GreenNet-Based AGM: Solving Elliptic PDEs using Axial Green Surrogates

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Abstract

The Axial Green's Function Method (AGM) efficiently solves multidimensional elliptic boundary-value problems by decomposing them into one-dimensional subproblems. However, its utility is often limited when analytic 1-D Green's functions are unavailable due to complex variable coefficients. We address this gap with a neural approach, termed GreenNet, which learns the requisite one-dimensional Green's functions within the AGM framework.

The neural Green's function $G(x, s)$ is constructed to satisfy homogeneous Dirichlet boundaries and the differential operator in a weak sense through a hybrid architecture. We adopt a superposition of an analytically defined singular kernel—based on classical Poisson primitives to explicitly treat boundary traces and flux jumps—and an MLP-based residual that captures the smooth, non-singular component. Trained on supervised source–solution pairs (f, u), the learned kernel is inserted into AGM integral representations to efficiently reconstruct solutions. Benchmarks with high-contrast variable coefficients demonstrate that the method preserves AGM's structure and convergence while significantly outperforming standard neural networks in accuracy, thereby extending AGM's applicability to cases lacking closed-form 1-D Green's functions. The result is a flexible, scalable synthesis of data-driven modeling and analytic numerical methods.