Positivity-Preserving Stationarity Preserving PAMPA-DG Scheme with Global Flux Quadrature

Remi Abgrall[†], Yongle Liu[†] and Mario Ricchiuto[‡]
[†] Institute of Mathematics, University of Zürich
(remi.abgrall@math.uzh.ch, yongle.liu@math.uzh.ch)
[‡] University of Bordeaux (mario.ricchiuto@inria.fr)

Abstract

In this talk, we present a novel stationarity preserving Point-Average-Moment PolynomiAl-interpreted (PAMPA) method for solving the nonlinear hyperbolic problems. We begin with the one-dimensional setting and the mathematical model considered is the shallow water equations. The idea is based on a global flux quadrature formulation, in which the discretization of the source terms is obtained from the derivative of and additional flux function computed via high order quadrature of the source term. The reformulated system is quasi-conservative with global integral terms computed using Gauss–Lobatto quadrature nodes. The resulting method is capable of preserving a large family of smooth moving equilibria: supercritical and subcritical flows, in a super-convergent manner. We also show that, by an appropriate quadrature strategy for the source, we can exactly preserve the still water states. Moreover, to guarantee the positivity of water depth and eliminate the spurious oscillations near shocks, we blend the high-order PAMPA schemes with the first order local Lax–Friedrichs schemes using the method developed in [1]. The first-order schemes are designed to preserve the still water equilibria and positivity of water height, as well as to deal with wet-dry fronts. We then extend the method to the two-dimensional setting on Cartesian meshes using the PAMPA-DG framework [2]. Similar to the 1D, the div$\mathbf{f}$ will be represented using some global flux. The numerical scheme will be tested on both the 2D Euler equations and shallow water equations. Extensive numerical experiments are tested to validate the advantages and robustness of the proposed scheme.

References

[1] Rémi Abgrall, Miaosen Jiao, Yongle Liu, and Kailiang Wu, "Bound-preserving Point-Average-Moment PolynomiAl-interpreted (PAMPA) scheme: One-dimensional Case", *Communications in Computational Physics*, 39, pp. 29–58, 2026.

[2] Rémi Abgrall and Yongle Liu, Robust PAMPA Scheme in the DG Formulation on Unstructured Triangular Meshes: Bound Preservation, Oscillation Elimination, and Boundary Conditions, in the revision of Journal of Computational Physics, https://doi.org/10.48550/arXiv.2511.16180