The Effect of Quadrature on the Convergence of Policy Iteration for Hamilton–Jacobi–Bellman Equations

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Abstract

Modern finite element libraries allow users to express partial differential equations directly in variational form, with the added convenience of automatic selection of numerical quadrature. In the context of Hamilton–Jacobi–Bellman equations, we show that the choice of quadrature plays a significant role in the observed performance of the policy iteration solver for the nonlinear problem, which is known from theory to converge locally superlinearly in the absence of quadrature error.

In particular, we show that automatic quadrature selection can result in inconsistent choices in the assembly of matrix and right-hand side, thereby causing stagnation and, in some cases, loss of convergence of the policy iteration. We also show that the simple remedy of enforcing consistent choices of quadrature for both matrix and right-hand side recovers the expected superlinear convergence. These results reveal a subtle but significant interaction between quadrature and nonlinear iteration in finite element implementations.