L^2-Stability of Explicit Runge–Kutta Methods with SUPG Stabilization for Transient Transport Problems

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Abstract

We analyze the approximation of linear transport problems using continuous finite elements in space with SUPG stabilization and explicit Runge–Kutta (ERK) schemes in time. We establish stability for ERK3 and ERK4 schemes in the L^2-norm augmented by the mesh-dependent weighted advective derivative induced by the SUPG stabilization. We provide two proofs. The first proof hinges on the stability of Taylor polynomials and covers both ERK3 and ERK4 schemes [1, 2]. The second proof uses more classical energy arguments and covers only ERK3 schemes. This second proof is presented to provide a more direct connection to previous work in the literature [3, 4]. We present numerical experiments illustrating the theoretical findings, including a Fourier analysis in 1D and the approximation of smooth solutions and solutions in BV in 2D. This is joint work with Jean-Luc Guermond (TAMU).

References

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