Linearization in Elastodynamics

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Abstract

Nonlinear viscoelastodynamics predicts the deformation and internal stresses of a solid body under the action of applied forces. The state of the specimen is described by a *deformation* $y : [0,T] \times \Omega \to \mathbb{R}^d$ such that $y(t,\cdot) : \Omega \to \mathbb{R}^d$ is one-to-one and continuous. Denoting by $\varrho : \Omega \to \mathbb{R}$ the density of the body, the deformation fulfills the following force balance.

$$\varrho \partial_{tt} y - \operatorname{div} \mathcal{T} = \tilde{f}, \tag{1}$$

in the time-space cylinder $[0,T] \times \Omega$; here $\mathcal{T}$ is a stress tensor and $\tilde{f}$ is the volume density of applied body forces.

The stress tensor $\mathcal{T}$ in (1) must be defined constitutively. We will consider *generalized standard materials* for which it is assumed that the stress tensor can be determined by taking an appropriate functional derivative of two underlying functionals: The so-called elastic energy functional $y \mapsto \mathcal{E}(y)$ and the dissipation functional $y \mapsto \mathcal{R}(y, \partial_t y)$. Written abstractly, the stress tensor is assumed to be obtained via the additive rule

$$-\operatorname{div}\mathcal{T} = D\mathcal{E}(y) + D_2\mathcal{R}(y, \partial_t y), \tag{2}$$

which is also referred to as the *Kelvin-Voigt rheology.*

It has been shown recently [1] can actually be approximated by suitable quasistatic and static equations; that is by

$$-\operatorname{div} \mathcal{T} = \tilde{f}, \qquad \text{and} \qquad D\mathcal{E}(y) = \tilde{f}, \tag{3}$$

with appropriately changed $\mathcal{E}$ and $\mathcal{R}$.

Problem (3) has been extensively studied in the past years in the context of model reduction, i.e. passing to a simpler model in solid mechanics. A prominent example is linearization [2] when a model reduction is sought for very small strains.

On the example of linearization we show how these results can be extended to the dynamic setting.

This is a joint work with Malte Kampschulte (Charles University, Prague) and Martin Kružík (ÚTIA, Prague).

References

[1] B. Benešová, M. Kampschulte, and S. Schwarzacher, A variational approach to hyperbolic evolutions and fluid-structure interactions, *Journal of the European Mathematical Society*, 26(12):4615–4697, 2024.

[2] G. Dal Maso, M. Negri, and D. Percivale, Linearized elasticity as Γ-limit of finite elasticity, *Set-Valued Analysis*, 10(2–3):165–183, 2002.