On Schauder Estimates for Nonlocal Viscous Hamilton Jacobi Equations

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Abstract

Viscous Hamilton–Jacobi equations arise as dynamic programming equations in optimal control and differential games, where the unknown function represents the value function of an optimally controlled process. We use a heat-kernel and semigroup approach to study the spatial regularity for such equations with a nonlocal diffusion term, specifically Lévy-type diffusion operators such as the fractional Laplacian. Using the Duhamel (mild) formulation together with quantitative smoothing estimates in Hölder spaces, we explain how the order of the diffusion determines the spatial smoothing effect: for any positive time the solution gains the optimal amount of Hölder regularity in space, and we can quantify how the relevant norms behave as time approaches zero. We focus on two canonical regimes: solutions with uniformly bounded gradients, and solutions whose gradients may blow up as time approaches zero. In addition to regularity, we discuss existence, uniqueness, and conditions under which solutions are classical. Our results are obtained under mild assumptions on the heat-kernel and the regularity of the data. Our motivation comes from Mean Field Games, and our goal is to extend our results to the full nonlocal Mean Field Game system with low-regularity data. The talk is based on our paper "On Schauder estimates for nonlocal and mixed local–nonlocal viscous Hamilton–Jacobi equations".