Some Remarks About an Effective Description of High-Frequency Non-Polarized Wave-Packet Propagation

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Abstract

We consider systems of the form

$$\partial_\tau \mathcal{U} + \mathcal{A}(\partial_\xi)\mathcal{U} + \frac{1}{\varepsilon}\mathcal{E}\mathcal{U} = \mathcal{T}_2(\mathcal{U},\mathcal{U}) + \varepsilon \mathcal{T}_3(\mathcal{U},\mathcal{U},\mathcal{U}),$$

with $0 < \varepsilon \ll 1$ a small perturbation parameter. We are interested in an effective description of high-frequency wave-packet propagation associated to highly oscillatory initial conditions

$$\mathcal{U}(\xi, 0) = \mathcal{U}_*(\xi)e^{ik_0\xi/\varepsilon} + c.c..$$

For polarized initial conditions classical perturbation analysis yields NLS approximations up to an arbitrary order. For non-polarized initial conditions, in lowest order a system of decoupled NLS equations can be derived as an approximate description of the associated solutions. Assuming non-resonance conditions, we prove estimates that rigorously control the error between these formal approximations and true solutions of the original system. The estimates imply that solutions to non-polarized initial conditions split into polarized wave packets. The interaction of these wave packets results in phase and envelope shifts, for which explicit formulas are provided. The result improves results from the existing literature in at least two directions, firstly, the handling of higher order approximations in case of quadratic nonlinearities $\mathcal{T}_2(\mathcal{U},\mathcal{U})$ and secondly, the handling of non-polarized initial conditions which is the main novelty of the present work.

This is a joint work with X. Meng and G. Schneider.