Qualitative Dynamics of Nonlinear Discrete p-Laplacian Models with Delay

Ondřej Suchánek

Department of Mathematics and Statistics, Faculty of Science,
Masaryk University, Brno, Czech Republic
SUCHANEKO@MAIL.MUNI.CZ

Abstract

We study qualitative dynamics of the nonlinear second-order difference equation with deviating argument

$$\Delta\big(a_k\Phi_p(\Delta x_k)\big) + b_k\Phi_q(x_{k+\tau}) = 0, \qquad \tau \in \mathbb{N}\cup\{0\},$$

where $\Phi_r(s) = |s|^{r-2}s$, $p, q > 1$, and $a_k, b_k > 0$. This equation can be viewed as a discrete analogue of delayed p-Laplacian-type differential equations and as a nonlinear discretization scheme for evolutionary models with memory effects. We investigate how the presence of a deviating argument influences the long-time behavior and oscillatory structure of solutions.

Using a reciprocity (duality) principle adapted to the non-half-linear setting, we derive sharp necessary and sufficient conditions governing the existence of nonoscillatory regimes. In the cases $\tau \in \{0, 1\}$, we obtain a complete classification of asymptotic behaviors in terms of the convergence or divergence of two explicitly constructed characteristic series. Our results reveal a structural dichotomy between canonical and non-canonical forms and demonstrate that the discrete delay may generate dynamical phenomena absent in the continuous theory. These findings contribute to the understanding of qualitative stability and asymptotic preservation in nonlinear discrete dynamical systems.