Global Solvability for a Stochastic Hyperbolic Keller - Segel System

Nikolai V. Chemetov

Department of Computing and Mathematics, University of São Paulo,
Ribeirão Preto - SP, Brazil
NVCHEMETOV@USP.BR

Abstract

In this work we study the stochastic Stratonovich hyperbolic Keller-Segel model with an infinite dimensional multiplicative noise in a bounded domain $\mathcal{O}$ in $\mathbb{R}^2$. The evolution system on a time interval $(0,T)$ reads for a.s. in Ω as

$$\begin{cases} du = -\mathrm{div}(g(u)\ \nabla h)\,dt + \sigma(u) \circ d\mathcal{W}_t, \\ \\ -\Delta h + h = u, \end{cases} \qquad \text{for} \quad (t,\mathbf{x}) \in (0,T) \times \mathcal{O}$$

where $u = u(t,\mathbf{x})$ denotes the cell density and $g(u) = u - u^2$; $\mathcal{W}_t$ is a Wiener process given on the probability space $(\Omega, \mathcal{F}, P)$ and $\sigma = \sigma(u)$ is the diffusion coefficient.

This system is closed by the boundary condition

$$h = a \qquad \text{on} \quad (0,T) \times \partial\mathcal{O}$$

on the boundary $\partial\mathcal{O}$ of the domain $\mathcal{O}$, the influx boundary condition

$$u = b \quad \text{on} \quad (0,T) \times \partial\mathcal{O}^-$$

and the initial condition

$$u = u_0 \qquad \text{in} \quad \mathcal{O} \quad \text{at} \quad t = 0,$$

where

$$\partial\mathcal{O}^- = \{\mathbf{x} \in \partial\mathcal{O} : \quad g'(u)\,(\nabla h \cdot \mathbf{n})\,(t,\mathbf{x}) < 0\}$$

is the influx part of $\partial\mathcal{O}$ and $\mathbf{n} = \mathbf{n}(\mathbf{x})$ is the external normal at $\mathbf{x} \in \partial\mathcal{O}$.

We establish the global-in-time existence result for weak entropy martingale solutions to this stochastic Keller-Segel model with general boundary-initial data (a, b, u_0).

The proof combines the stochastic a priori estimates with compactness techniques based on the kinetic theory [1] and on the Jakubowski-Skorokhod representation theorem [2, 3]. The dependence of the noise function makes more difficult for the analysis of a priori estimates, giving rise to nonlinear terms induced by the martingale part of the equation and the Stratonovich correction term.

References

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