Structure-Preserving LDG Methods for Linear and Nonlinear Transport Equations with Gradient Noise

Thomas Christiansen

Department of Mathematics, University of Oslo, Norway
THOMCHR@MATH.UIO.NO

Abstract

We develop a framework for constructing local discontinuous Galerkin (LDG) discretizations that preserve the underlying hyperbolic structure of stochastic partial differential equations of the form

$$du + \mathrm{div}_x f(x,u)\,dt + \sum_{\ell \in L} \mathrm{div}_x\big(\sigma_\ell(x) g_\ell(u)\big) \circ dW_t^\ell = 0, \qquad \text{in } (0,T] \times \mathbb{R}^d.$$

Here $f = (f_1, ..., f_d) : \mathbb{R}^d \times \mathbb{R} \to \mathbb{R}^d$ is a given deterministic heterogenous flux vector and the stochastic perturbations enter through Stratonovich gradient noise (or as stochastic heterogenous fluxes), where each $\sigma_\ell : \mathbb{R}^d \to \mathbb{R}^d$ is a prescribed spatial vector field and $g_\ell : \mathbb{R} \to \mathbb{R}$ is a scalar flux function. Moreover, L is a finite index set and the driving processes $\{W^\ell\}_{\ell \in L}$ are mutually independent real-valued Wiener processes.

Starting from the Itô formulation we design semi-discretizations that reproduce the exact cancellation between the quadratic variation and the Itô–Stratonovich correction term (modulo fluxes) which is the key hyperbolic stability mechanism of gradient noise. By selecting suitable numerical fluxes, we obtain discrete energy conservation or energy dissipation, valid either pathwise or in expectation. Furthermore, by combining this energy stability with a Khasminskii-type argument we can prove well-posedness of the resulting high-order schemes without relying on commonly adopted global linear growth assumptions. We end this talk by presenting a few numerical results that showcase the stability of the LDG schemes and how various numerical fluxes influence the approximation accuracy.

The talk is based on joint work with Kenneth H. Karlsen (UiO).