Existence and Selection in the Solutions in the Energy-Variational Framework

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Abstract

We introduce the concept of energy-variational solutions for a broad class of nonlinear evolutionary partial differential equations (PDEs), with a particular emphasis on low-regularity settings [1]. This framework, based on a variational inequality, provides a robust solution theory applicable to diverse problems—from fluid dynamics and elasticity to geometric flows [3]. A key advantage of this approach lies in its ability to handle nonlinear and singular evolutionary PDEs, where classical solution concepts may fail [2]. We establish existence of solutions via a minimizing movements scheme and systematically analyze their properties. Finally, we discuss selection criteria within this framework to ensure uniqueness and physical relevance of solutions [4].

References

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