Global Solutions to Cross-Diffusion Systems with Independent Advections in One Dimension

Jakub Skrzeczkowski

St John's College & Mathematical Institute, University of Oxford,
Oxford, United Kingdom
JAKUB.SKRZECZKOWSKI@MATHS.OX.AC.UK

Abstract

We consider cross-diffusion systems describing evolution of two species u and v moving according to the Darcy's law with the pressure law $p(s) = \frac{1}{\alpha-1}s^{\alpha-1}$ where $s = u + v$. One of the most challenging questions in the field is the construction of solutions to the problem in the presence of additional advection fields, without imposing any artificial structure (for instance, preventing the species from mixing). Although advection arises naturally in these models, it breaks the symmetry of the system and prevents application of techniques developed in recent years. We solve the problem in one space dimension in a unified way for all pressure exponents $\alpha \in (0, \infty)$ and arbitrary initial data (segregated, mixed, or partially mixed). In the regime $\alpha \in (0, 1]$ this extends existing results to general initial conditions, while for $\alpha > 1$, it provides the first existence result without structural assumptions. We construct the solutions as a limit of a vanishing viscosity approximation $(u_\varepsilon, v_\varepsilon)$. The main challenge is to identify the limit of the product $u_\varepsilon \, \partial_x p(s_\varepsilon)$ with $s_\varepsilon = u_\varepsilon + v_\varepsilon$ and the key new insight is that possible oscillations of u_ε and $\partial_x p(s_\varepsilon)$ are correlated, which simplifies the analysis of the associated Young measures in the compensated compactness argument and yields strong convergence of both quantities. Quite surprisingly, the argument relies on only three entropies, which is especially helpful for $\alpha > 2$, where it is unclear whether additional entropies are available.