Unified Numerical Analysis of Moving Boundary Problems via the Stabilized SAV Approach

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Abstract

This talk presents a unified mathematical framework for moving boundary problems described as gradient flows, using the stabilized Scalar Auxiliary Variable (SAV) method. The geometric evolution of interfaces presents significant numerical challenges due to severe stiffness and the deterioration of mesh quality during large deformations. To overcome these issues, we establish a fully discrete, linear, and unconditionally energy-stable scheme.

The main focus of this presentation will be on the Curve Shortening Flow (CSF) and the Area-Preserving Curve Shortening Flow (APCSF) as fundamental examples to establish our framework. Special attention is paid to the rigorous treatment of the tangential velocity field and the linearized area constraint, ensuring that the discrete energy dissipation structure is exactly preserved. We introduce a relaxation flow utilizing a weighted Laplacian, and carefully project this tangential velocity onto the unit tangent vector. Furthermore, our adaptive weighting strategies automatically concentrate vertices in high-curvature regions, effectively controlling mesh quality even for non-convex domains.