Numerical Construction of Multiple Positive Solutions of an Elliptic Problem with Concave and Convex Nonlinearity

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Abstract

We are interested in positive solutions of the semilinear elliptic Dirichlet problem

$$\begin{aligned} -\Delta u &= \lambda u(x)^{q(x)-1} && \text{for } x \in \Omega, \\ u &= 0 && \text{on } \partial\Omega \end{aligned}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with a $C^{1,\alpha}$ boundary, $\lambda \in \mathbb{R}$ is a spectral parameter and $q(x) > 1$. The convexity/concavity of the reaction term depends on the spatial variable x, i.e., it is convex in $\Omega_+ \stackrel{\text{def}}{=} \{x \in \Omega : q(x) > 2\} \neq \emptyset$ and concave in $\Omega_- \stackrel{\text{def}}{=} \{x \in \Omega : q(x) < 2\} \neq \emptyset$.

In the case of one spatial dimension $N = 1$, namely $\Omega = (-1, 1)$, we numerically construct multiple (two or more) positive solutions, provided that $\lambda \in (0, \lambda^*)$ for a certain $\lambda^* > 0$. One of them (the minimal one) bifurcates from zero, whereas the others bifurcate from infinity as $\lambda \to 0+$.

References

[1] BENEDIKT, J., GIRG, P., KOTRLA, L., AND TAKÁČ, P. *On the Existence of Two Positive Solutions for Spatially Nonhomogeneous Elliptic Problems.* To appear in Mathematica Slovaca (2026).