

Heteroclinic Networks in Coupled Cell Systems

Liliana Sofia Garrido da Silva

Universidade Católica Portuguesa, Católica Porto Business School,
CEGE & CMUP, Portugal
LSSILVA@UCP.PT

Abstract

A coupled cell system is an ODE system associated with a coupled cell network, where the dimension is determined by the number of cells. A heteroclinic connection is a set of solution trajectories between two equilibria of an ODE system. A realization of a heteroclinic network is an ODE system that exhibits equilibria corresponding to the nodes and heteroclinic connections between them according to the heteroclinic network. This paper investigates the realization of heteroclinic networks within coupled cell systems, focusing on embedding heteroclinic connections in 2D and 3D invariant subspaces. We adapt Field's method of embedding each heteroclinic connection in distinct 2D synchrony subspaces to support multiple connections within the same subspace. Using the concept of book embedding from graph theory, we demonstrate that any heteroclinic network can be realized using a coupled cell system with a number of cells proportional to the network's book-thickness. Additionally, we extend our analysis to 3D synchrony subspaces, allowing for more complex realizations. In this case, the number of cells necessary for a realization is proportional to the number of nodes in the heteroclinic network.

For a deeper discussion of the subject we refer the reader to [1].

References

- [1] Garrido-da-Silva, L., and Soares, P. (2026). Heteroclinic networks in coupled cell systems. *arXiv preprint arXiv:2601.03370*.