A Spectral Approach to Computer-Assisted Proofs in Dynamics

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Abstract

Finding solutions of problems in nonlinear dynamics often involves simulations. Turning these numerical computations into mathematical theorems requires computer assistance. We will start by explaining the main idea of a computer-assisted proof and give examples of recent results for various ordinary, delay, and partial differential equations. Such computer-assisted proofs focus on specific solutions (or invariant objects more generally) to specific equations. Nevertheless, in this talk we take a more universal viewpoint and describe a relatively simple framework in which many of these problems can be cast. In particular, since these systems and their solutions are usually locally analytic, one can recast the problem into one concerning sequence spaces of rapidly decaying coefficients, say of a Taylor, Fourier, or Chebyshev series. The core of the analysis is then to manipulate such sequences, e.g., evaluating derivatives and nonlinearities, by computer, while keeping track of truncation and rounding errors. We discuss how one can use basic complex and Fourier analysis to accomplish this task for a wide variety of problems in nonlinear dynamics. Such a unified approach can simplify both the analysis and the accompanying implementation.