

Think Global, Act Local: Inducing Fully Localised Patterns via Spatial Heterogeneity

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Abstract

The existence of localised two-dimensional patterns has been observed and studied in numerous experiments and simulations: ranging from optical solitons, to patches of desert vegetation, to fluid convection. And yet, our mathematical understanding of these emerging structures remains extremely limited beyond one-dimensional examples.

In this talk I will discuss how adding a compact region of spatial heterogeneity to a PDE model can not only induce the emergence of fully localised 2D patterns, but also allows us to rigorously prove and characterise their bifurcation [1]. The idea is inspired by experimental and numerical studies of magnetic fluids and tornados, where our compact heterogeneity corresponds to a local spike in the magnetic field and temperature gradient, respectively. In particular, we obtain local bifurcation results for fully localised patterns both with and without radial or dihedral symmetry, and rigorously continue these solutions to large amplitude. Notably, the initial bifurcating solution (which can be stable at bifurcation) varies between a radially-symmetric spot and a 'dipole' solution as the width of the spatial heterogeneity increases. This work is in collaboration with David J.B. Lloyd and Matthew R. Turner (both University of Surrey).

References

- [1] D.J. Hill, D.L.B. Lloyd & M.R. Turner, *Global bifurcation of localised 2D patterns emerging from spatial heterogeneity* (2025), arXiv:2510.02867