joint works with Franz Gmeineder, Patrizio Neff, Stefan Müller and Jean Van Schaftingen, [5, 4, 3, 2, 1].

### References

[1] F. Gmeineder, P. Lewintan, J. Van Schaftingen: Limiting Korn-Maxwell-Sobolev inequalities for general incompatibilities. arXiv Preprint: 2405.10349.

[2] F. Gmeineder, P. Lewintan, P. Neff: Korn-Maxwell-Sobolev inequalities for general incompatibilities, Math. Models Methods Appl. Sci. (M3AS) 34, 03 (2024), 523-570.

[3] F. Gmeineder, P. Lewintan, P. Neff: Optimal incompatible Korn-Maxwell-Sobolev inequalities in all dimensions, Calc. Var. PDE 62, 182 (2023).

[4] P. Lewintan, S. Müller, P. Neff: Korn inequalities for incompatible tensor fields in three space dimensions with conformally invariant dislocation energy, Calc. Var. PDE 60, 150 (2021).

[5] P. Lewintan, P. Neff: Nečas-Lions lemma revisited: An $L^p$-version of the generalized Korn inequality for incompatible tensor fields, Math. Meth. Appl. Sci. 44 (2021); 11392-11403.

### On the Optimal Rate of Vortex Stretching for Axisymmetric Euler Flows without Swirl

Deokwoo Lim and In-Jee Jeong

Seoul National University, Seoul, Republic of Korea
dwlim95@snu.ac.kr

**Abstract**

We consider incompressible Euler flows with axisymmetry and without swirl. In $\mathbb{R}^3$, we prove the $t^{4/3}$-upper bound for the growth of the vorticity maximum. This was conjectured by Childress [1] and supported by numerical computations from Childress—Gilbert—Valiant [2]. The key idea is to estimate the velocity maximum by the kinetic energy and conserved quantities related to the vorticity. This is a joint work with In-Jee Jeong (SNU) [3].

**References**

[1] Stephen Childress. Growth of anti-parallel vorticity in Euler flows. *Phys. D*, 237(14-17):1921–1925, 2008.

[2] Stephen Childress, Andrew D. Gilbert, and Paul Valiant. Eroding dipoles and vorticity growth for Euler flows in $\mathbb{R}^3$: axisymmetric flow without swirl. *J. Fluid Mech.*, 805:1–30, 2016.

[3] Deokwoo Lim and In-Jee Jeong. On the optimal rate of vortex stretching for axisymmetric Euler flows without swirl. *Arch. Ration. Mech. Anal.*, 249(32), 2025.

## Qualitative and Quantitative Homogenization of Some Non-Newtonian Flows in Perforated Domains

Yong Lu

Nanjing University, Nanjing, China
luyong@nju.edu.cn

### Abstract

We consider the homogenization of some viscous incompressible non-Newtonian flows in perforated domains. With certain general assumptions on the nonlinear stress tensor, we show the limit system is the Darcy's law for the case of 'small holes', and system remain unchanged for the case of 'large holes'. Quantitative convergence rates are also given for both the velocity field and the pressure.

### References

[1] R. M. Höfer, Y. Lu, F. Oschmann. Qualitative/quantitative homogenization of some non-Newtonian flows in perforated domains, to appear in Math. Ann. 2026.

[2] Y. Lu, Z. Qian, C. Zhang. Qualitative and quantitative homogenization of some non-Newtonian flows in perforated domains with very small holes. Submitted 2026.

## Multiscale Hyperbolic-Parabolic Models for Nonlinear Reactive Transport in Heterogeneously Fractured Porous Media

Taras Mel'nyk

Institute of Applied Analysis and Numerical Simulation, Suttgart University, Germany
TARAS.MELNYK@MATHEMATIK.UNI-STUTTGART.DE

### Abstract

This talk concerns the asymptotic analysis of nonlinear reactive transport in a layered porous medium separated by a thin, geometrically heterogeneous fracture. The fracture is represented as an $\varepsilon$-thin channel whose aperture and internal obstacle pattern vary periodically on the microscale. Transport in the bulk domains is described by parabolic reaction–diffusion equations, whereas the fracture supports a convection–diffusion–reaction process with nonlinear reactions on the walls and obstacle boundaries. A key feature is that transport inside the fracture is convection dominated, with a Péclet number of order $\varepsilon^{-1}$.

Using multiscale asymptotics as $\varepsilon \to 0$, we derive a reduced homogenized model in which the thin fracture collapses to a flat interface. The homogenized limit consists of classical diffusion–reaction equations in the bulk domains coupled through homogenized nonlinear interface conditions and a first-order semilinear hyperbolic system posed on the flat interface. The latter encodes the effective fracture dynamics produced by the strong advection and the microscale geometry. The following aspects are addressed in this study: firstly, the well-posedness of the homogenized system is proven, including the derivation of regularity results; secondly, an explicit multiscale approximation incorporating boundary-layer correctors is constructed; and thirdly, quantitative error estimates in appropriate energy norms are established. These three aspects rigorously justify the reduced model and quantify the approximation accuracy with respect to the small parameter $\varepsilon$. The presentation is based on new results currently available in [1].

### References

[1] Taras Mel'nyk, Sorin Pop, Christian Rohde (2026): *Multiscale hyperbolic-parabolic models for nonlinear reactive transport in heterogeneously fractured porous media*, Preprint: arXiv:2602.16439v1, https://doi.org/10.48550/arXiv.2602.16439

### Regularity for a Denoising Model on Manifolds

Salvador Moll

Universiat de València, Burjassot, Spain
J.SALVADOR.MOLL@UV.ES

#### Abstract

In this joint work with Esther Cabezas-Rivas and Vicent Pallardó-Julià, we study existence, uniqueness, and regularity of minimizers for a manifold-constrained variant of the Rudin-Osher-Fatemi (ROF) model for image denoising. Despite its frequent appearance in the applied literature, this model has so far lacked a rigorous analytical foundation. The associated Euler–Lagrange equations give rise to a system of elliptic PDEs with Neumann boundary conditions.

Our results can be viewed as an extension of the regularity theory for p-harmonic maps, originating in the seminal works of Eells–Sampson and Schoen–Uhlenbeck, to the critical case $p = 1$. We establish optimal regularity for the manifold-constrained model, generalizing the known Euclidean scalar theory to vector-valued maps, to manifold targets, and to curved domains, without requiring convexity of the boundary. The proofs hinge on a deep interplay between geometric and analytical techniques. We further present regularity results of independent interest, including the one-dimensional setting (with applications to signal denoising), local Lipschitz estimates relevant to image processing, and global Lipschitz regularity for a perturbed model motivated by fluid mechanics.

## Per Aquam ad Astra: on Stars (and) Boiling Water

Florian Oschmann

Charles University, Faculty of Mathematics and Physics, Czech Republic
florian.oschmann@matfyz.cuni.cz

### Abstract

We will derive the so-called (magneto-)Oberbeck-Boussinesq approximation from the compressible (MHD-)Navier-Stokes-Fourier system by means of singular limits. This approximation tells that, in a certain regime, the density deviations can be approximated linearly by temperature deviations. Instead of taking Neumann boundary values for the temperature, Dirichlet boundary conditions are imposed, giving rise to an additional non-local forcing term in the limiting heat equation. This is joint work with Peter Bella (TU Dortmund), Eduard Feireisl (CAS), Piotr Gwiazda (PAN) and Aneta Wróblewska-Kamińska (PAN).

### References

[1] Bella, P.; Feireisl, E.; Oschmann, F.: Rigorous derivation of the Oberbeck–Boussinesq approximation revealing unexpected term. *Communications in Mathematical Physics*, 403.3 (2023): 1245-1273.

[2] Gwiazda, P.; Oschmann; F., Wróblewska-Kamińska, A.: Rigorous derivation of magneto-Oberbeck-Boussinesq approximation with non-local temperature term. *arXiv preprint arXiv:2504.13525* (2025).

## Global Solutions to a Keller–Segel–Navier–Stokes System with Potential Consumption

Gabriela Planas

Universidade Estadual de Campinas, Brazil
gplanas@unicamp.br

### Abstract

The present work deals with a Keller–Segel–Navier–Stokes system with potential consumption, under homogeneous Neumann boundary conditions for cell density and chemical signal, and Dirichlet type for the velocity field, over a bounded three-dimensional domain. The paper aims to develop a time discretization scheme converging to weak solutions of the system, which are uniformly bounded at infinite time. While global existence results are already known for simplified cases, either in absence of fluid flow or for linear consumption, the existence of global weak solutions for the fully coupled system with potential consumption has remained as an open problem.

Joint work with D. Barbosa (Brazil) and F. Guillén-González (Spain)

**Temple-Type Systems with Discontinuous Flux**

Massimiliano D. Rosini

Faculty of Mathematics and Computer Science,
Maria Curie-Skłodowska University,
Plac Marii Curie-Skłodowskiej 1, Lublin, 20031, Poland,
and
Department of Management and Business Administration,
University "G. d'Annunzio" of Chieti-Pescara,
viale Pindaro, 42, Pescara, 65127, Italy

MASSIMILIANO.ROSINI@UNICH.IT

## Abstract

This talk addresses a one-dimensional $2 \times 2$ Temple-type system of conservation laws with discontinuous flux functions, motivated by models of vehicular traffic. Such discontinuities naturally arise when road conditions or driving regimes change abruptly, leading to interfaces in the governing dynamics. We examine the analytical structure of the problem and prove the existence of entropy solutions under appropriate assumptions on the flux discontinuity.

Our approach is based on wave-front tracking, which provides a constructive framework for building approximate solutions and passing to the limit in a rigorous manner. Within this setting, we introduce an explicit Riemann solver tailored to the discontinuous-flux system and investigate its key properties, including well-posedness, consistency, and admissibility criteria for shocks and contact discontinuities.

To illustrate the theoretical results, we present numerical simulations that capture the qualitative features of the solutions and demonstrate the impact of flux discontinuities on wave propagation. These examples emphasize the relevance of the model for traffic applications and highlight the effects of discontinuities in traffic dynamics.

## Nonstationary Non-Newtonian Fluid Flow in a Thin Tube Structure

Borja Rukavina

Department of Mathematics, Faculty of Science, University of Zagreb, Croatia
RUBORJA@MATH.HR

### Abstract

We consider the nonstationary non-Newtonian flow in a thin tube structure with a strain rate dependent viscosity. A thin tube structure is defined as a smoothened finite union of thin cylinders with diameters of order $\varepsilon$, where $\varepsilon > 0$ is a small parameter. In the described domain we observe a non-stationary Stokes system of equations where viscosity depends on the gradient of the solution, with a no-slip boundary condition, a homogeneous initial condition and a given velocity $\varepsilon\mathbf{g}$ on the inflow/outflow parts of the boundary.

We study the existence and uniqueness of a solution, its high order regularity, and derive a priori estimates. A new technical tool is introduced to prove the existence of a solution with high regularity for the non-Newtonian version of the Stokes equations, which may be useful for other non-linear problems. This is joint work with Grigory Panasenko, Igor Pažanin and Konstantinas Pileckas.

## The Brezis-Nirenberg Problem: from the Local to the Nonlocal Setting

Raffaella Servadei

Università degli Studi di Urbino Carlo Bo, Urbino, Italy
RAFFAELLA.SERVADEI@UNIURB.IT

### Abstract

This talk is devoted to the famous Brezis-Nirenberg problem, firstly studied by these authors in their celebrated paper appeared in Commun. Pure Appl. Anal., 36 (1983). Since then, critical equations have been widely studied from many perspectives and in various contexts, including, among others, general local operators, nonlocal operators, mixed local and nonlocal operators, higher-order operators, and operators in non-Euclidean contexts. It is interesting to note that, regardless of the context, most results concerning critical Dirichlet problems are obtained through suitable adaptations of the original argument of Brezis and Nirenberg. The purpose of this talk is to consider the nonlocal counterpart of the Brezis-Nirenberg problem (in which the classical Laplace operator is replaced by its fractional version), focusing on the similarities and differences between the local and nonlocal cases, highlighting the innovations and adaptations to consider in the treatment of the fractional case.

### Extinction, Persistence and Pattern Formation in Non-Linear Models with Non-Local Dispersal on Flat Landscapes

Maciej Tadej

Institute of Mathematics, University of Wrocław, pl. Grunwaldzki 2, 50-384 Wrocław, Poland
MACIEJ.TADEJ@MATH.UNI.WROC.PL

**Abstract**

In this talk, we first explore a general mathematical framework for non-linear models describing the evolution of species density through non-local dispersal. We focus on the spectral analysis of non-local operators that serve as scaled analogues to the classical Laplacian with Neumann or Dirichlet boundary conditions. We establish explicit formulas and asymptotic behavior for eigenvalues and eigenfunctions, which allow us to determine the well-posedness of the system, the existence of invariant regions, and the long-time asymptotic behavior of solutions, including convergence to the mean mass and the construction of spatially heterogeneous steady states [2].

Building upon these theoretical foundations, we investigate the dynamics of vegetation patterns in water-limited ecosystems using a generalized Klausmeier model. By incorporating non-local plant dispersal within finite habitats, we identify rigorous criteria for vegetation survival and extinction governed by the trade-off between local growth and boundary losses. Our results reveal the existence of a critical patch size and a critical maximal biomass density, below which the population inevitably collapses to a desert state. Furthermore, we demonstrate that non-local dispersal mechanisms—particularly those characterized by fat-tailed kernels—significantly enhance ecosystem resilience, allowing vegetation to persist in smaller and more fragmented habitats than predicted by classical local reaction-diffusion models [1].

#### References

[1] M. Tadej, R. Martinez-Garcia, M. Hecht, *Extinction and persistence criteria in non-local Klausmeier model of vegetation dynamics on flat landscapes*, Physica D: Nonlinear Phenomena (submitted, 2026).

[2] M. Tadej, *Long time behaviour of solutions to non-local and non-linear dispersal problems*, Journal of Differential Equations, 416 (2025), 2043–2064.

## About the Occurrence of Akhmediev Breathers in General Dispersive Systems

Nils Thorin

Universität Stuttgart, Stuttgart, Germany
NILS.THORIN@MATHEMATIK.UNI-STUTTGART.DE

### Abstract

The Nonlinear Schrödinger Equation (NLS equation) can be derived by multiple scaling perturbation analysis for the description of slow modulations in time and space of the envelope of spatially and temporarily oscillating wave packets, as they appear in nonlinear optics, water wave theory, plasma physics, waves in DNA, Bose-Einstein condensates, etc.

The NLS possesses a number of special solutions such as solitons, Peregrine solutions, Kuznetsov-Ma solitons, and Akhmediev breathers. In previous publications it has been shown that there are solutions of the original physical systems which behave as predicted by the NLS equation. The existing theorems cover the solitons, cf. [1, 2] and recently also the Peregrine solutions and the Kuznetsov-Ma solitons [3], but not the Akhmediev breathers, which are solutions who are spatially periodic and localized in time above a time-periodic spatially constant background state [4]. In this talk we present new methods to bridge this gap, where the main point of consideration in the choice of a suitable space for our approximations.

### References

[1] L. A. Kalyakin. Asymptotic decay of a one-dimensional wave- packet in a nonlinear dispersive medium. *Math. USSR, Sb.*, 60(2):457–483, 1988.

[2] Guido Schneider and Hannes Uecker. *Nonlinear PDEs. A dynamical systems approach*, volume 182 of *Grad. Stud. Math.* Providence, RI: American Mathematical Society (AMS), 2017.

[3] Guido Schneider and Nils Thorin. Approximate Peregrine solitons in dispersive nonlinear wave equations. *Water Waves*, 7(3):557– 570, 2025.

[4] N. Akhmediev, A. Ankiewicz, and M. Taki. Waves that appear from nowhere and disappear without a trace. *Physics Letters A*, 373(6):675–678, 2009.

## On the Modeling of Thermal Effects in Forchheimer Flow

Marko Radulović, Karol Hajduk, Luka Tolj*

Department of Mathematics, Faculty of Science, University of Zagreb,
Bijenička 30, 10000, Zagreb, Croatia
luka.tolj@math.hr

### Abstract

In this paper, we study the Forchheimer–extended Darcy–Brinkman–Boussinesq fluid flow through a thin channel filled with porous medium using methods of asymptotic analysis. The fluid inside the channel is cooled (or heated) by the surrounding medium and the flow is governed by the prescribed pressure drop between the pipe's ends. Employing asymptotic analysis with respect to the small parameter representing the channel's thickness, we derive a first–order asymptotic approximation for the velocity, pressure and temperature. The velocity approximation explicitly acknowledges the thermal effects as well as the inertial effects. These effects are clearly visualized in the provided numerical examples. Finally, we rigorously justify the obtained asymptotic model via the error estimates in suitable norms in order to indicate the order of accuracy of the proposed approximate solution.

### References

[1] M. Radulović, K. Hajduk, L. Tolj, *Modelling of the Forchheimer-Extended Darcy–Brinkman–Boussinesq Flow Through a Thin Channel*, Transport in Porous Media 152, 94, 279–318 (2025).

[2] I. Pažanin, M. Radulović, *On the Forchheimer-extended Darcy-Brinkman flow through a thin fracture*, Zeitschrift für Angewandte Mathematik und Mechanik 104, 3 (2024).

[3] M. Beneš, I. Pažanin, *Rigorous derivation of the effective model describing a non–isothermal fluid flow in a vertical pipe filled with porous medium*, Continuum Mechanics and Thermodynamics 30, 301–317 (2018).

## Exact Low Multiplicity Results for Positive Solutions of Some Sublinear Problem with Indefinite Weights under Robin Boundary Conditions

Kenichiro Umezu

Faculty of Basic Natural Science, Ibaraki University, Japan
KENICHIRO.UMEZU.MATH@VC.IBARAKI.AC.JP

### Abstract

In this talk, we consider positive solutions of the sublinear elliptic equation with Robin boundary conditions

$$\begin{cases} -\Delta u = a(x)u^q & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \nu} = \alpha u & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 1$, is a smooth bounded domain, $a \in C^\theta(\overline{\Omega})$, $\theta \in (0,1)$, changes sign, $q \in (0,1)$, $\alpha \geq 0$ is a parameter, and $\nu$ is the unit outer normal to $\partial\Omega$. We focus on positive solutions lying in $P^\circ = \{u \in C(\overline{\Omega}) : u > 0 \text{ in } \overline{\Omega}\}$. First, we observe that if there exists a positive solution in $P^\circ$, then $\int_\Omega a < 0$. Next, in a specific case that $a$ and $q$ satisfy that $\int_\Omega a \simeq 0$ and $q \simeq 1$, respectively, we prove the existence of $\alpha_s > 0$ such that:

- at $\alpha = 0$ and $\alpha_s$, there exists a *unique* positive solution in $P^\circ$;
- for every $\alpha \in (0, \alpha_s)$, there exist *exactly two* positive solutions ordered in $P^\circ$;
- for any $\alpha > \alpha_s$, there is *no* positive solutions in $P^\circ$.

The analysis uses bifurcation methods and the spectral theory for the associated linearized eigenvalue problem. This is a joint work with Uriel Kaufmann and Humberto Ramos Quoirin.

### References

[1] U. Kaufmann, H. Ramos Quoirin, K. Umezu, Nonnegative solutions of an indefinite sublinear Robin problem II: local and global exactness results. Israel J. Math., **247**, (2022), 661–696.

**Lie Group Analysis, Group-Invariant Solutions and Conservation Laws of a Family of Coupled Nonlinear Schrödinger Equations**

Vassil Vassilev

Institute of Mechanics, Bulgarian Academy of Sciences
Acad. G. Bonchev St., Building 4, 1113 Sofia, Bulgaria
VASILVAS@IMBM.BAS.BG

**Abstract**

In this talk we present results concerning the symmetry properties of the following family of coupled nonlinear Schrödinger equations

$$i\frac{\partial u}{\partial t} + p_1\frac{\partial^2 u}{\partial x^2} + p_{11}u + p_{12}v + \left(q_{11}|u|^2 + q_{12}|v|^2\right)u + q_{13}v^2u^* + q_{14}u^2v^* = 0,$$

$$i\frac{\partial v}{\partial t} + p_2\frac{\partial^2 v}{\partial x^2} + p_{21}u + p_{22}v + \left(q_{21}|u|^2 + q_{22}|v|^2\right)v + q_{23}u^2v^* + q_{24}v^2u^* = 0,$$

under Lie groups of point transformations of the respective space of real-valued independent variables $(t, x)$ and complex-valued dependent variables $(u, v)$. Here, $p_1$, $p_2$, $p_{11}$, $p_{12}$, $p_{21}$, $p_{22}$, $q_{11}$, $q_{12}$, $q_{21}$, $q_{22}$, $q_{13}$, $q_{23}$, $q_{14}$, $q_{24}$ are real numbers and the asterisk $(^*)$ denotes the complex conjugate of a complex function. Systems of equations of this type are used to describe the propagation of waves (polarized optical pulses in weakly birefringent waveguides, for instance) in nonlinear, dispersive and dissipative media (see, e.g., [1]). This family comprises and slightly extends the set of the systems of coupled nonlinear Schrödinger equations whose group properties have been studied in [2, 3, 4, 5].

First of all, the current study focuses on determining the point Lie symmetry groups admitted by the system of equations under consideration with respect to the set of parameters involved, i.e., the related group classification problem is solved. Then, several types of group-invariant solutions are identified and presented in explicit form. Finally, cases in which systems belonging to the regarded family of evolution partial differential equations admit an exact variational formulation are considered. Their variational and divergence symmetries are determined, and the corresponding conservation laws are obtained via Noether's theorem (c.f. [6]).

## References

[1] N. N. Akhmediev and A. Ankiewicz, *Solitons: Nonlinear Pulses and Beams*, Springer, New York, 1997.

[2] M. Euler et al., *J. Nonlinear Math. Phys.* **1** (1994) 358–379.

[3] E. Alfinito et al., *Phys. Rev. E* **52** (1995) 3159–3165.

[4] V. Pulov, I. Uzunov, E. Chacarov, *Phys. Rev. E* **57** (1998) 3468–3477.

[5] J. C. Ndogmo, H. Y. Donkeng, *Wave Motion* **130** (2024) 103356.

[6] P. J. Olver, *Applications of Lie Groups to Differential Equations*, 2nd ed., Springer, New York, 1993.

### Homogenisation of High-Contrast Integral Operators

Igor Velčić

University of Zagreb, Faculty of Electrical Engineering and Computing, Zagreb, Croatia
igor.velcic@fer.hr

#### Abstract

We deal with homogenisation problems for high-contrast symmetric convolution type operators with integrable kernels in media with a periodic microstructure. We adapt the two-scale convergence method to nonlocal convolution-type operators and obtain the homogenisation result both for problems stated in the whole space and in bounded domains with the homogeneous Dirichlet boundary condition. Then our focus is on homogenisation of the corresponding spectral problems. We describe the spectrum of the effective operator and characterize the limit behaviour of the spectrum of the original problem as the microstructure period tends to zero. It is shown that the spectrum of the limit operator belongs to the Hausdorf limit of the spectrum of the original operator, and that they need not coincide. This is a joint work with M. Cherdamtsev (University of Cardiff) and A. Piatnitski (UIT Narvick).

#### References

[1] Mikhail Cherdantsev, Andrey Piatnitski, Igor Velčić, Homogenisation and spectral convergence of high-contrast convolution type operators, accepted for publication in Journal of Differential Equations.

### Solvability of Nonlinear Coupled Systems Including Katugampola Fractional and p-Laplacian Operators

Pratibha Verma[1] and Wojciech Sumelka [2]

[1,2]Poznan University of Technology, Piotrowo 5 Street, Poznan 60-965, Poland
[1]pratibha.verma@put.poznan.pl; [2]wojciech.sumelka@put.poznan.pl

**Abstract**

We consider a coupled system of nonlinear fractional differential equations involving the Katugampola fractional derivative of variable-order and the $p$-Laplacian operator on the interval $[0,1]$. The system is studied together with fractional boundary conditions. The purpose of this work is to examine whether the system has solutions, and if so, under what conditions those solutions are unique. To do this, the problem is transformed into an equivalent integral form using known properties of the Katugampola operator. Fixed-point methods are then applied to obtain the main results. We also investigate the Hyers–Ulam stability of the solutions, which shows that the solutions remain stable when small changes are made to the equation. The results presented here extend known results on fractional differential equations and demonstrate that the Katugampola fractional derivative can be effectively used to study nonlinear coupled systems involving the $p$-Laplacian operator.
**Keywords:** Fractional derivatives; $p$-Laplacian operator; Variable-order Caputo–Fabrizio derivative; Boundary value problems; Hyers–Ulam Stability

## Acknowledgment

This research was funded in whole by the National Science Center, Poland, grant number 2023/51/B/ST8/01062. For the purpose of Open Access, the authors have applied a CC-BY public copyright license to any Author Accepted Manuscript (AAM) version arising from this submission.

**References**

[1] B. Łupińska, *Existence of solutions to nonlinear Katugampola fractional differential equations with mixed fractional boundary conditions*, Mathematical Methods in the Applied Sciences, 2023, 46(11), 12007-12017.

[2] Y. Arioua and A. Ma, *On criteria of existence for nonlinear Katugampola fractional differential equations with p–Laplacian operator*, Fractional Differential Calculus, 2021, 11(1), 55–68.

[3] B Łupińska, T Odzijewicz, *A Lyapunov-type inequality with the Katugampola fractional derivative*, Fractional Differential Calculus, 2018, 41(18), 8985-8996.

## Weak-Strong Uniqueness and Relaxation Limit for a Parabolic Relaxation of the Navier-Stokes-Korteweg System

Florian Wendt

Institute of Applied Analysis and Numerical Simulation, University of Stuttgart, Germany
FLORIAN.WENDT@MATHEMATIK.UNI-STUTTGART.DE

### Abstract

In this talk, we consider a parabolic relaxation formulation for the compressible Navier–Stokes–Korteweg system. This relaxation model depends on two positive relaxation parameters $\alpha, \beta > 0$ and approximates formally the compressible Navier–Stokes–Korteweg system in the relaxation limit $\alpha \to \infty$ and $\beta \to 0$. In the first part of this talk, we introduce the class of finite energy weak solutions to the initial-boundary value problem of the relaxation system and derive a corresponding relative energy inequality. In the second part of this talk, we use the relative energy inequality as a tool to prove two new analytical results for the relaxation model. First, we prove for fixed relaxation parameters that the relaxation model satisfies the weak-strong uniqueness property in the class of finite energy weak solutions. Second, we show that a sequence of finite energy weak solutions converges in the relaxation limit to a classical solution of the compressible Navier–Stokes–Korteweg system in strong norms, provided such a solution exists. As a byproduct we identify a convergence rate for the corresponding norms. This is joint work with Nilasis Chaudhuri (University of Warsaw, Warsaw, Poland) and Christian Rohde (University of Stuttgart, Stuttgart, Germany).

**Existence and Weak-Strong Uniqueness of Measure Solutions to Euler-Alignment/Aw–Rascle–Zhang Models**

Jakub Woźnicki

University of Warsaw and Polish Academy of Sciences, Warsaw, Poland
JW.WOZNICKI@STUDENT.UW.EDU.PL

**Abstract**

We study the multi-dimensional Euler–alignment system with a matrix-valued communication kernel, motivated by models of anticipation dynamics in collective behaviour. A key feature of this system is its formal equivalence to a nonlocal variant of the Aw–Rascle–Zhang (ARZ) traffic model, in which the desired velocity is modified by a nonlocal gradient interaction. We prove the global-in-time existence of measure solutions to both formulations, obtained via a single degenerate pressureless Navier–Stokes approximation. Furthermore, we establish a weak–strong uniqueness principle adapted to the pressureless setting and to nonlocal alignment forces. As a consequence, we rigorously justify the formal correspondence between the nonlocal ARZ and Euler–alignment models: they arise from the same inviscid limit, and the weak–strong uniqueness property ensures that, whenever a classical solution exists, both formulations coincide with it.

### A Variational View on Constitutive Laws in Parabolic Problems

Espen Xylander

Institute of Analysis, Dynamics and Modeling, Universtity of Stuttgart, Stuttgart, Germany
ESPEN.XYLANDER@IADM.UNI-STUTTGART.DE

**Abstract**

We consider a variational approach to the modelling of viscous fluids. The physical conservation laws of fluid dynamics are posed as side constrains for a functional that measures how much a given flow deviates from the underlying viscous properties of the fluid. Finding minimisers of this constrained functional can be interpreted as a generalised solution concept, even in cases where the constitutive laws are not known analytically.

Based on results on anisotropic quasiconvexity, we discuss parameter regimes where we can prove existence of minimisers, and where we can recover well-known results through our variational methods.

#### References

[1] Schiffer, Stefan and Xylander, Espen. "A variational view on constitutive laws in parabolic problems" *Advances in Calculus of Variations.* https://doi.org/10.1515/acv-2025-0076

**Traveling Waves in Reaction-Diffusion-Convection Problems with $p$-Laplacian-Type Diffusion**

Michaela Zahradníková

University of South Bohemia in České Budějovice, České Budějovice, Czech Republic
zahram05@prf.jcu.cz

**Abstract**

We study the existence and properties of traveling fronts for the equation

$$v_t + h(v)v_x = \left[D(v)|v_x|^{p-2}v_x\right]_x + g(v), \quad x \in \mathbb{R},\, t \geq 0,$$

with $p > 1$. The diffusion coefficient $D = D(u)$ is assumed to be strictly positive in $(0, 1)$ with possible degenerations or singularities at one or both endpoints. We consider three types of nonlinear reaction terms $g$, commonly referred to as types A, B, and C. We present sufficient conditions for the existence and non-existence of continuous, possibly non-smooth wave profiles defined on the whole real line. Our approach is based on comparison arguments for an equivalent non-Lipschitz first-order ODE on a bounded interval.

This is a joint work with Pavel Drábek (University of West Bohemia, Czech Republic), Soyeun Jung (Kongju National University, South Korea) and Eunkyung Ko (Keimyung University, South Korea).

**Parameter Estimation for a Cahn-Hilliard Type Tumor Growth Model with Nutrient Dynamics**

Leonardo Beretta, Abramo Agosti, Giancarlo Sangalli

University of Pavia, Pavia, Italy
l.beretta38@campus.unimib.it

**Abstract**

The Cahn-Hilliard equation is widely used to describe the dynamics of two interacting phases separated by a thin interfacial region and provides a natural framework for modeling tumor growth.

In this work we study parameter estimation for a phase-field model of tumor evolution coupled with nutrient dynamics. The model consists of a Cahn-Hilliard type equation for the tumor phase field coupled with a diffusion equation for the nutrient concentration, including chemotactic effects and reaction terms. This system captures key mechanisms such as tumor proliferation, nutrient consumption, and chemotactic interactions.

The forward problem is solved numerically using isogeometric analysis with second-order B-spline basis functions, which naturally provide the $C^1$ regularity required by the fourth-order Cahn-Hilliard operator. Time integration is performed using a generalized-$\alpha$ scheme combined with Newton iterations.

The inverse problem is formulated as a PDE-constrained optimization problem for identifying model parameters from observed tumor configurations. The gradient of the objective functional is computed through sensitivity equations derived from the linearized system. The resulting optimization algorithm combines weighted gradient descent with a quasi-Newton strategy based on a Gauss-Newton approximation of the Hessian.

This framework provides a computational tool for calibrating phase-field tumor growth models and moves toward data-driven, patient-specific tumor simulations.

**Green's Function Sign-Constancy in Fractional Differential Equations: A Subordination Approach**

Rajarshi Dey

Department of Mathematics, Ariel University, Ariel-40700, Israel
rajarshidey1729@gmail.com

**Abstract**

This talk investigates sign-constancy criteria for Green's functions associated with a broad class of fractional differential boundary value problems. The equations under consideration are driven by the Riemann–Liouville fractional derivative of order $\beta \in (n-1, n]$ and include linear continuous Volterra operators acting on the unknown function together with its derivatives.

The central idea developed in the paper is a subordination principle for the sign behavior of Green's functions across different types of boundary value problems. Starting from focal-type problems, we show how sign-constancy results can be transferred to more general settings, including multipoint and nonlocal boundary conditions.

For focal problems, we establish that the sign-constancy of the Green's function and its derivatives is equivalent to the requirement that the spectral radius of a corresponding compact operator is less than one. We also derive a Vallée–Poussin-type comparison theorem within this fractional framework.

To demonstrate the scope of the theory, several examples are presented, including equations involving integral operators and deviating arguments. In this way, the talk extends classical Vallée–Poussin theory to fractional differential equations with functional perturbations and nonlocal effects.

*Key Words and Phrases*: fractional differential equations; Green's function; Volterra operators; nonlocal boundary conditions; sign constancy of Green's functions; focal boundary value problems.

## Axisymmetric Compressible Micropolar Real Gas Flow in a Cylindrical Annulus: Mathematical Modeling and Galerkin Approximation

Nelida Črnjarić, Ivan Dražić* and Valentino Marković

University of Rijeka, Faculty of Engineergin, Rijeka, Croatia
*IVAN.DRAZIC@UNIRI.HR

**Abstract**

We develop an axisymmetric model describing the motion of a viscous, heat-conducting compressible micropolar real gas in the region between two concentric cylinders. The pressure is governed by the real gas equation of state

$$p = \rho^{\gamma}\theta, \qquad \gamma \geq 1,$$

where $\rho$ denotes the density and $\theta$ the temperature. Starting from the three-dimensional balance laws for mass, linear momentum, angular momentum, and energy, the system is reformulated under the assumption of cylindrical symmetry. Although the spatial dependence reduces to the radial variable, the model retains its full three-dimensional character, since all components of the velocity and microrotation fields remain present and nonlinearly coupled through the cylindrical geometry. The resulting equations contain additional radial terms characteristic of the cylindrical setting, distinguishing this framework from the planar one-dimensional case.

Related micropolar gas models involving derivation and numerical approximation have been previously investigated in the axisymmetric ideal setting [1] and in the one-dimensional compressible real gas framework [2]. The present formulation combines these approaches within a unified cylindrical real gas model. For the associated initial-boundary value problem with homogeneous boundary conditions and sufficiently regular initial data, we construct approximate solutions by means of the Faedo–Galerkin method in function spaces adapted to the axisymmetric geometry. Numerical experiments are performed to examine convergence of the approximation procedure and to analyze the qualitative and long-time behavior of solutions.

### References

[1] Dražić, I., Črnjarić-Žic, N., Mujaković, N., *Three-dimensional compressible viscous micropolar fluid with cylindrical symmetry: derivation of the model and a numerical solution*, Mathematics and Computers in Simulation, 140, 107–124 (2017), doi:10.1016/j.matcom.2017.03.006.

[2] Bašić-Šiško, A., Dražić, I., Simčić, L., *One-dimensional model and numerical solution to the viscous and heat-conducting micropolar real gas flow with homogeneous boundary conditions*, Mathematics and Computers in Simulation, 195, 71–81 (2022), doi:10.1016/j.matcom.2021.11.012.

## Space–Time Additive Schwarz Preconditioners for Optimal Control of the Heat Equation

Gobinda Garai

Institute of Mathematics, Czech Academy of Sciences, Prague, Czech Republic
GARAI@MATH.CAS.CZ

### Abstract

We develop one and two-level additive Schwarz preconditioners for the all-at-once optimal control formulation of the heat equation. The problem is discretized using a space–time Petrov–Galerkin finite element method, leading to a coupled KKT system. Our approach considers domain decomposition in both space and time, resulting in fully space–time local subproblems.

We construct a one-level space–time additive Schwarz method and extend it to two-level variants by incorporating coarse spaces. In particular, we investigate two types of coarse corrections: one based on spatial coarsening only, and another based on both space and time. Numerical experiments demonstrate that the proposed preconditioners are effective.

### References

[1] Beranek, N., Reinhold, M.A. & Urban, K. A space–time variational method for optimal control problems: well-posedness, stability and numerical solution. Comput Optim Appl 86, 767–794 (2023). https://doi.org/10.1007/s10589-023-00507-x

[2] Garai, G., & Mandal, B. C. (2026). Domain decomposition-based space-time preconditioner for the optimal control problem with constraints imposed by the Cahn-Hilliard equation. International Journal of Computer Mathematics, 1–23. https://doi.org/10.1080/00207160.2026.2614077

### GreenNet-Based AGM: Solving Elliptic PDEs using Axial Green Surrogates

Taeyoung Ha

National Institute for Mathematical Sciences, Daejeon, South Korea
tha@nims.re.kr

#### Abstract

The Axial Green's Function Method (AGM) efficiently solves multidimensional elliptic boundary-value problems by decomposing them into one-dimensional subproblems. However, its utility is often limited when analytic 1-D Green's functions are unavailable due to complex variable coefficients. We address this gap with a neural approach, termed GreenNet, which learns the requisite one-dimensional Green's functions within the AGM framework.

The neural Green's function $G(x, s)$ is constructed to satisfy homogeneous Dirichlet boundaries and the differential operator in a weak sense through a hybrid architecture. We adopt a superposition of an analytically defined singular kernel—based on classical Poisson primitives to explicitly treat boundary traces and flux jumps—and an MLP-based residual that captures the smooth, non-singular component. Trained on supervised source–solution pairs $(f, u)$, the learned kernel is inserted into AGM integral representations to efficiently reconstruct solutions. Benchmarks with high-contrast variable coefficients demonstrate that the method preserves AGM's structure and convergence while significantly outperforming standard neural networks in accuracy, thereby extending AGM's applicability to cases lacking closed-form 1-D Green's functions. The result is a flexible, scalable synthesis of data-driven modeling and analytic numerical methods.

## Sufficient Conditions for Exact Bifurcation Curves in Minkowski Curvature Problems and Their Applications

Shao-Yuan Huang and Wei-Hsun Lee

National Taipei University of Education, Taipei, Taiwan
SYHUANG@MAIL.NTUE.EDU.TW

### Abstract

In this paper, we study the bifurcation curve $S_L$ of positive solutions for the Minkowski curvature problem

$$\begin{cases} -\left(u'/\sqrt{1-u'^2}\right)' = \lambda f(u), & \text{in } (-L, L), \\ u(-L) = u(L) = 0, \end{cases} \tag{35}$$

where $\lambda, L > 0$ and $f \in C^2(0,\infty)$ satisfies that $f(0^+) < \infty$ together with certain additional assumptions where

$$S_L \equiv \{(\lambda, \|u_\lambda\|_\infty) : \lambda > 0 \text{ and } u_\lambda \in C^2(-L,L) \cap C[-L,L] \text{ is a positive solution of (35)}\} \quad \text{for } L > 0.$$

Notice that we allow $f(0^+) = -\infty$. The bifurcation curve $S_L$ describes the multiplicity of positive solutions and plays a central role in understanding the global structure of the solution set.

Earlier studies [1, 2, 3] classified the possible shapes of $S_L$ (monotone increasing, S-like shaped, or $\subset$-like shaped) under various assumptions on $f$, but these results were not sufficiently precise. Thus, we extend those works by establishing *sufficient conditions* to determine the exact shape of $S_L$. Among all examples, the most important application is the *diffusive logistic equation with a Holling type-II functional response*:

$$\begin{cases} -\left(u'/\sqrt{1-u'^2}\right)' = \lambda u \left(k - u - \frac{1}{1+mu}\right), & \text{in } (-L, L), \\ u(-L) = u(L) = 0, \end{cases}$$

which models predator–prey dynamics. For this model, we explicitly determine the parameter regimes $(k, m)$ leading to either monotone increasing or $\subset$-shaped bifurcation curves.

### References

[1] S.-Y. Huang, Classification and evolution of bifurcation curves for the one-dimensional Minkowski-curvature problem and its applications, J. Differential Equations, 264 (2018), 5977–6011.

[2] S.-Y. Huang, Bifurcation diagrams of positive solutions for one-dimensional Minkowski-curvature problem and its applications, Discrete Contin. Dyn. Syst. A., 39 (2019), 3443–3462.

[3] S.-Y. Huang, Classification and evolution of bifurcation curves of semipositone problem with Minkowski-curvature operator and its applications, J. Differential Equations, 400 (2024), 278–311.

### Numerical Simulation of Slope-Selection Epitaxial Thin-Film Growth

Hyun Geun Lee

Dongguk University, Seoul, Republic of Korea
LEEH1@DONGGUK.EDU

### Abstract

Slope-selection epitaxial thin-film growth gives rise to a variationally structured, energy-dissipative dynamical system. When this system is formulated within a phase-field framework, the resulting evolution equation exhibits complex coarsening dynamics that unfold over very long time scales and are closely related to experimentally observed hill-and-valley structures. Capturing such dynamics poses significant challenges for numerical simulation, particularly in maintaining correct energy dissipation and structural consistency over long times. In this work, we develop a second-order, linear, and energy-stable scheme that preserves the intrinsic dissipative structure of the model at the discrete level. Numerical simulations demonstrate that the scheme robustly resolves long-time coarsening dynamics and accurately recovers physically relevant scaling laws.

## Bifurcations of Limit Cycles in DDEs: Theory & Software

Lentjes Bram

Department of Mathematics, Hasselt University, Diepenbeek Campus, Belgium
BRAM.LENTJES@UHASSELT.BE

### Abstract

Until now, no robust and efficient (numerical) method has been available to study bifurcations of limit cycles in delay differential equations (DDEs). In this poster presentation, I will present explicit computational formulas for the critical normal form coefficients of all codimension 1 bifurcations of limit cycles (fold, period-doubling and Neimark-Sacker) in DDEs. These formulas are essential for detecting codimension 2 points and distinguishing between sub- and supercritical bifurcations.

The approach consists of four steps. First, we prove the existence of a smooth periodic finite-dimensional center manifold near a nonhyperbolic cycle [1]. Second, we show that the dynamics near such a cycle on the periodic center manifold can be described in terms of so-called periodic normal forms [2]. Third, we derive explicit formulas for the critical normal form coefficients in terms of the periodic (adjoint) (generalized) eigenfunctions of the characteristic operator, a generalization of the well-known characteristic matrix [3]. Finally, we illustrate the effectiveness of the method with numerical examples [4].

### References

[1] B. Lentjes, L. Spek, M. M. Bosschaert, and Yu. A. Kuznetsov, *Periodic Center Manifolds for DDEs in the Light of Suns and Stars*, Journal of Dynamics and Differential Equations, vol. 37, no. 1, pp. 815–858, 2023, doi:10.1007/s10884-023-10289-9.

[2] B. Lentjes, L. Spek, M. M. Bosschaert, and Yu. A. Kuznetsov, *Periodic normal forms for bifurcations of limit cycles in DDEs*, Journal of Differential Equations, vol. 423, pp. 631–694, 2025, doi:10.1016/j.jde.2025.01.064.

[3] M. M. Bosschaert, B. Lentjes, L. Spek, and Yu. A. Kuznetsov, *Numerical Periodic Normalization at Codim 1 Bifurcations of Limit Cycles in DDEs*, arXiv preprint, 2025, doi:10.48550/arXiv.2505.19786.

[4] M. M. Bosschaert, B. Lentjes, L. Spek, and Yu. A. Kuznetsov, *PeriodicNormalizationDDEs*, GitHub repository, 2024. Available at: https://github.com/mmbosschaert/PeriodicNormalizationDDEs

### Synchronisation of Delay Induced Oscillations

Simone Milesi

University of Hamburg, Germany
SIMONE.MILESI@UNI-HAMBURG.DE

#### Abstract

Networks of coupled oscillators in which evolution depends on delays are often studied for their applications in neural models (e.g., the delayed Wilson-Cowan oscillators [1]). The main interest from the neural sciences point of view is the study of those synchronisation processes that are supposed to happen after introducing the coupling. More precisely, in the case where the equations of the uncoupled system are all equal to each other and the coupled system can be seen as a perturbation of the uncoupled one, the aim is to verify whether and in which way the phases are going to synchronise (i.e., becoming asymptotically synchronous or anti-phase synchronous) after the coupling.

A phase reduction on a proper invariant submanifold is required to deal with the problem. In the case of networks of coupled oscillators described by ODEs [2], and when a delay makes its appearance only at the level of the coupling [3], the uncoupled dynamic is finite dimensional and thus an application of the Floquet Theorem provides quite straightforwardly proper coordinates for the submanifold. The natural next step is to introduce a delay also in the uncoupled case, as required by applications in neural sciences. Now Floquet Theorem is in general no more applicable, but in some particular cases (such as a delayed version of Stuart-Landau oscillators) it is still possible to obtain a phase reduction by other ways, for example exploiting some structural symmetries. With such a phase reduction, one can then pass to analyse the synchronisation behaviour in favourable coordinates.

#### References

[1] Pinder I, Nelson MR, Crofts JJ. Bifurcations and synchrony in a ring of delayed Wilson–Cowan oscillators. Proc. R. Soc. A 479: 20230313, https://doi.org/10.1098/rspa.2023.0313 (2023).

[2] Sören von der Gracht, Eddie Nijholt, Bob Rink. A parametrisation method for high-order phase reduction in coupled oscillator networks. https://arxiv.org/pdf/2306.03320, (2023).

[3] Christian Bick, Bob W. Rink, Babette A. J. de Wolff. Higher-order phase reduction for delay-coupled oscillators beyond the phase-shift approximation. https://doi.org/10.48550/arXiv.2510.27524, (2025).

## A Posteriori Error Estimate for Hp-version Discontinuous Galerkin Method for Convection-Diffusion-Reaction Equation on Polytopic Meshes

Prabhat Mishra, Scott Congreve

Charles University, Prague, Czech Republic
MISHRA@KARLIN.MFF.CUNI.CZ

### Abstract

We derive a hp-version aposteriori error estimator for interior penalty discontinuous Galerkin discretizations of general convection diffusion reaction equations on polygonal/polyhedral meshes. The method employs polynomial approximation spaces on agglomerated meshes generated by a recently introduced R-tree based construction [1]. The proposed estimator provides a reliable global upper bound for the discretization error measured in a norm combining the standard energy norm and a seminorm associated with the convection term. In addition, the interpolation results of Karakashian and Pascal are extended to the present discontinuous Galerkin framework on general agglomerated meshes. Numerical experiments confirm the robustness and effectiveness of the proposed method with respect to both the mesh size and the degree of the polynomial.

### References

[1] M. Feder, A. Cangiani, and L. Heltai, R3MG: R-tree based agglomeration of polytopal grids with applications to multilevel methods, *Journal of Computational Physics*, vol. 526, p. 113773, 2025.

## A Posteriori Error Analysis for a Nonconforming Virtual Element Method for the Monge-Ampère Equation

Anwesh Pradhan*, Scott Congreve

Department of Numerical Mathematics, Charles University, Prague - Czech Republic
ANWESHPRADHANGP@GMAIL.COM

### Abstract

Fully nonlinear partial differential equations (PDEs) are known as the class of nonlinear PDEs that are nonlinear in the highest order derivatives of the unknown function. A well known example of a fully nonlinear second-order PDE is the Monge-Ampère equation. Using the vanishing moment method, the fully nonlinear Monge-Ampère equation is approximated by a fourth order quasilinear PDE. The $C^1$-nonconforming – $C^0$-conforming Virtual Element Method is applied to this quasilinear PDE. Extending [1], which performed the a priori analysis for this problem, we present the a posteriori error analysis and derive a residual based error estimator. The estimator is fully computable, and we prove upper and lower bounds of the error, which are explicit in the local mesh size. We use this estimator to drive an adaptive mesh refinement algorithm, and present numerical experiments to validate the error bound.

### References

[1] Scott Congreve, Alice Hodson, Anwesh Pradhan. *Nonconforming virtual element method for the Monge-Ampère equation*, in preparation.

**Boundary Value Problem for Equations with a Fractional Derivative and Power-Type Singular Coefficients**

Volodymyr Ilkiv[1], Yaroslav Slonovskyi[2], Zinovii Nytrebych[3], Mykhailo Symotiuk[4]

[1,2,3]Lviv Polytechnic National University, Lviv, Ukraine,
[2]Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, Lviv
[1]ilkivvv@i.ua, [2]yaroslav.o.slonovskyi@lpnu.ua, [3]znytrebych@gmail.com

**Abstract**

The work is devoted to the presentation of the results obtained in the study of a model boundary value problem for the partial differential equation

$$D^{\alpha}_{0+,\,t}u(t,x) = at^{\beta}u_{xx}(t,x), \quad t\in(0,T), \quad a>0, \quad x\in\mathbb{T}\equiv\mathbb{R}/2\pi\mathbb{Z}, \tag{36}$$

$$D^{\alpha-k}_{0+,\,t}\, u(t,x)\big|_{t=0} = b_k(x), \quad k=m+1,\dots,n, \quad x\in\mathbb{T}, \tag{37}$$

$$D^{\alpha-k}_{0+,\,t}\, u(t,x)\big|_{t=T} = b_k(x), \quad k=1,\dots,m, \quad x\in\mathbb{T}, \tag{38}$$

where $\alpha\in(n-1,n)$, $n\in\mathbb{N}$, $\beta>-\alpha$, $1<m<n-1$. The fractional derivative $D^{\alpha}_{0+,\,t}$ with respect to the variable $t$ is defined [1] by means of the Riemann–Liouville fractional integral operator of fractional order by the relations

$$D^{\alpha}_{0+}y(t) = \Big(\frac{d}{dt}\Big)^n I^{n-\alpha}_{0+}y(t), \quad t>0, \quad I^{\alpha}_{0+}y(t) = \frac{1}{\Gamma(\alpha)}\int_0^t \frac{y(\tau)\,d\tau}{(t-\tau)^{1-\alpha}}, \quad t>0.$$

The functions $b_1(x),\dots,b_n(x)$ and the unknown function $u(t,x)$ are assumed to be $2\pi$-periodic with respect to the variable $x$. Depending on the values of the parameter $\alpha$, equation (36) admits different interpretations: as a fractional diffusion equation for $\alpha\in(0,1)$ and as a fractional oscillation equation for $\alpha\in(1,2)$. The time-fractional diffusion-wave equation describes many important physical phenomena in different media [2].

For solving a problem (36)–(38), the representation in the form of a Puiseux–Fourier series was obtained. To estimate the small denominators [3], which arise in the study of the convergence of this series in an appropriate functional space, the Hausdorff fractal dimension [4] was applied.

**References**

[1] S.G. Samko, A.A. Kilbas, O.I. Marichev, Fractional Integrals and Derivatives: Theory and Applications, Gordon and Breach, Amsterdam (1993).

[2] Yu. Povstenko, Linear Fractional Diffusion-Wave Equation for Scientists and Engineers, Birkhäuser (2015).

[3] V. I. Arnold, Small divisor problems in classical and celestial mechanics. *Russian Math. Survey* **18** (1963), 85–191.

[4] K. Falconer, Fractal Geometry: Mathematical Foundations and Applications, 3rd Ed., J. Wiley & Sons (2014).

## Qualitative Dynamics of Nonlinear Discrete $p$-Laplacian Models with Delay

Ondřej Suchánek

Department of Mathematics and Statistics, Faculty of Science, Masaryk University,
Brno, Czech Republic
SUCHANEKO@MAIL.MUNI.CZ

### Abstract

We study qualitative dynamics of the nonlinear second-order difference equation with deviating argument

$$\Delta\big(a_k\Phi_p(\Delta x_k)\big) + b_k\Phi_q(x_{k+\tau}) = 0, \qquad \tau \in \mathbb{N}\cup\{0\},$$

where $\Phi_r(s) = |s|^{r-2}s$, $p, q > 1$, and $a_k, b_k > 0$. This equation can be viewed as a discrete analogue of delayed $p$-Laplacian-type differential equations and as a nonlinear discretization scheme for evolutionary models with memory effects. We investigate how the presence of a deviating argument influences the long-time behavior and oscillatory structure of solutions.

Using a reciprocity (duality) principle adapted to the non-half-linear setting, we derive sharp necessary and sufficient conditions governing the existence of nonoscillatory regimes. In the cases $\tau \in \{0, 1\}$, we obtain a complete classification of asymptotic behaviors in terms of the convergence or divergence of two explicitly constructed characteristic series. Our results reveal a structural dichotomy between canonical and non-canonical forms and demonstrate that the discrete delay may generate dynamical phenomena absent in the continuous theory. These findings contribute to the understanding of qualitative stability and asymptotic preservation in nonlinear discrete dynamical systems.

## Unconditionally Energy Stable Cahn-Hilliard Solvers with Adaptive Free Energy Potentials

Abdul Wahab

Department of Mathematics, King Fahd University of Petroleum and Minerals (KFUPM),
Dhahran 31261, Saudi Arabia.
g202216920@kfupm.edu.sa

### Abstract

The Cahn-Hilliard equation is a cornerstone of nonequilibrium thermodynamics, modeling diffusion-driven phase separation, coarsening, and interfacial dynamics in binary mixtures [1, 2]. It captures the fundamental physics of mass conservation and free energy minimization, with wide-ranging applications in materials science, soft matter physics, and condensed matter systems such as alloys, polymer blends [3], pattern formation in multiphase flows [4], binary fluids[5, 6], and tumor growth dynamics [7]. Due to the equation's strong nonlinearity and stiffness, analytical solutions are rarely attainable, necessitating accurate and stable numerical methods. The choice of free energy potential significantly influences both physical fidelity and numerical stability. In this study, we develop an adaptive free energy approach that switches pointwise between the Flory-Huggins logarithmic potential, which preserves thermodynamic realism, and a polynomial approximation, which prevents singularities near phase limits. The switching ensures physical consistency within the bounded range of the order parameter. Time integration is performed using Eyre's convex-concave splitting scheme combined with first- and second-order backward differentiation formulas, guaranteeing unconditional energy stability, while Fourier cosine spectral discretization with Neumann boundary conditions ensures mass conservation and monotonic energy dissipation. The proposed framework enables stable and physically consistent simulations of phase separation and morphological evolution, offering valuable insights into pattern formation, interfacial motion, and coarsening kinetics that are crucial for understanding microstructure development and nanoscale self-organization in advanced materials.

### References

[1] Cahn, J. W., & Hilliard, J. E. (1958). Free energy of a nonuniform system. I. Interfacial free energy. The Journal of chemical physics, 28(2), 258-267.

[2] Luo, J., Manafian, J., Aghazadeh, A., Ilhan, O. A., Eslami, B., Fattah, A. A., ... & Aghdaei, M. F. (2025). Study of phase separation process in multi-component mixtures using analytical methods and decomposition variational iteration method for the fourth-order Cahn–Hilliard equation. Scientific Reports, 15(1), 38789.

[3] Bruder, F., & Brenn, R. (1992). Spinodal decomposition in thin films of a polymer blend. Physical review letters, 69(4), 624.

[4] Cheng, L., Ribatski, G., & Thome, J. R. (2008). Two-phase flow patterns and flow-pattern maps: fundamentals and applications.

[5] Elder, K. R., & Grant, M. (2004). Modeling elastic and plastic deformations in nonequilibrium processing using phase field crystals. Physical Review E-Statistical, Nonlinear, and Soft Matter Physics, 70(5), 051605.

[6] Shah, A., Saeed, S., & Khan, S. A. (2018). Numerical investigation of bubbles coalescence in a shear flow with diffuse-interface model. Heliyon, 4(12).

[7] Garcke, H., Kovács, B., & Trautwein, D. (2022). Viscoelastic Cahn-Hilliard models for tumor growth. Mathematical Models and Methods in Applied Sciences, 32(13), 2673-2758.

## Homogenization of Non-Divergence Type Equation with Oscillating Coefficients Defined on a Highly Oscillating Obstacles.

Minha Yoo

National Institute for Mathematical Sciences, Daejeon, South Korea
MHYOO@NIMS.RE.KR

### Abstract

In this talk, we discuss the homogenization of a highly oscillating obstacle problem using the viscosity method. The equation we deal with is a non-divergence type equation with oscillating coefficients. To analyze the behavior of solutions in the obstacle problem, we construct a corrector function, periodic function when the obstacle is given as 1. By utilizing this corrector, we identify the so-called "strange term behavior" when the size of the domain where the obstacle is defined reaches a critical value. We then modify the corrector for critical size and analyze the solution's behavior when the size of the obstacle is either larger or smaller than the critical value.

# Index