On the Multiplicity of Positive Solutions for Spatially Nonhomogeneous Elliptic Problems

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Abstract

We consider the semilinear Dirichlet problem

$$-\Delta u = \lambda f(x, u(x)) \quad \text{for } x \in \Omega; \quad u = 0 \quad \text{on } \partial\Omega$$

with a nonlinear reaction term $f(x, \cdot) : s \longmapsto |s|^{q(x)-2}s$. For $s \geq 0$, this term is convex and concave in the nonempty open subsets

$$\Omega_+ \stackrel{\text{def}}{=} \{x \in \Omega : q(x) > 2\}, \qquad \Omega_- \stackrel{\text{def}}{=} \{x \in \Omega : q(x) < 2\}$$

of a bounded domain $\Omega \subset \mathbb{R}^N$ with a sufficiently smooth boundary, respectively. Here, $\lambda \in \mathbb{R}$ is a nonnegative spectral parameter that determines the existence and multiplicity of positive weak solutions (at least two).

For $0 < \lambda \leq \lambda^*$, we obtain a C^1-solution $u \equiv u_\lambda$ by monotone iterations. For $0 < \lambda < \lambda^*$, we obtain another C^1-solution, v, by the Leray-Schauder degree theory, which satisfies $v(x) > u_\lambda(x)$ for all $x \in \Omega$. Our a priori estimates are obtained by Young's inequality. Our main contribution is a method for handling the interplay between convex and concave nonlinearities in two disjoint, nonempty open subsets of Ω. For $\Omega = (-1, 1)$, we will discuss higher multiplicity of positive solutions bifurcating from infinity as $\lambda \to 0+$ by means of the rescaling method. These multiplicity results were motivated by extensive numerical experiments performed by J. Benedikt. We will also mention mathematical models from combustion theory and explain the role of the spectral parameter λ and the variable exponent $q(x)$ in these models.

References

[1] Benedikt, J., Girg, P., Kotrla, L., and Takáč, P. *On the Existence of Two Positive Solutions for Spatially Nonhomogeneous Elliptic Problems.* Mathematica Slovaca, to appear.