Derivation of the Vlasov-Stokes Equation – the Monokinetic and the Small-Inertia Case

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Abstract

We consider the sedimentation of N spherical particles with identical radii R in a Stokes flow in $\mathbb{R}^3$. The particles satisfy a no-slip boundary condition and are subject to constant gravity. The dynamics of the particles is modeled by Newton's law. In a mean-field scaling with $N \to \infty$ $NR \sim 1$, one expects that the evolution of the N-particle system is well approximated by the Vlasov-Stokes equation. The non-binary and singular nature of the implicit particle interaction through the fluid poses a major challenge for its rigorous derivation; as for the Vlasov-Poisson equation, it is still open whether particle clustering might prevent the validity of the mean-field limit.

We show that clustering does not occur and derive the Vlasov-Stokes equation in the following two cases. Firstly, in the monokinetic case, where we show a short time result. Secondly, in the case where the particle inertia is vanishing as $N \to \infty$. In this case, the microscopic system is to leading order well-approximated by the transport-Stokes equation. However, the Vlasov-Stokes equation takes into account the (small) inertia and therefore provides a better error estimate. The proof is based on stability estimates for the Vlasov-Stokes equation in the 2-Wasserstein distance and on an adaptation of Hauray's work for mean-field limits to 2-Wasserstein distances.