Variational Approximation of the Heat flow of Harmonic Maps

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Abstract

Harmonic maps play a central role in both differential geometry and the analysis of partial differential equations. In particular, harmonic maps and their associated heat flow naturally arise in the Oseen–Frank model for nematic liquid crystals, where they describe the local orientation of the molecules.

In this talk, I will present a variational approximation of the harmonic map heat flow. The approach is based on the Weighted Energy–Dissipation (WED) scheme, which relies on a functional defined over entire trajectories and depending on a small parameter ε. As $\varepsilon \searrow 0$, minimizers of this functional are shown to converge to weak solutions of the heat flow.

For smooth target manifolds with nonpositive sectional curvature, we recover—through a purely variational argument—the classical theorem of Eells and Sampson.

Moreover, the variational structure of the scheme allows us to extend the analysis to CAT(0) targets. In this more general setting, we prove that solutions to the flow are Lipschitz continuous in the space variable, thereby settling a long-standing open problem in the area. This is a joint work with Fang-Hua Lin, Yannick Sire and Changyou Wang.