Towards a Proof of Chaotic Dynamics in the Mackey–Glass Equation

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Abstract

In this talk, we present a constructive strategy to prove the existence of a transverse intersection between the stable and unstable manifolds of a periodic orbit in the Mackey–Glass equation

$$\frac{\mathrm{d}}{\mathrm{d}t}u(t) = -au(t) + b\frac{u(t-\tau)}{1+u(t-\tau)^{\rho}}, \qquad a, b, \rho \geq 0,\ \tau > 0.$$

Such a phenomenon is generally referred to as the Poincaré scenario, and constitutes a classical geometric mechanism for the existence of chaotic dynamics via a Smale horseshoe.

The approach begins with reformulating the delay differential equation as a discrete dynamical system, defined a function space, through the time-τ map. The phase space is discretized using functions admitting convergent Chebyshev expansions, whose coefficients are modeled in a weighted ℓ^1 Banach space. The transverse homoclinic orbit is then characterized as the zero of a nonlinear operator $F(x) = 0$, arising from a projected shooting method on the invariant stable and unstable manifolds. The strategy requires (*i*) computing a (non-degenerate) periodic orbit, (*ii*) determining its Morse index, and (*iii*) retrieving a local graph of its (infinite-dimensional) stable manifold. With this in place, we verify the existence of a transverse intersection via a computer-assisted proof based on the contraction of a quasi-Newton operator.