Adaptive Domain Decomposition Preconditioners for Discontinuous Galerkin Discretization

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Abstract

We developed efficient adaptive strategy solving time-dependent partial differential equations with the aid of space-time discontinuous Galerkin (STDG) method in combination with anisotropic *hp*-mesh adaptation, cf. [4, 3]. To make the STDG approach competitive, efficient algebraic solvers are required. The iterative solution of nonlinear algebraic systems leads to sequences of linear systems that are frequently solved iteratively by Krylov methods with suitable preconditioners. A prominent role is played by the *domain decomposition* (DD) techniques, their combination with discontinuous Galerkin discretization leads to a very comfortable setting. Particularly, two-level Schwarz preconditioners, which accelerate the transfer of information through the whole system, can be constructed in a natural way, see e.g., [1, 2].

We consider additive and hybrid two-level Schwarz preconditioners, we study the convergence of the linear solver in dependence on the number of subdomains and the number of elements of the coarse grid. We propose a simplified cost model measuring the computational costs in terms of floating-point operations, the speed of computation, and the wall-clock time for communications among computer cores. Moreover, the cost model serves as a base of the presented adaptive domain decomposition method which chooses the number of subdomains and the number of elements of the coarse grid in order to minimize the computational costs. The efficiency of the proposed technique is demonstrated by two benchmark problems of compressible flow simulations.

References

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