Mathematical Models for Hemorheology and Hemolysis

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Abstract

Blood is a suspension of blood cells in blood plasma. It can be seen as a complex fluid with non-Newtonian rheology. Under specific flow conditions, such as in Ventricular Assist Devices, blood is exposed to high mechanical stress, which may lead to damage of Red Blood Cells, the hemolysis. Mathematical modeling of both, the visoelastic rheology of blood and the hemolysis requires solution of tensorial equations related to microstructure of the fluid. The aim of the present talk is to provide comparative analysis of specific tensorial models of hemorheology and hemolysis, pointing out their formal similarity based on shared physical concepts.

All models are formulated in a unified manner, enabling a systematic comparison. The analysis is complemented by numerical simulations of blood flow in a multiply corrugated tube, where both flow characteristics and hemolysis indicators are evaluated. The main contributions of this work are the development of a unified framework for strain- (stretch-) based models of rheology and hemolysis, the formulation of a combined model integrating the approaches of Rajagopal[1, 3] and Arora[2, 4], and the introduction of a new stretch-tensor invariant-based indicator of hemolysis.

This is a part of an ongoing joint work with Anna Lancmanová, Nico Dirkes, Alberto Girelli, Adelia Sequeira and Giulia Giantesio.

References

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