The Hopf Bifurcation for Neutral Delay Differential Equations with State-Dependent Delay

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Abstract

We consider neutral delay differential equations (NDDEs) with state-dependent delays of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}[x(t) + g(x_t, a)] = f(x_t, a),$$

where $x_t(s) = x(t + s) \in \mathbb{R}^n$, and $a \in \mathbb{R}$ is a parameter. The functionals f and g do not satisfy the classical smoothness properties that are assumed for DDEs with constant delay. Consequently, the Hopf bifurcation theorem cannot be applied directly. In this research, we study periodic boundary-value problems for NDDEs using a weaker differentiability concept (mild differentiability) and prove the Hopf bifurcation theorem (existence of a smooth branch of periodic orbits near an equilibrium $x_{\mathrm{eq}}(a)$ changing stability at $x_{\mathrm{eq}}(a_{\mathrm{H}})$) under standard conditions with the following additional restriction on the neutral term:

$$|\partial_x g(x, a)z| \leq \kappa \|z\|_0 \quad \text{for some } \kappa < 1$$

and all a and continuously differentiable x near $(a_{\mathrm{H}}, x_{\mathrm{eq}}(a_{\mathrm{H}}))$.