Numerical Solution of Allen-Cahn Equation for Planar Curvature Flow with Constraints

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Abstract

In the contribution we discuss numerical solution of the planar curvature flow with the area constraint (see [1]) and its applications. The underlying motion law is captured by the phase-field method using the non-local Allen-Cahn equation (as in [3]), and compared to the parametric method (as in [4]) in selected cases. We summarize the theoretical background of the non-local Allen- Cahn equation with respect to the existence, uniqueness and matched asymptotics recovering the sharp-interface law. The numerical solution used for both approaches is based on the method of lines, with space discretization using the finite-difference method and time discretization using a higher-order time solver (see [2]). In our computational studies, we investigate numerical convergence and demonstrated how the selected examples apply to various problems in natural sciences, such as in the aerosol dynamics, recrystallization, droplet shapes and the cell motion.

References

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