Synchronisation of Delay Induced Oscillations

Simone Milesi

University of Hamburg, Germany
SIMONE.MILESI@UNI-HAMBURG.DE

Abstract

Networks of coupled oscillators in which evolution depends on delays are often studied for their applications in neural models (e.g., the delayed Wilson-Cowan oscillators [1]). The main interest from the neural sciences point of view is the study of those synchronisation processes that are supposed to happen after introducing the coupling. More precisely, in the case where the equations of the uncoupled system are all equal to each other and the coupled system can be seen as a perturbation of the uncoupled one, the aim is to verify whether and in which way the phases are going to synchronise (i.e., becoming asymptotically synchronous or anti-phase synchronous) after the coupling.

A phase reduction on a proper invariant submanifold is required to deal with the problem. In the case of networks of coupled oscillators described by ODEs [2], and when a delay makes its appearance only at the level of the coupling [3], the uncoupled dynamic is finite dimensional and thus an application of the Floquet Theorem provides quite straightforwardly proper coordinates for the submanifold. The natural next step is to introduce a delay also in the uncoupled case, as required by applications in neural sciences. Now Floquet Theorem is in general no more applicable, but in some particular cases (such as a delayed version of Stuart-Landau oscillators) it is still possible to obtain a phase reduction by other ways, for example exploiting some structural symmetries. With such a phase reduction, one can then pass to analyse the synchronisation behaviour in favourable coordinates.

References

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