Extracting Tent Maps from the Mackey-Glass Attractor

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Abstract

Since its appearance in the celebrated paper [5], the Mackey–Glass equation has become an essential reference in the field of delay differential equations (DDEs). Its importance goes well beyond being a prototype model for investigating dynamical diseases in biology and medicine or for investigating the effects of delayed feedback in control and relevant applications. Even from a strictly mathematical point of view, it represents a cornerstone benchmark to understand emerging analytical techniques or to test advanced computational methods. Despite the great attention it has received in almost half a century, a rigorous mathematical proof of chaos in the Mackey-Glass equation is still an open problem. However, several numerical simulations highlight the chaotic nature of the Mackey-Glass system.

Inspired by the pioneering work of Lorenz [2], in this talk we present several computational results showing the presence of tent-like scalar maps in the dynamics of the Mackey-Glass equation. Up to our knowledge, an in-depth study on tent maps in the dynamics of this system has not been provided yet. The numerical evidence of the presence of such maps can be a strong indicator of chaotic behavior, since their iteration gives rise to chaotic dynamics. We provide an interpretation of the adopted tools on the basis of the arguments proposed by Siegberg in [4] to reduce the dynamics of simpler DDEs to tent maps in order to apply standard results on chaotic maps [1]. The emerging key mechanism is that of a double reduction: first from continuous to discrete time, second from infinite to finite dimensional state stapce.

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References

[1] T. Y. Li, J. A. Yorke, *Period Three Implies Chaos*, The American Mathematical Monthly, Dec., 1975, Vol. 82, No. 10 (Dec., 1975), pp. 985-992. https://doi.org/10.2307/2318254

[2] E. N. Lorenz, *Deterministic Nonperiodic Flow*, Journal of the atmospheric sciences, Massachussetts Insistute of Technology, 1963.

[3] M. C. Mackey, L. Glass, *Oscillation and Chaos in Physiological Control Systems*, Science, New Series, Volume 197, Issue 4300 (Jul. 15, 1977), 287-289.

[4] H. W. Siegberg, *Chaotic Behavior of a Class of Nonlinear Differential Delay Equations*, Bremen, West Germany.