A Threshold-Type Algorithm for Fourth Order Geometric Motions

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Abstract

In this talk, we are concerned with an algorithm for the motion of hypersurfaces/curves $\{\Gamma(t)\}_{t\geq 0}$ given by the following equation:

$$V = \begin{cases} -\Delta_{\Gamma(t)} H - H\left(|A|^2 - \frac{1}{2}H^2\right), & (N = 2, 3), \\ -\Delta_{\Gamma(t)} H - H\left(|A|^2 - \frac{1}{2}H^2\right) - 2\sum_{1\leq i<j<k\leq N-1} \kappa_i\kappa_j\kappa_k, & (N \geq 4), \end{cases} \tag{1}$$

on $\Gamma(t)$, $t > 0$. Here V is the outward normal velocity of $\Gamma(t)$, κ_i is the principal curvature of the hypersurface $\Gamma(t)$ with respect to the outer unit normal ($i = 1, 2, \ldots, N-1$), $H := \kappa_1 + \kappa_2 + \cdots + \kappa_{N-1}$, $|A|^2 := \kappa_1^2 + \kappa_2^2 + \cdots + \kappa_{N-1}^2$, and $\Delta_{\Gamma(t)}$ denotes the Laplace–Beltrami operator on $\Gamma(t)$.

In this talk, we introduce a threshold-type algorithm for the motion defined by (1). This type of algorithm was first proposed by Bence, Merriman, and Osher in 1991 to numerically compute mean curvature flows. Our algorithm is an application of theirs, but the main difference is that we use the fourth order heat equation although they use the second order heat equation.

The main purposes are to define our algorithm and to show the consistency result.

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