Iterative Methods for Fractional Differential Equations via the $\star$-Product

Fabio Durastante, Pierre-Louis Giscard, Stefano Pozza

Charles University, Prague, Czech Republic.
POZZA@KARLIN.MFF.CUNI.CZ

Abstract

We address the numerical solution of non-autonomous linear fractional differential equations of the form

$$D_t^\alpha u(t) = \widehat{A}(t)\, u(t), \qquad u(0) = v,$$

where $0 < \alpha < 1$ and $\widehat{A}(t)$ is analytic in time. These problems arise in applications involving memory effects and nonlocal dynamics. Our new method expands the solution on a suitable polynomial basis and reformulates the problem as a structured matrix equation that links the fractional dynamics across time; see [1]. The approach is related to the $\star$-product formalism, a generalization of the Volterra convolution that enables a tractable discretization of the fractional operator and, in certain cases, even yields closed-form expressions.

We also present preliminary results on the extension of this framework to partial differential equations, showing that similar structured systems appear and can be exploited for numerical computations.

References

[1] Fabio Durastante, Pierre-Louis Giscard, Stefano Pozza, *A $\star$-Product Approach for Analytical and Numerical Solutions of Nonautonomous Linear Fractional Differential Equations*, arXiv:2507.16652 [math.NA], 2025.