Variational Derivation of the Flamant Solution for a Nonlinear Elastic Wedge

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Abstract

When a planar kirigami metamaterial is deformed by slowly modulating a mechanism motion across the pattern, its individual elastic panels still rotate predominantly, but they must also stretch slightly to accommodate the modulation. The resulting strain is not confined to a small neighborhood of the hinges, nor does it spread uniformly over the whole panel. Instead, it spreads at leading order in a manner consistent with the Flamant solution, which is a classical singular solution to linear elasticity arising from concentrated forces acting at the tip of a wedge.

In this talk, we provide a rigorous variational derivation of this behavior, starting from a generic family of hyperelastic energy functionals, by studying energy minimization problems on wedge-shaped domains truncated at a small radius and under small loading, with both parameters tending to zero simultaneously. We address displacement-controlled boundary conditions as well as pure-traction problems. By introducing logarithmic polar coordinates, we map the varying domains to a fixed reference domain and obtain a rescaling consistent with the total linear elastic energy of the Flamant solution; moreover, these variables induce an anisotropic scaling of derivatives akin to those arising in thin structure limits. In this framework, we prove convergence of minimal energies and strong convergence of the associated low-energy strains, including an explicit quantitative rate depending on the scaling of cut-off and loading. A key ingredient is a geometric rigidity estimate with a constant uniform across the family of truncated wedge domains. This is joint work with Paul Plucinsky and Ian Tobasco.