

Space–Time Additive Schwarz Preconditioners for Optimal Control of the Heat Equation

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Abstract

We develop one and two-level additive Schwarz preconditioners for the all-at-once optimal control formulation of the heat equation. The problem is discretized using a space–time Petrov–Galerkin finite element method, leading to a coupled KKT system. Our approach considers domain decomposition in both space and time, resulting in fully space–time local subproblems.

We construct a one-level space–time additive Schwarz method and extend it to two-level variants by incorporating coarse spaces. In particular, we investigate two types of coarse corrections: one based on spatial coarsening only, and another based on both space and time. Numerical experiments demonstrate that the proposed preconditioners are effective.

References

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