A Relative Energy Inequality for Electrolytes with Liquid-Crystal-Like Solvent: Weak-Strong Uniqueness and a Posteriori Error Estimates

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Abstract

We study an anisotropic Navier–Stokes–Nernst–Planck–Poisson (NSNPP) system whose constant in time anisotropy is motivated by continuum models for liquid crystals. Building on the concept of suitable weak solutions [1], we derive a relative energy inequality for the NSNPP system [2]. This inequality yields a weak–strong uniqueness principle and provides a robust analytical tool for a posteriori error estimation. We further investigate the high-viscosity (low-Reynolds number) regime, which leads to an anisotropic Stokes–Nernst–Planck–Poisson (SNPP) limit system. Using the relative energy framework, we obtain quantitative error estimates measuring the distance between solutions of the NSNPP and SNPP models in natural energy and dissipation norms.

References

[1] D. Hömberg, R. Lasarzik and L. Plato. Existence of suitable weak solutions to an anisotropic electrokinetic flow model. *J. Differ. Equations, 428:511-584, 2025*

[2] R. Lasarzik and L. Plato. A relative energy inequality for an anisotropic Navier–Stokes–Nernst–Planck–Poisson system — Weak-strong uniqueness and *a posteriori* error estimates. WIAS *Preprint No. 3208 Berlin, 2025.* 37 pages.