$\mathcal{H}$-LU Preconditioners for the RBF-FD Discretized Oseen Equations

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Abstract

The possibility of using $\mathcal{H}$-matrices as preconditioners for the Oseen equations has been known for a while. They have been shown to be robust for finite elements as well as meshless discretizations [1, 2]. However, despite having an asymptotic runtime complexity of $\mathcal{O}(N \log(N)^2)$, they may in practice be quite slow when compared to competing methods due to large constants hidden in the $\mathcal{O}$-notation.

We present some recent improvements to reduce the runtime for setting up these preconditioners. These improvements are due to a recently introduced arithmetic for $\mathcal{H}$-matrices as well as clusterings that are adapted to the structure of the Oseen problem. These changes can be applied regardless of the underlying discretization.

Lastly, we present one more improvement that is possible if we discretize the Oseen equations using the meshless Radial Basis Function - Finite Difference method (RBF-FD) [3] by utilizing the flexibility of this method in the construction of the preconditioner.

References

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[2] Le Borne, Sabine (2008) *Hierarchical matrix preconditioners for the Oseen equations*, Computing and Visualization in Science

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