Axisymmetric Compressible Micropolar Real Gas Flow in a Cylindrical Annulus: Mathematical Modeling and Galerkin Approximation

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Abstract

We develop an axisymmetric model describing the motion of a viscous, heat-conducting compressible micropolar real gas in the region between two concentric cylinders. The pressure is governed by the real gas equation of state

$$p = \rho^{\gamma}\theta, \qquad \gamma \geq 1,$$

where ρ denotes the density and θ the temperature. Starting from the three-dimensional balance laws for mass, linear momentum, angular momentum, and energy, the system is reformulated under the assumption of cylindrical symmetry. Although the spatial dependence reduces to the radial variable, the model retains its full three-dimensional character, since all components of the velocity and microrotation fields remain present and nonlinearly coupled through the cylindrical geometry. The resulting equations contain additional radial terms characteristic of the cylindrical setting, distinguishing this framework from the planar one-dimensional case.

Related micropolar gas models involving derivation and numerical approximation have been previously investigated in the axisymmetric ideal setting [1] and in the one-dimensional compressible real gas framework [2]. The present formulation combines these approaches within a unified cylindrical real gas model. For the associated initial-boundary value problem with homogeneous boundary conditions and sufficiently regular initial data, we construct approximate solutions by means of the Faedo–Galerkin method in function spaces adapted to the axisymmetric geometry. Numerical experiments are performed to examine convergence of the approximation procedure and to analyze the qualitative and long-time behavior of solutions.

References

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[2] Bašić-Šiško, A., Dražić, I., Simčić, L., *One-dimensional model and numerical solution to the viscous and heat-conducting micropolar real gas flow with homogeneous boundary conditions*, Mathematics and Computers in Simulation, 195, 71–81 (2022), doi:10.1016/j.matcom.2021.11.012.