Sufficient Conditions for Exact Bifurcation Curves in Minkowski Curvature Problems and Their Applications

Shao-Yuan Huang and Wei-Hsun Lee

National Taipei University of Education, Taipei, Taiwan
SYHUANG@MAIL.NTUE.EDU.TW

Abstract

In this paper, we study the bifurcation curve S_L of positive solutions for the Minkowski curvature problem

$$\begin{cases} -\left(u'/\sqrt{1-u'^2}\right)' = \lambda f(u), \ \text{ in } (-L,L), \\ u(-L) = u(L) = 0, \end{cases} \tag{1}$$

where $\lambda, L > 0$ and $f \in C^2(0,\infty)$ satisfies that $f(0^+) < \infty$ together with certain additional assumptions where

$$\begin{aligned} S_L \ \equiv \ & \big\{(\lambda, \|u_\lambda\|_\infty) : \lambda > 0 \text{ and } u_\lambda \in C^2(-L,L) \cap C[-L,L] \\ & \text{is a positive solution of (1)}\big\} \ \text{ for } L > 0. \end{aligned}$$

Notice that we allow $f(0^+) = -\infty$. The bifurcation curve S_L describes the multiplicity of positive solutions and plays a central role in understanding the global structure of the solution set.

Earlier studies [1, 2, 3] classified the possible shapes of S_L (monotone increasing, S-like shaped, or $\subset$-like shaped) under various assumptions on f, but these results were not sufficiently precise. Thus, we extend those works by establishing *sufficient conditions* to determine the exact shape of S_L. Among all examples, the most important application is the *diffusive logistic equation with a Holling type-II functional response*:

$$\begin{cases} -\left(u'/\sqrt{1-u'^2}\right)' = \lambda u\left(k - u - \frac{1}{1+mu}\right), \ \text{ in } (-L,L), \\ u(-L) = u(L) = 0, \end{cases}$$

which models predator–prey dynamics. For this model, we explicitly determine the parameter regimes (k,m) leading to either monotone increasing or $\subset$-shaped bifurcation curves.

References

[1] S.-Y. Huang, Classification and evolution of bifurcation curves for the one-dimensional Minkowski-curvature problem and its applications, J. Differential Equations, 264 (2018), 5977–6011.

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[3] S.-Y. Huang, Classification and evolution of bifurcation curves of semipositone problem with Minkowski-curvature operator and its applications, J. Differential Equations, 400 (2024), 278–311.