

Non-Uniqueness of Locally-Minimizing Clusters

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Abstract

Optimal bubble cluster problems concern the study of partitions of the $\mathbb{R}^n$ into a finite collection of chambers, some with finite volume and some with infinite volume. One looks for local minimizers of interfacial area subject to volume constraints on the finite-volume chambers. The case of one infinite-volume chamber is the classical multiple bubble problem and has received much attention in recent decades, with a well-known conjecture of Sullivan predicting the existence of a unique minimizing configuration when there are not too many chambers.

We study a variant of the multiple bubble problem in $\mathbb{R}^n$ with more than one infinite-volume chamber, with particular focus on the simplest case of 1 finite-volume chamber and 2 infinite-volume chambers. Here, Bronsard & Novack showed that uniqueness of local minimizers also holds in low dimensions $n \leq 7$. In stark contrast, we show that uniqueness fails in a large number of dimensions $n \geq 8$, and we provide some particular surprising phenomena that local minimizers can exhibit in higher dimensions. This is joint work with Lia Bronsard, Robin Neumayer and Michael Novack.