Uniform Stabilization of the Boussinesq System in Besov Spaces with Boundary-Based, Localized, Finite Dimensional, Fluid and Thermal Feedback Controllers

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Abstract

The Boussinesq system models heat transfer in a viscous incompressible heat conducting fluid. In an open, connected, bounded domain Ω, dimension d =2, 3, we consider an uncontrolled Boussinesq system which we assume to be unstable. We then seek boundary-based, localized, finite dimensional, feedback controllers that stabilize the Boussinesq system in the vicinity of an unstable solution. For d=3, the setting is a suitable Besov space by necessity, as the Sobolev setting is not adequate; even in the case of the Navier-Stokes equations (d=3), a component of the Boussinesq system. This was recently shown by the authors. The proof is constructive. It has no assumptions. The number of controls is minimal; ie equal to the largest geometric multiplicity of the unstable eigenvalues. The corresponding linear feedback problem defines a strongly continuous, uniformly stable, analytic semigroup, in fact of maximal regularity up to $T = \infty$. This latter property is critically used in the analysis of well-posedness and uniform stabilization of the non-linear feedback problem in the vicinity of the unstable equilibrium. A key effort is in showing controllability (Kalman algebraic conditions) for the finite dimensional projection on the unstable subspace. For this, the "ignition key" is based on two Unique Continuation Properties of suitable overdetermined adjoint eigenproblems, one of which was recently proved by Carleman-type estimates. This paper is joint work with Irena Lasiecka and Buddhika Priyasad.

References

[1] Uniform Stabilization of 3D Navier-Stokes Equations in Low Regularity Besov Spaces with Finite Dimensional, Tangential -Like Boundary, Localized Feedback Controllers(I.Lasiecka and B. Priyasad, R. Triggiani) , *Archive of Rational Mechanics and Analysis* 241,1575-1654 (2021)

[2] Uniques Continuation Properties of static fluid equations critical for the uniform stabilization of dynamic fluid equations with boundary-based finite dimentional feedback controls (R. Triggiani), *Fluids under Control", Chapter 2*,invited paper, 75 pages , Birkhauser, to appear in 2023