Dynamics of a Rimming Flow in a Cylinder of Finite Length

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Abstract

A rimming flow is the flow of a fluid partially filling a rotating cylinder. We assume that the fluid forms a capillary-driven thin film coating the inner wall. The film height h of the surface above the cylinder can be modelled by the following fourth-order degenerate-parabolic evolution equation [1]:

$$h_t + h_\theta + \gamma \operatorname{div}\left(h^3 \nabla(\Delta h + h)\right) = \delta(h^3 \cos(\theta))_\theta$$

with time $t > 0$, cylindrical coordinates $(\theta, z) \in \mathbb{T} \times (0, L)$ and Neumann-type boundary conditions on the cylinder covers. Competing effects relevant to this equation are the cylinder rotation, surface tension $\gamma > 0$, viscosity and gravity $\delta \geq 0$. We investigate the stability of positive steady states. For small gravity $0 < \delta \ll 1$ and short cylinders ($L < \pi$) we show that certain steady states are stable and while they are unstable for long cylinders ($L > \pi$). For cylinders of critical length $L = \pi$ we investigate solutions of the form

$$h(t, \theta, z) = m + a_1(\delta^2 t)e^{i(\theta - t)} + a_{-1}(\delta^2 t)e^{-i(\theta - t)} + b(\delta^2 t)\cos(z)$$

and formally derive an ODE for the (slow) coefficients $a_1, a_{-1} \in \mathbb{C}, b \in \mathbb{R}$, thus describing the dynamics of such solutions on the slow time scale $\tau = \delta^2 t$.

This talk is based on joint work with Janne Laudien (Stuttgart), Christina Lienstromberg (Stuttgart) and Juan J. L. Velázquez (Bonn).

References

[1] Pukhnachov, V. V. "Capillary/gravity film flows on the surface of a rotating cylinder." Journal of Mathematical Sciences 130.4 (2005): 4871-4883.