Domain Decomposition with Neural Model Order Reduction for Multiscale Mixed-Dimensional Problems

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Abstract

Multiscale and multiphysics problems involving heterogeneous media and coupled processes across different spatial dimensions arise in a wide range of scientific and engineering applications, including transport phenomena in biological tissues, porous media, and vascularized systems [1, 2]. The numerical approximation of such problems is particularly challenging due to geometric complexity, strong scale separation, and the high computational cost associated with repeated simulations in parametrized or data-rich settings. In this work, we present a computational framework that combines non-overlapping domain decomposition methods with non-intrusive, data-driven reduced order models to efficiently approximate multiscale mixed-dimensional partial differential equations, with a particular focus on coupled 3D–1D formulations.

The proposed approach relies on a localization strategy in which the global problem is decomposed into parametrized local subproblems, coupled through optimized Schwarz-type transmission conditions. Reduced order models, constructed from localized high-fidelity data and implemented using neural-network-based surrogates [2], are embedded within domain decomposition iterations to accelerate the solution of the most computationally demanding subdomain problems. We discuss the mathematical structure of the method, including the notion of local representability and the impact of surrogate-induced perturbations on convergence and stability. Numerical results on representative mixed-dimensional diffusion and transport problems illustrate the effectiveness of the proposed domain decomposition–neural reduced order model paradigm in achieving significant computational savings while preserving accuracy and global coherence. The presented framework provides a flexible foundation for scalable, learning-augmented solvers for complex multiphysics systems.

References

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