# Entropy Stable DGSEM for Multi-Fluid 5-Moment GLM-Maxwell Systems

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**Abstract**

Multi-fluid 5-moment GLM-Maxwell systems [1] simulate plasma flows using a set of Euler equations for each plasma species, coupled via Lorentz force source terms to a full set of Maxwell's equations extended with the Generalized Lagrange Multiplier (GLM) [2] ansatz to reduce divergence errors occurring during a numerical simulation. Our goal is to derive and construct suitable high order approximations that are provably entropy-stable and in the future we aim to exactly satisfy $\text{div}B = 0$. In this talk we give an overview of the status quo of our research and demonstrate an entropy analysis for multi-fluid 5-moment systems with an arbitrary number of species. We show that modifications of the original system are necessary to obtain an entropy inequality. We mimic the continuous entropy analysis based on recent developments for the Discontinuous Galerkin Spectral Element Method (DGSEM) with the Summation-by-Parts (SBP) property [3]. A key ingredient is the derivation of an entropy-conserving numerical flux function for the system. We will demonstrate our theoretical findings with numerical simulations.

## References

[1] Entropy Stable Numerical Schemes for Two-Fluid Plasma Equations, H. Kumar and S. Mishra, Journal of Scientific Computing 52, 401–425 (2012).

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[3] An entropy stable nodal discontinuous Galerkin method for the resistive MHD equations. Part I: Theory and numerical verification, Marvin Bohm and Andrew R. Winters and Gregor J. Gassner and Dominik Derigs and Florian Hindenlang and Joachim Saur, Journal of Computational Physics, 2020