Heterogeneity in Reaction–Diffusion Systems: A Feature, Not a Perturbation

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Abstract

Spatial heterogeneities are frequently regarded as an inconvenience: they break translation invariance, make spectral analysis more intricate, and can obscure the clarity of bifurcation scenarios familiar from homogeneous media. In this talk, we contend that heterogeneity is not merely a complication, but also a valuable tool offering a controlled mechanism to generate, choose, and stabilize coherent structures. We develop an analytical framework addressing existence, stability, and bifurcations of solutions in reaction–diffusion systems with spatially dependent coefficients, with particular emphasis on front and wave train dynamics. The presentation is structured around two representative case studies. First, we examine heterogeneous front solutions in a FitzHugh–Nagumo equation, where spatial variability produces fronts that propagate at non-constant speeds through stationary heterogeneous background states. Second, we investigate wave trains governed by a Ginzburg–Landau amplitude equation arising as slow modulations within a Swift–Hohenberg model, highlighting how spatially non-uniform coefficients influence wave-number selection. A common thread throughout is the use of perturbation techniques involving a small parameter - but importantly, this parameter does not quantify the magnitude of the spatial heterogeneity. Rather, it reflects a scale separation or nearness to a critical regime, which permits order-one heterogeneities while preserving analytical tractability. The key novelty is an extension of classical perturbative approaches to systematically include non-autonomous terms, leading to solvability conditions and reduced modulation equations in spatially varying environments.