Homogenization of a Stokes-Transport System in an Anisotropic Porous Medium with Thin, Evolving Tubes

Kilian Hacker

Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany
KILIAN.HACKER@FAU.DE

Abstract

In this contribution, we study the asymptotic behavior of a Stokes flow in an array of thin, evolving tubes, coupled with advective-diffusive transport within the tubes and in the surrounding medium. The domain evolution is induced by the transport process via chemical reactions at the microscopic lateral boundaries of the tubes, leading to a fully coupled problem. Our approach aims to model the thermal control of blood flow in human skin, including temperature-dependent production of biochemical substances and the subsequent dilation and constriction of blood vessels.
After a transformation of the microscopic problem onto a periodic microscopic reference domain, we rigorously derive a fully coupled effective model, by means of the methods of homogenization. It consists of an anisotropic Darcy model coupled to an effective reaction-diffusion-advection system of partial and ordinary differential equations. The effective coefficients (permeability, diffusivity) of the macroscopic model are computed from solutions to standard cell problems involving the time and space dependent macroscopic deformation gradient. The anisotropies induced by the special features of the geometry (array of thin tubes) are exhibited in the effective model. This is joint work with Markus Gahn and Maria Neuss-Radu.