Numerical Approximation of Fluid-Structure Interaction: From Mathematical Modeling to Applications

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Abstract

This talk is devoted to the mathematical modeling and numerical approximation of fluid-structure interaction (FSI) problems. Mutual interactions between flowing fluids and vibrating structures are critical across a wide range of engineering and scientific disciplines. A prominent historical example of such phenomena is the collapse of the Tacoma Narrows suspension bridge due to aeroelastic instability. Today, FSI must be rigorously accounted for in the design of bridges, tall buildings, and lightweight membrane structures, as well as in mechanical engineering, where aircraft wings or turbomachinery blades may oscillate dangerously due to fluid-structure coupling. Furthermore, FSI is essential in biomechanics, notably in the study of blood flow through elastic arteries and the self-induced vibrations of vocal folds during human phonation. While traditional aeroelasticity and hydroelasticity often rely on linearized models limited to small deformations to determine stability regions, these approaches fail to capture nonlinear phenomena such as post-flutter behavior or limit cycle oscillations. Numerical simulations represent a practical tool for exploring these complex interactions.

Moving from modeling to numerical implementation, we provide a survey of several key aspects of the numerical approximation of FSI problems in the context of incompressible flows. To treat the domain motion, the Arbitrary Lagrangian-Eulerian (ALE) formulation is introduced, and the application of ALE techniques across different spatial discretization methods — including the finite volume and finite element methods — are examined. In particular, the effects of time discretization schemes on the ALE framework are explored, evaluating the overall stability and accuracy of the numerical solution. Coupling strategies, such as monolithic and partitioned approaches, are reviewed. To satisfy the divergence constraint, the choice of suitable finite element pairs is addressed in relation to stabilization methods for convection-dominated flows at high Reynolds numbers. Finally, numerical examples are presented, ranging from classical aeroelastic problems to the complex simulation of human phonation.

References

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